DEVELOPMENT AND VALIDATION OF MODULES ON EXPONENTS AND RADICALS IN MATHEMATICS III

A Thesis

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Master of Arts in Teaching Mathematics

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APPROVAL SHEET

This thesis entitled "**DEVELOPMENT AND VALIDATION OF MODULE ON EXPONENTS AND RADICALS IN MATHEMATICS III**," has been prepared and submitted by **DANILO ALANDINO**, who having passed the comprehensive examination, is hereby recommended for oral examination.

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DRA

D E D I C A T I O N

To My loving and everdearest wife

ANGELITA

and daughters

DARYLL and DANISSE

the constant source of inspiration...

this humble work is

wholeheartedly dedicated

DRA

ABSTRACT

The main problem of the study is to develop and validate module in high school Mathematics III based on the difficulties encountered by third year students of La Milagrosa Academy, Calbayog City, during the school year 1995-1996. The computed t-value in module 1 is 3.15 and 3.14 in module 2 were very much greater than the tabular t-value of 2.064 at 0.05 level of significance and 48 degree of freedom, indicating the rejection of number 5 null hypothesis. The rejection of the null hypothesis shows that there is a significant difference between the retention of the control group and the experimental group. This finding shows that the experimental group whose members were taught using the modularized instruction has better retention than the control group which was taught using the lecture-discussion method. In the light of the finding of this study, the following conclusions were drawn. The modular approach of teaching is more effective than the lecture-discussion method in so far as the sub-topics exponents and radicals are concerned. The fact is that students can go through the module even outside the classroom, they can repeat some sections of the work if needed, discover the lessons by themselves and progresses at his own rate until the feeling of self satisfaction and accomplishment are attained. The module are appropriate and interesting to the third year high school students in terms of readability level. Students in the experimental group who were taught using the modularized instruction have better retention than the students in the control group who were taught using the lecture discussion method.

TABLE OF CONTENT

Chapter 1

THE PROBLEM

Introduction

Mathematics is a foundation subject involved in almost all fields of human activity and endeavor, private or public life. This could easily be seen in the kind of transaction people encounter in their work wherein simple mathematical equation and computation are used. In this regard, the subject mathematics becomes indispensable.

Basically, the teaching of mathematics in the elementary grade learners essentially begins with the mastery of the four fundamental operations. Without this skill, pupils will encounter difficulties in the study of higher mathematics. Workably, the inculcation of this mathematical skill among students in both secondary and tertiary levels is basically necessary because many of them consider mathematics as the most difficult subject to hurdle in their studies.

According to Lorenzo (1988:122) mathematics has been called the "Queen of Science". The discoveries and advancement of today's civilization underwent statistical analysis thus, mathematics had contributed to the workable and functional implementation of the programs of both the government and private institutions.

In this particular case, qualification of data derived from the conduct of research work become very useful as to arrive on sound interpretation of its findings. Therefore, the conduct of research study becomes wasteful in the absence of statistical analysis and interpretation of the data and information.

In an added setting, there were several attempts, to simplify the teaching of mathematics most especially in the elementary and secondary school teachers all over the country.

Smith (1987:101) brought out this concern that it can easily be perceived that in the elementary school level, teachers were encouraged by their immediate superior to make use of instructional aids and devices in the teaching of mathematics. The primordial purpose is to make the subject better understood by the pupils.

It is sad to note that in the secondary school level, the use and employment of instructional aids and devices in the teaching of mathematics is the least observed by the teachers.

It was firmly established that one could hardly teach in the secondary and tertiary levels without majoring a subject like mathematics. In this regard, mathematics teachers are very confident that they could handle the same subject effectively relying more on their cognitive expertise and knowledge.

Today, mathematics continue to be one of the basic subjects in the school curriculum in both private and public educational institutions. In this regard, it is even a wiser move if teachers should come up with a methodology or approach to facilitate the easy and understandable teaching of mathematics in the classroom for the benefit and welfare of the students.

One approach to facilitate learning and to improve the learner's abilities is through the use of instructional materials, especially modules. It is an innovation which is very popular in the area of mathematics.

This study will focus on the development of instructional materials in the form of a module based on the difficulties encountered by the students in high school Mathematics III.

Theoretical Framework

This study is anchored on the learning theory of Gregorio (1979:138-139) that the principles and practices of education under the new society are based on the pragmatic philosophy of John Dewey...

> Which states that "education is life, education is growth, education is social process, and education is a reconstruction of human experiences."

He further explained that based on the pragmatic concept of education, the child is made the center of educative process rather than the subject matter, and that the child acts as a unified whole or acts as a unit. The total growth and development of the child is the aim of education in the new society, learning by doing, reacting, or experiencing is emphasized. This concept utilizes the theory of self activity and this is considered the basis of all learning

Conceptual Framework

The conceptual model of the study comprises five components. Namely; 1.) The conceptual model for the preparation and validation of diagnistic test, 2.) The conceptual model for the identification of difficulties in Mathematics III, 3) The conceptual model for the development of module, 4.) The conceptual model for the determining of the readability level of the module, 5) The conceptual model for the validation of module.

PREPARATION AND VALIDATION OF DIAGNOSTIC TEST

INPUT	THOUGHPUT	OUTPUT
1. Subject syllabus in Math III 2. Third year H.S. Students at TTMIST 3. Objectives of the study 4. Textbooks & other references	Preparation and Validation of Diagnostic test	Valid and Reliable Diagnostic Test

Figure 1

Figure 1. Model for the preparation and validation of diagnostic test in Mathematics III

The input in this phase includes the subject syllabus in high school Mathematics III, third year high school students of Tiburcio Tancinco Memorial Institute of Science and Technology, the objectives of the study, the textbooks and other relevant references.

The throughput shows the preparation and validation of diagnostic test.

The output is the valid and reliable diagnostic test in Mathematics III.

CONCEPTUAL MODEL FOR THE IDENTIFICATION OF DIFFICULTIES IN MATHEMATICS III

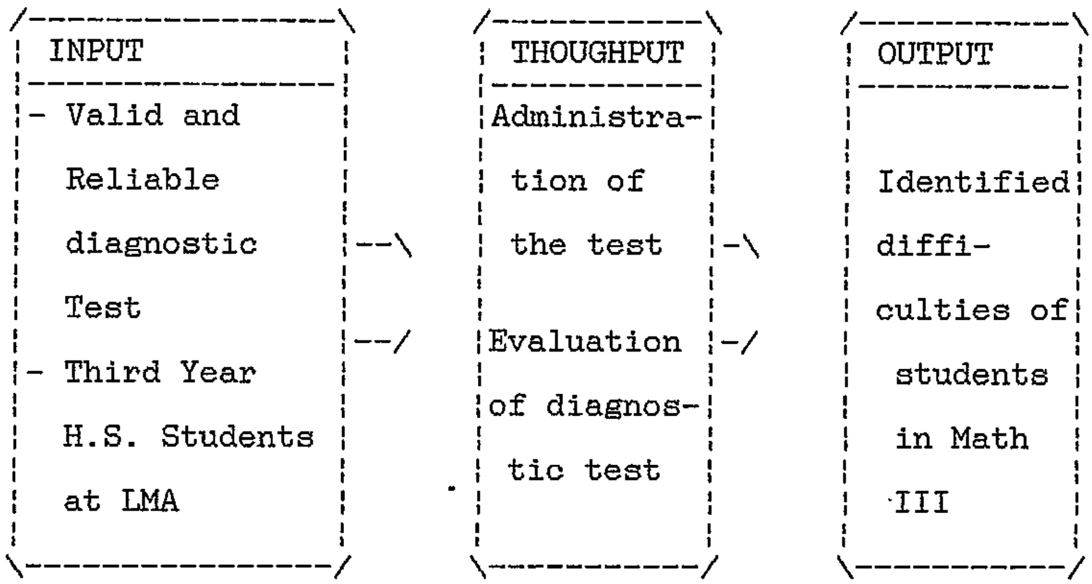

Figure 2

Figure 2. The Conceptual Model for the Identification of Difficulties in Mathematics III

The input of the conceptual model are the validated diagnostic test, and the third year high school students at La Milagrosa Academy, Calbayog City.

The throughput shows the administration of the test to third year students, and the evaluation of the results gathered.

The output indicates the identified difficulties of the students in Math III

DEVELOPMENT OF MODULES

```
/-----------------\     /--------------\     /------------\
| INPUT           |     | THOUGHPUT    |     | OUTPUT     |
|-----------------|     |--------------|     |------------|
|* identified     |     |Development   |     |Developed   |
|  difficulties   |--\  |of modules in |-\   |modules in  |
|* References &   |     |two difficult |     |Math III on |
|  Textbooks in   |--/  |topics in     |-/   | Exponents  |
|  Math III       |     | Math III     |     | & Radicals |
\-----------------/     \--------------/     \------------/
```

Figure 3

Figure 3. The Step for the Development of Modules on Exponents and Radicals

The inputs are the identified difficulties, references, and the textbooks and other references.

The throughput consists of the development of modules in two difficult topics in Math III the Exponents and Radicals.

The output is the developed modules in Mathematics III along with Exponents and Radicals.

CONCEPTUAL MODEL FOR DETERMINING THE READABILITY LEVEL OF THE MODULES

```
/-------------\     /-----------\     /------------\
| INPUT       |     | THOUGHPUT |     | OUTPUT     |
|-------------|     |-----------|     |------------|
|             |     | Flesch    |     |Readability |
|  Module     |     | Formula   |     |  Level     |
\-------------/     \-----------/     \------------/
```

Figure 4

Figure 4. Conceptual Model for Determining the Readability Level of the Modules

The input is the module and the throughput is the Flesch formula. The result is the readability level of the modules.

CONCEPTUAL MODEL FOR THE VALIDATION OF THE MODULES

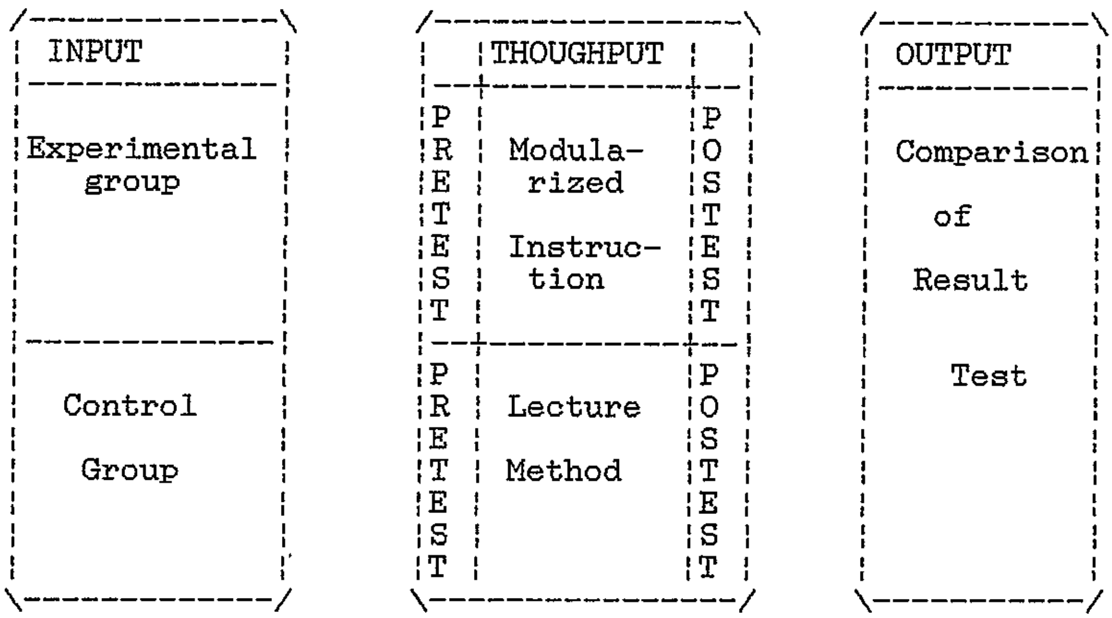

Figure 5

Figure 5. The Conceptual Model for the Validation of Modules

The inputs are the experimental and the control groups. The throughput are the method of instruction to be used in each group. The experimental group was given the module while the control group was taught using the lecture-discussion method. The pretest and posttest were administered to both group of respondents and the result of achievements were compared.

<u>Statement of the Problem</u>

The main problem of the study is to develop and validate module in high school Mathematics III based on the difficulties encountered by third year students of La Milagrosa Academy, Calbayog City, during the school year 1995-96.

Specifically, the study sought to answer the following questions:

1. What are the difficulties of the third year high school students in Mathematics III?

2. On the basis of the pretest and posttest result, how effective are the developed module?

   2.1 Is there a significant difference between the pretest results of the control and experimental groups?

   2.2 Is there a significant difference between the pretest and posttest results of the control group?

   2.3 Is there a significant difference between pretest and posttest results of the experimental group?

   2.4 Is there a significant difference between the posttest results of the control and experimental groups?

3. What is the readability level of the module? Are the develop module appropriate and interesting to the third year high school students?

4. Is there a significant difference between the retention of the control and experimental groups?

Research Hypotheses

The following are the research hypotheses of the study.

1. There is no significant difference between the pretest mean scores of the control and experimental groups.

2. There is no significant difference between the pretest and posttest mean scores of the control group.

3. There is no significant difference between the pretest and posttest mean scores of the experimental group.

4. There is no significant difference between the posttest mean scores of the control and experimental groups.

5. There is no significant difference between the retention of the control and experimental groups.

Significance of the study

An awareness on the use of instructional module can help a lot in the efforts towards the quality performance and achievement in mathematics. In the context of this situation, it cannot be denied that students look upon the teachers in the school as persons who are knowledgeable in the art of teaching and therefore expects them to display excellence in the teaching of the subject in conformity with their instructional needs, demands and convenience.

This study which is focused on the development and validation of modules on exponents and radicals will be beneficial not only to third year high school students but to various groups of individual because of its relevance and applicability in our times today.

To the students, this module will develop them to be independent learners, develop their potential and improve their learning capabilities through self-discovery and self-reliazation at their own rate. This also develop and enhances the student interest in subject matter and remedies the difficulties or deficiencies. The prepared module enhances the understanding of the students because it is within their level of understanding.

To the parent, the angle that should be taken into consideration is the economic status of the parents and the present economic crises. Textbooks and instructional materials are so expensive that parents cannot afford to buy for their students. In this regard, since the developed modules can be mass-produced in school, it is then expected that the cost will be less than the commercial ones. The parents feel a responsibility for helping their students with homeworks.

To the teachers, the modular approach means lesser work in preparing daily lesson plan and other instructional materials and so as to give particular attention to the slow learners. This would also mean a good reference and an additional source of information for the subject.

To the school administration, the module provides a solution to over increasing students population which creates shortage of classrooms and building, textbooks, reference materials and even competent teachers. With the use of modules one teacher can possibly manage 60 students or more in one class and the modules minimizes such problem stated above.

Scope and Delimitation of the Study

This study is concerned mainly with the development and validation of modules in high school Mathematics III, specifically on Exponents and Radical which were found difficult topics by the students based on the result of a diagnostic test.

There were fifty third year high school students of La Milagrosa Academy, Calbayog City for school year 1995-1996 taken as respondents of the study. The respondents consisted of two groups: twenty-five students constituted the experimental group and another twenty-five students constituted thecontrol groups which were selected by purposive randomization technique according to their second year high school mathematics grades.

Definition of Terms

Some terms that constantly appeared in most educational writings in recent time need to be clarified for the purpose of the conduct of the study. Some of the terms have been modified to give its meaning in the manner they are being used in a given situation or condition.

Control Group This term refers to a group composed of twenty five (25) students to be taught by lecture-discussion method.

Diagnostic Test This term refers to a 75 item test based on the syllabus for high school Mathematics III. It measures the skills or knowledge gained by students in the subjects.

Experimental Group This term refers to a group composed of twenty five (25) students to be taught using the module.

Exponents This term refers to any number or symbol placed as a superscript to the right of a quantity to indicate a power. The exponent may be negative, zero, or positive number.

Flesch Formula This term refers to the instrument use in determining the readability level of the developed instructional materials. It consists of human interest score and reading ease score.

Module. This term refers to a self-contained independent unit of instruction with primary focus on well defined objectives,

Modular Approach Refers to the strategy of teaching wherein modules are used in the teaching learning process.

Posttest Refers to the 30 item multiple type of test given to both control and experimental groups after learning each module

Pretest This refers to the 30 item multiple type of test given to both control and experimental groups before the lesson.

Radicals This refers to an indicated roots of any number or algebraic expressions.

Validation This pertains to the assessment of the quality of instructional modules produced for possible use in classroom teaching.

Chapter 2

REVIEW OF RELATED LITERATURE AND STUDIES

This chapter contains the review of related literature, related studies and the relationship with the present study which give emphasis on effectivenees and apropriateness on modular approach used in teaching-learning proccess.

Related Literature

There is no doubt that a workable knowledge in mathematics is a requisite tool for an individual to survive, economically in this world. "Similar fact", in the words of Rouke (1971:129) that ordinarily, the average person manifest a lack of ability in applying his allocated resources. It could be unwise to disregard this knowledge if one has to progress in life.

In the context of his situational condition, there is a dearth to bridge this evident gap between the persons theoretical knowledge in mathematics and his real everyday tasks of spending wisely. The same author holds an added idea that although it is not the first attempt to provide answer to the mathematical problems in daily chores of life, some research work studies are novel in presenting a workable opinions and ideas within the realm of personal practices, needs and aspirations of individuals.

We are even sure to pinpoint that meeting problems is akin to human existence. In this regard Hoffman (1973:95) advanced this idea that time and again, human beings are confronted with problems in their day to day existence. Some of these problems are simple, other are complex. In a clear analysis of all these problems, solutions can be reached if they will only make use of their heads on.

One of the most important skill, knowledge and information that should be acquired by individual in the study of mathematics is the mastery of the four fundamentals and the understanding of mathematical vocabularies. In the realm of this situation, Pedalino (1976:12) asserts that the acquisition and mastery of the four fundamentals is as basic as highly applied in problem solving. We can't expect the pupils and students to be more skillful and knowledgeable in problem solving unless they possess and acquire a workable knowledge in the operation of numbers.

In an added dimension, the procedural ways highly involved in mathematical problems is an avenue to develop pupils' interest and creativity in the study of mathematics. In the opinion of Silva (1978:5) one can even improvise his study habits to response to the solution of mathematical problem which eventually, will lead him to develop critical thinking which is most useful and relevant in real life situation.

Socrates (1976:209) states that modular instruction is one of the more recent outgrowths with the concept of individualized instruction. In fact according to him modules in themselves are part of the multi-media approaches to individualized instruction. This statement clearly signifies that we can use modular instruction to provide for the individual differences of students.

Gregorio (1976:422-423)pointed out that the thrust towards attempting to meet the needs of the individual pupils has been largely an outgrowth of investigation of individual difference among pupils. The nature and scope of individual growth under the varying conditions of present-day life in which no two pupils have the same experiences, would make it necessary in learning. Today there is a growing demand for individualized teaching. He also added that there are general suggestions to be observed in individualized instructions which are the following:

1. The promotion should be according to the work or subject completed. It should be individual rather than general.

2. The pupils should have accurate and well kept records of his accomplishment or achievements.

3. The pupils should be allowed to work at his own

rate of speed and ability.

4. The pupils should be given enough time for completion of the unit.

5. The curriculum should be graded into units of increasing difficulty each measured by achievement test.

6. Every pupil should be furnished with complete instructional materials which can be accomplished individually, and with corresponding administering test.

Just what instructional aids a teacher uses depends on his or her knowledge and experiences, the availability of the materials, the lesson assignment, the subject and the students. Instructional aids are made for situations in general, it is the teacher's job to tailor them to the needs of the students. Ornstein (1991:326-327) suggested some basic guidelines for using instructional aids, and guides for selecting and using instructional materials:

Some basic guidelines for using instructional aids:

1. Purpose. Ask yourself what you are trying to accomplish and why this instructional aid is important.

2. Define objectives. Clearly defined objectives are essential for planning the lesson and selecting and using instructional aids.

3. Flexibility. The same instructional aid can satisfy many different purposes.

4. Diversity. Use a variety of materials, media, and resources to develop and maintain students interests.

5. Development. Instructional aids must be related to the age, maturity, ability, and interest of students.

6. Content. You must know the content of the instructional aids to determine how to use them and how to make the best use of them.

7. Guide learners. Focus students' attention on specific things to attend while viewing, listening, or reading the materials.

8. Evaluate results. Check students reactions and consider your own reactions to the instructional aids.

Some guides for selecting, using, and even developing instructional materials, with emphasis on reading and subject-related tasks.

1. Materials should be relevant to the instruction that is going on in the rest of the unit or lesson.

2. A portion of the materials should provide for a systematic and cumulative review of what has already been taught.

3. Materials should reflect the most important aspect of what is being taught in the course or subject.

4. Materials should contain, in a form that is readily accessible to students and teachers, extra task for

students who need extra practice.

5. The vocabulary and concept level of materials should relate to that of the rest of the subject.

6. The language used in the materials must be consistent with the use in the rest of the lesson and in the rest of the textbook.

7. Instructions to students should be clear, un-ambiguous, and easy to follow, brevity is a virtue.

8. The layout of pages should combine attractiveness with utility.

9. Materials should contain enough content so that there is a chance for students to learn something and not simply be exposed to somthing.

10. Tasks that require students to make discriminations must be preceded by a sufficients of the discriminations.

11. The content of materials must be accurate and precise, tasks must not present wrong information or be presented in language that contain grammatical errors and incorrectly used words.

12. At least some tasks should be fun and have an obvious payoff to them.

13. Student response modes should be consistent from task and should be the closest possible to reading and

writing.

14. The instructional design of individual tasks and of task sequences should be carefully planned.

15. There should be a limit on the number of different materials so as not to overload or confuse students.

16. Artwork in the materials must be consistent with the text of the materials.

17. Cute, nonfunctional, space and time-consuming material should be avoided.

18. When appropriate, materials should be accomplished by brief explanations of purpose for both teachers and students.

Lardizabal (1977:291) states that the effectiveness of the teaching-learning process can be increased greatly through the proper use of instructional aids. Among these aids are printed materials, audio, audio-visual aids, demo-stration, community resources, and autoinstructional materials. These aids are commonly referred to as audio-visual aids because they are sensory objects and images utilized to promote meaningful communication.

Instructional aids cannot teach by themselves. They need a skillful teacher to make them effective. To get the most from the use of any of these aids, the teacher must take into account four basic consideration: 1) selecting the materials, 2) preparing the class for the audio-visual experience, 3) guiding the class through it, and 4) following up the experience after its completion.

Related Studies

Modern teaching believed to be more effective, gives much importance to students activities in the classroom. A large number of students in the past deal with the development of instructional materials. The use of instructional materials become imperative.

Perez (1985:51-55) focus her study in the development and validation of the instructional materials in the form of a module based on the identified difficulties in progression. The study comprised five phases, namely; 1) development and validation of diagnostic test. 2) Identification of difficulties. 3) Development of instructional materials through difficulties. 4) Validation of modules. 5) Determining of the readability level of the module.

The main problem of her study is how can a module be best developed and validated to meet the identified difficulties in progression.

In her findings, she found out that out of the 12 subtopics included in the diagnostic test, 11 were found difficult based on the index of the difficulties were as follows; Infinite geometric progression, repeating decimal. geometric series, summation symbols, geometric mean, harmonic mean, harmonic progression.

The Reading Ease Score obtained from the module on progression is 58.216. It follows that the readability level of the materials are fairly difficult. It is appropriate for the first year to second year college students based on the Flesh Reading Ease Scale.

In the light of the findings of her study, the following conclusions were drawn:

1. College students of Samar State Polytechnic College have varying degrees of difficulty in progression.

2. There is a significant difference between the pretest and posttest mean score of the experimental and control group in the same learning content.

3. There is a significant difference between the posttest mean score of the experimental and control groups in favor of the modular instructional method.

4. The instructional materials is appropriate for the second year college students in terms of readability level.

On the bases of the conclusion made, she recommended the following:

1. Students with identified difficulties should be

given learning materials like modules to give them time to catch up with the lesson not well learned from the classroom.

2. Workshop on module preparation and construction should be conducted to provide basic knowledge to teachers with the end in view of producng modules in oter related subjects which should be finaced by the administration.

3. Students should be exposed to modular instruction to develop in them the feeling of independence and self-confidence in learning a lesson without the teacher's aid.

4. Teachers should be motivated and supported to undertake further researches on the effectiveness of modular instruction to improve teaching-learning process.

Lacambra (1985:48-54) in her study the "Development and Validation of modules in Electrochemistry" aimed to answer the question on the appropriateness and effectiveness of the modules to be used in the classroom.

She found out that the last two in the rank of the different sub-topic included in electrochemistry are oxidation-reduction and chemical effect of an eletric current This was based on the average correct responses of the students in the different sub-topics. The difficulty of the students in understanding oxidation-reduction and chemical effect of an electric current may be due to the fact that these sub-topics deal with mathematical analysis and computation as well as practical applications which may have not been taught or emphasized fully in their high school chemistry and physics.

Based on the findings, the following conclusions were arrived at:

1. The college students of SSPC encountered difficulty in the sub-topic in electrochemistry specially oxidation reduction and chemical effect of an electric current.

2. a. The control group and experimental group had the same level of entry behavior.

   b. Students gained knowledge about oxidation-reduction and chemical effect of an electric current through the lecture method.

   c. Students learned oxidation-reduction and chemical effects on an electric current with the use of the modular approach.

   d. The modular approach of teaching is more effective than the lecture method in so far as the sub-topic oxidation-reduction and chemical effects of an electric current are concerned. The fact is that students can go through the module at his own pace, repeat some sections of the work if needed, and progresses at his own rate until the feeling of self-satisfaction is attained.

3. The modules are appropriate and interesting to the first year DIT students.

In the light of the conclusion of her study, the following recommendations were made:

1. Teachers/instructors should be motivated to prepare other instructional materials aside from the modules to alleviate the problem on the inadequacy of instructional materials in most schools.

2. Teachers/instructors should be encouraged to prepare modules on other areas of chemistry.

3. The developed modules on electrochemistry, particularly oxidation-reduction and chemical effect of an electric current, should be used and evaluated in other schools to further confirm its effectiveness.

4. School principal/superintendents should give full support to their teachers/instructors in the development of any instructional materials for the school.

The study conducted by Uy (1991:60-64) focus to the "Construction and Validation of Modules on Circular Trigonometric Functions and Fundamental Identities" aimed to develop modules appropriate and interesting to the first year BSIE students at Samar State Polytechnic College. The data gathering procedure also consists of five major parts, namely: 1) Construction and validation of diagnostic test, 2) Identification of difficulties, 3) Development of modules, 4) Validation of modules, 5) Validation/Evaluation of the Readability of the modules.

She constructed a table of specification using course syllabus in Plane Trigonometry. Based from this syllabus a 60 item multiple type of teacher-made test was constructed and validated using the student who have already taken the course. The result were used as bases for determining of the topic to be modularized.

In her findings, she found out that the students of SSPC encountered difficulties in Circular Trigonometric Functions and Fundamental Identities due to insufficient background of the students in Trigonometry. She concluded that students gained knowledge on Circular Trigonometric Functions and Fundamental Identities with the use of the lecture-discussion and modular approach but modular approach is more effective.

In the light of her findings, she recommended that the developed modules on Circular Trigonometric Function and Fundamental Identities should be used in SSPC and other schools to further confirm its effectivity. She added that the use of module will serve as an effective remedial resource materials for them even if they are away from classes.

Oliva (1980) in her study, "Modular Approach in Solving System of Linear Equations by Relaxation Method," aimed to evaluate the effectiveness of modular instruction by relaxation method and to determine the degree of acceptance of modular instruction by the students.

She found out that there was a significant difference between the mean of the pretest and the mean of the posttest for each of the experimental and control groups.

She recommended the use of modular instruction to students with above average intelligence as often as possible in order to maximize the learning process and output. For average and poor students, modules can be used provided that they will go hand in hand with traditional instruction. She also suggested to transform every possible subject matter into modular form.

Lucero (1985) in her study entitled "A Modular Approach in Mastery of the Concept of Operation with Integers" designed to find out if the mastery of the concept of operation on integers could be achieved through the modular approach. The study was conducted and administered to selected groups of freshmen of the Tiburcio Tancinco Memorial Institute of Science and Technology (TTMIST), in

Calbayog City.

The objective of her study is to determine the appropriateness and effectiveness of the modular approach in achieving mastery of the concept of operations of integers as compared to that of the lecture-discussion approach.

The finding are as follows:

1. The seventy four freshmen of TTMIST involved in the study, thirty six belong to the average mental ability level.

2. The students belong to the experimental and control group were the same in class standing in terms of the entering behavior necessary before taking up the lesson on the concepts of operations with integers.

3. There is a significant difference in the performance of the experimental and control groups. It appears that the module is as effective as the traditional lecture-discussion method in improving the student's achievements.

4. The average mental ability group the traditional method of teaching is more effective than the modular approach is improving their performance. The below average mental ability group the module is just as effective as the traditional method of teaching in improving student's performance.

She concluded that the students belong to experimental and the control group did not differ in their entering behavior in relation to the learning content of the module but the module is effective in improving the students achievement of bòth expêrimental and control group. The lecture discussion method is more effective than the modular approach for the average mental ability group. The module is as effective as the traditional lecture discussion method of teaching in improving the students achievement of the below average mental ability group.

Based on the conclusion made, she had the following recommendation to make:

Based on the findings, the following conclusions she has the following recommendation.

1. Module should be used as teaching aids to supplement and/or complement the traditional lecture-discussion method of teaching mathematics, especially the most difficult concepts.

2. School system should undertake training of teacher in module construction, especially in mathematics.

3. Module should be constructed and validated in all the topics in mathematics.

4. Modularized instruction should be started from the lowest level of formal education in order to familiarize the

learners with the module even as a supplementary material only.

Bohol (1982) in her study on the "Effectiveness of a proposed Instructional Module in the Study of Motion in high school Physics," found out that developed instructional module is effective in the teaching of some concept in motion related to typhoon in the high school physics, because students achieved better than the students who were taught the conventional way.

Reyes (1984) as cited by Perez, in her study on the deficiencies in Mathematics I of first year high school of four Agricultural schools in the division of Cagayan.

She found that most deficient in computing fractions, finding the ratio, proportion, and percent, and solving areas, circumference, and time. The skills which obtained the three highest ranks were selected as the most serious deficiencies. Modules were developed based on the most deficient skills.

She concluded that the use of the module produced significant differences in the scores of the students.

She recommended that teachers should give greater emphasis on the areas in mathematics I were the students are weak. More guide activities should be provided to improve student's computational skills. She also recommended that teachers should adjust instruction to the needs and characteristics of individual learners. Development and the use of instructional materials like modules are strongly recommended to cope with these deficiencies.

Gordove ((1993) in his study on "Effectiveness of Self-learning Kits in Grade V Mathematics," found out that the experimental group performed better than control group. Moreover the inidividual approach in teaching Goemetry through self-learning kits to the grade V pupils in better than the use of the traditional lecture method.

He concluded that teaching with the use of self-learning kit is more effective than the lecture-discussion method.

He recommended the use of self-learning kits since it develops proper acquisition of mathematical abilities and skills of the pupils.

Dacula (1995) in her study on "Development and Validation of module on Precent and Ratio for Mathematics I" found out that the experimental group shows a significant amount of learning after the respondents were exposed to modularized instruction. She also found out that the developed module was fairly easy and appropriate for the first year high school students and the module is interesting based on the results of the test.

She concluded that the modular approach of teaching is more effective than the traditional lecture-discussion method as far as the topic Percent and Ratio is concerned, because the students can go through the modules and learn its contents of their works if needed, discover process and techniques in learning the lesson until the feeling of self-satisfaction is attained.

She recommemded that modular instruction help the students to learn to be independent, responsible, self-reliant and hardworking.

Relationship with the Present Study

The aforementioned review of related studies showed the effectiveness of module as they relate to students achievement in mathematics. The studies made by Perez, Lacambra, Uy are related to the present study because they are all concerned with the construction of instructional materials that can be possibly used in the classroom activities. The studies mentioned above and the present study make use of instructional material development as well as the pretest and posttest results. Moreover, the development of module is made based on the identified difficulties or weaknesses of the students as revealed in the diagnostic test.

The work of Bohol, Lucero, Oliva, have some similarities to the present study because it deals with the effectiveness and appropriateness of the use of instructional materials.

The present study differs from the cited studies in terms of the score of the problem, subject treated, instrument and the setting of the study.

Chapter 3

METHODOLOGY

This chapter describe the methods and research design, instrumentation, sampling procedure, data gathering procedure and statistical treatment of data.

The Research Design

The experimental method of research was utilized in this study using the randomized pretest and posttest design. The study shall consist of two groups namely: the control and experimental group. The control group receive no treatment except that the respondents was taught using the lecture-discussion method, while the experimental group receive treatment in the form of module as the meduim instrument used.

In this design the pretest was given to both control and experimental group before the experimental begins. This provide the researcher with a means of checking whether or not the two groups have at least similar characteristics. After the experimention the posttest was given to determine its significant difference.

The research design is shown in the table form on the next page.

Group	Pretest	Treatment	Posttest
Experimental	E_1	Module	E_2
Control	C_1	Lecture	C_2

Experimental $D = E_1 - E_1$ (difference between pretest and posttest mean scores)

Control $C = C_2 - C_1$ (difference between pretest and posttest mean score)

After the respondents have been exposed to the treatment, the two groups were subjected to posttest. This assess if learning took place. The difference between the pretest and posttest mean score of the experimental group was compared to the control group's mean scores. The appropriate statistical procedure to ascertain whether the difference in the mean scores of the two groups of respondents are sufficiently great to be a real difference, or whether it is only a chance occurence was applied.

Instrumentation

The data gathering instruments used in this research work are the diagnostic test, progress report card, pretest and posttest.

Diagnostic Test. A 75 item test based on the Mathematics III syllabus was constructed and validated to third year high school students enrolled at Tiburcio Tancinco Memorial Institute of Science and Technology, Calbayog City School year 1994-95. After the test has been validated, the modified test was administered to third year high school students of La Milagrosa Academy. The test was designed to determine the difficulties encountered by the students in Mathematics III.

Progress Report Card or DECS Form 138. The grades obtained by the students in Mathematics II were secured from the form 138 which is the student's report card. Form 138 was taken from the section advisers of the students. The grades of the students in Mathematics II were used for the randomization of choosing the respondents of the control and experimental group.

Pretest. A 30 item multiple choice teacher made type of test which was given to both control and experimental groups before the experimentation. It determine the present knowledge of the students in exponents and radicals.

Posttest. It made use of the pretest. This was given to both control and experimental group after the conduct of the experiment. It determine the amount of knowledge acquired by the respondents from the study.

Retest. This is a test given to both the control and the experimental groups two weeks after the administration of the module and after the posttest. It determines the retention of knowledge acquired by the respondents of the study. This test is the same test used as in the posttest, but the order of the items was rearrange.

Sampling Procedures.

The respondents of the study consisted of fifty third year high school students at La Milagrosa Academy. Twenty five (25) of them constitute the control group and the remaining twenty five (25) constitute the experimental group. The second year mathematics grades were the basis of distributing the members of the control and experimental group using the purposive randomization technique. To avoid bias in the experimental result, the mathematics grades of 80 percent and above were matched or equated correspondingly to assure equivalency. To illustrate further, the Mathematics II grade of student in the control group is 85 were matched correspondingly with a students in expeimental group whose grade is 85 also. The same procedure were done to other members of both groups. Since the mathematics grades are matched correspondingly, the mean grades of these two groups are equal.

Data Gathering Procedure

The data gathering procedure is divided into five phases, namely:

Preparation and Validation of Diagnostic Test. The test is designed to determine the specific weaknesses or difficulties of the students in Mathematics III. The content of the test are classified into knowledge, comprehension and application under each topic content. A multiple choice of 75 items was constructed based on the specific objectives of the course. The original pool of 75 items were tried out to 50 students of Tiburcio Tancinco Memorial Institute of Science and Technology, Calbayog City.

Revision of the test was done based on the result of the item analysis. Items with very low discriminating values are either improve, revise, or rejected using the steps suggested by Ebel. Difficulty and discriminating power of the diagnostic test was determined using the Kuder-Richardson Formula.

Identification of Difficulties. The required data in the phase were the result of the validated diagnostic test which was given to third year high school students of La Milagrosa Academy. The test papers were corrected by the researcher himself. The number of students who got wrong responses per sub-topic was determined and tabulated. Then the sub-topics were ranked from lowest to highest. The two highest ranked sub-topics were considered to be the most difficult for the respondents and were modularized.

Validation of Pretest. The original pool of fifty two items were shown to the research adviser and colleagues for their coOmments and suggestions for improvement. Incorporating their suggestions, the instruments was tried out to all second year high school students of La Milagrosa Academy. After the try out the items were subjected to item analysis to determine the index of discrimination and index of difficulty and therefore were revised based on the analysis. The original pool of fifty two items was reduced to thirty items test.

Development of Modules. The format followed in the development of module contained the following features: a) Overview, b) direction for use, c) objectives d) presentation, e) reference for further reading, f) evaluation, g) key to correction.

a. Overview. It consists of a general statement of the subject matter or content of the module, its connection with the previous lesson, and its importance in the subject.

b. Direction for Use. This includes information or direction on activities to be undertaken and feed-back instrument to be accomplished.

c. Objectives. These are the specific object-ives for each lesson in every module.

d. Presentation/Input. It consists of the procedure in case of skill lessons or discussion in case of theoretical lesson, together with necessary illustration, chart, diagram, etc. including activi-ties, exercises, or assignments designed to provide enrichment and/or opportunities to apply the new knowledge.

e. References for further reading

f. Evaluation or Feedback Instruments. This make use of the practice task, pretest, and posttest.

g. Key to Correction. This refers to the correct answer to practice task.

Validation and Evaluation of the Readability of the Module. The Flesch Formula was used to determine the reading ease score (RES) and human interest score (HIS) of the module.

The developed module was the main source of data. The 108 page-module consisted of 11 lessons, namely: Concept of Exponents, Laws of Exponents, Forms of Exponents, Simlifying and evaluating expressions with exponents, Equations involving exponents, Scientific Notation, Roots pf number, Changing radicals to simplest form, Multiplication and division of radicals, Addition and Subtraction of radicals, Equation involving radicals. Twenty percent of the total number of pages was randomly taken and used to determine the readability level of the module.

To determine the readability of the module, 9 pages were be randomly selected from 47 content pages and subjected to the steps in measuring the reading ease score of the module.

In measuring the reading ease score, the following steps were followed.

a. Choosing the sample pages. The samples selected from 47 content pages of the instructional material in exponents and radicals excluding the practice task, pretest, and posttest. Nine (9) pages were chosen at random from the module which represented 20 percent of the total number of pages, of which six (6) pages were taken from Module 1 (exponents) and three (3) pages from Module 2 (Radicals).

b. Counting the number of words. One hundred words were taken from each sample page by counting the first word of the first paragraph to the 100th word. In samples where there were no new paragraph, the first word of the sentence is considered.

c. <u>Counting the number of sentences</u>. The Total number of sentences in one hundred words in each sample was counted. If the 100th word fall after more than one-half of the words of the sentence, it was counted as one. Otherwise, it was not counted.

4. <u>Counting the number of syllables</u>. The syllables in the one hundred words in each sample were counted. The syllables were counted the way the word was pronounced.

The average sentence length and average word length was computed. The results from the computation was used in solving the reading ease score of the module.

In measuring the human interest score, the following steps were followed.

a. <u>Counting the personal words</u>. Using the sample, the total number of personal words were counted. The personal words include singular and plural forms of all first, second, and third person pronouns except the neuter they, them, their, themselves, if referring to things rather than people.

b. <u>Counting personal sentences</u>. The number of personal sentences in each of the 100 word samples includes spoken sentences marked by quotation marks, questions, command, request, and other sentences directly addressed to the reader.

From these data, the percentage personal words and percentage personal sentences were computed. The results were used in determining the Human Interest Score of the module.

Validation of Module. The researcher personally handled both groups. The lesson was scheduled daily with a time allotment of 40 minutes every group. In module 1, classes of the control group were done in the morning while the xeperimental group in the afternoon, while in module 2, the schedule of the control group was done in the afternoon and the experimental group in the morning.

A pretest was given to both groups before they were exposed to the method of teaching to determine the extent of knowledge they have on the topics being modularized.

A posttest was given to both group after they were exposed to the methods of teaching to evaluate the achievement of the group on the learning content taught.

Variables that could affect the try-out such as conduciveness of the classroom, ventilation, time of the day, materials, and teacher were controlled so as not to be biased with any one method.

A retest was given to both group two weeks after taking the posttest. It is designed to evaluate the retention of the students who were exposed to different teaching methods.

Treatment of the Data

In this particular study, the following statistical tools were used:

Kuder-Richardson Formula. The reliability coefficient of the diagnostic test were computed using this formula.

$$r = \left[\frac{n}{n - 1} \right] \left[\frac{s^2 - n\,pg}{s^2} \right]$$

Where: r = reliability coefficient of the test

n = number of items in the test

p = $\frac{\bar{x}}{N}$ the proportion of the group passing an items and where x is the mean of the test score.

q = $1-p$ the proportion of the group failing an item

s = standard proportion of the test scores.

T-test for uncorrelated means. This is used to test the hypothesis 1, 4, and 5 of the study.

$$t = \frac{\bar{x}_1 \qquad \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

where:

$\bar{x}_1$ = mean of posttest of experimental group

$\bar{x}_2$ = mean of posttest of control group

s_1 = standard deviation of experimental group

s_2 = standard deviation of control group

n_1 = number of samples in the experimental grou

n_2 = number of samples in the control group

Test for Correlated Mean. The t-test for correlated mean was used to test the hypothesis 2 and 3 of the study.

$$t = \frac{\bar{x}_1 \qquad \bar{x}_2}{\sqrt{\frac{S_1 + S_2 - 2r^{12}}{n} \quad \frac{S^1}{}}}$$

Where: $\bar{x}_1$ = mean of posttest of experimental group

x_2 = mean of posttest of control group

s_1 = standard deviation of experimental group

s_2 = standard deviation of control group

Rank Difference Method for Determining Coefficient of Correlation. This is used to determine if their exist a relation between the first and the second test results of the experimental and control group using the formula.

$$r = 1 - \frac{6(D^2)}{N(N^2 - 1)}$$

Where:

D^2 = sum of the square of the rank difference

N = number of cases

Flesch Formula. This is used to determine the readability level of the two modules.

a. Reading Ease Score (RES) = 206.835 - (1.015 x average sentence length + 0.846 x average word length

Where:

$$\text{Ave. sen length} = \frac{\text{No. of words in all samples}}{\text{Total no. of sentences}}$$

$$\text{Ave. Word length} = \frac{\text{No. of syllabus in all sample}}{\text{Total no. of sample pages}}$$

b. Human Interest Score = (% Personal words per 100 words x 3.635) + (% Personal sentences x .314)

where:

$$\text{\% personal words} = \frac{\text{Total no. of personal words in all samples}}{\text{Total no. of words in all samples pages.}}$$

$$\%\ \text{personal sen.} = \frac{\text{Total no. of personal sentences}}{\text{Total sentences in all samples}}$$

Chapter 4

PRESENTATION, ANALYSIS, AND INTERPRETATION OF DATA

This chapter presents the description and analysis of data in order to gain knowledge into the following phase of this study: Identification of Difficulties of students in Mathematics III, evaluation of the Readability Level of the Module, and the Validity of the modules conducted by the researcher.

Identification of Difficulties

The results of the evaluation of the diagnostic test with their analysis and interpretation are presented in this discussion.

Table 1 present the precentage of wrong responses per sub-topic of the students in the diagnostic test in Mathematics III.

From the table, it can be seen that the first and second sub-topics with high percentage of wrong responses were Radicals and Exponents respectively. It can be said that the students were difficient in this two sub-topics. This can be attributed to the fact that the topics on radical was new to the third year high school students. Some of them found it hard to understand and comprehend

Table I

Ranking of Sub-topics on the Diagnostic Test Results

List of Sub-topics	% of Wrong Responses	Rank
Exponents	59%	2
Radicals	62%	1
Factoring	48%	5
Rational Expression	57%	3
Quadratic Equation	51%	4

terms and symbols as well as the computational aspect of the lesson.

Evaluation of the Readability Level of the Modules

Table 2, gives the result of the Reading Ease Score (RES) and the Human Interest Score (HIS) obtained from the different pages chosen as samples.

As reflected in Table 2, the developed modules for sub-topics 1 and 2 are found to have a Reading Ease Score (RES) of 25.61 and 12.69 respectively. Based on these RES values, the modules are fairly difficult for the students but suited or appropriate for third year high school students.

Table 2

RES and HIS Results of Modules 1 and 2

Modules	RES	Interpretation	MIS	Interpertation
1	25.611	Fairly difficult	11.10	Interesting
2	12.69	Fairly difficult	5.04	Interesting

Likewise the computed HIS are 11.10 and 5.04 for modules 1 an 2 respectively. This is interpreted to be interesting. Thus, it can be said that the prepared instructional materials can really enhance the desire of the students to go through the module.

Based on this findings, it can be said that the developed modules were appropriate and interesting to the third year high school students. It can be mentioned also that these findings indicate the desire of the students to go through the module.

<u>Validation of the Module</u>

This part presents the data and result of the validation of the modules using the control and experimental groups in two modules; Exponents and Radicals. It also includes the analysis and interpretation of the findings;

Hypothesis 1: There is no significant difference between the pretest mean score of the control and experimental groups.

It can be seen from Table 3 that in module 1 which is about Exponents, the mean score of the experimental group and control groups are 6.12 and 6.00 respectively, with a difference of 0.12, while in module 2 which is about Radicals, the mean score of the experimental group and the

Table 3

Result of the t-test Between the Mean Scores of the Pretest of the Experimental Group and Control Group in Module 1 and 2

Module	Experimental Group X_1	Experimental Group S_1	N_1	Control Group X_2	Control Group S_2	N_2	Computed t	t value	Interpretation
1	6.12	2.55	25	6.00	2.27	25	.17	2.064	A
2	4.08	2.80	25	3.84	2.14	25	.34	2.064	A

Significant at 0.05 level of significant and 48 degree of freedom

Legend: A - accepted R - Rejected

control groups are 4.08 and 3.84 respectively, with a difference of 0.24. Quantitatively, it can be said that in the pretest, they have more or less equal mean scores.

Applying the t-test to check the findings statistically, it yield a computed t-value of 0.17 and .34 for module 1 and 2 respectively. This value is lesser than the tabular t-value of 2.064 at .05 level of significance with 24 degrees of freedom. Thus, the null hypothesis is accepted.

The acceptance of the null hypothesis implies that there is no significant difference between the pretest of the control and experimental groups. This also interprets that the experimental and control groups have the same level of entry comptencies for modules 1 and 2. It can be said that proper grouping was done.

Table 4 presents the data and results of the t-test between the mean scores of the pretest and posttest of the control group.

Hypothesis 2: There is no significant difference between the pretest and posttest mean score of the control groups.

As presented in Table 4, the result of the posttest for modules 1 and 2 is greater than the pretest. These are interpreted to have high marked relationship. It can be said that students with high score in the pretest took high score in the posttest. This means that learning took place when the students of the control group were subjected to the lecture-discussion method.

Table 4

Result of the t-test between the Mean Scores of the Pretest and Posttest of the Control Group

Module	Pretest X_1	S_1	N_1	Posttest X_2	S_2	N_2	Computed t	t value	Interpretation
1	6.00	2.27	25	12.48	10.08	25	6.69	2.021	R
2	3.84	2.14	25	2.72	3.05	25	2.49	2.021	R

Significant at 0.05 level of significant and 48 degree of freedom

Legend: A - accepted   R - Rejected

The t-test for correlated means was used to verify the significance of the difference between the pretest and posttest means of the control group.

The computed t-value of 9.67 in module 1 and 2.49 in module 2 were very much greater than the tabular t-value of 2.021 indicating the rejection of the null hypothesis. So this can be interpreted that there is a significant difference between the mean score of the pretest and posttest results in the control group with the posttest being significantly higher than the pretest mean score.

Moreover, the significant increase in the posttest is attributed to the lecture, discussion, modelling, seatworks, exercises, assignments and quizzes.

Therefore, the performance of the control group in the posttest was better than in the pretest. This shows that there was a change in the learning behavior of the students.

Table 5 presents the data and results of the pretest and posttest of the experimental group.

Hypothesis 3: There is no significant difference between the pretest and posttest mean score of the experimental groups.

It can be said from table 5 that the mean score of the posttest is very much greater than the mean score of the posttest in module 1 which were 6.12 and 14.28. respective-

Table 5

Result of the t-test Between the Mean Scores of the Pretest and Posttest of the Experimental Group

Module	Pretest X_1	Pretest S_1	Pretest N_1	Posttest X_2	Posttest S_2	Posttest N_2	Computed t	t value	Interpretation
1	6.12	2.55	25	14.28	2.60	25	18.13	2.021	R
2	4.08	2.80	25	12.88	2.55	25	19.13	2.021	R

Significant at 0.05 level of significant and 24 degree of freedom

Legend: A - accepted R - Rejected

ly, and in module 2 were 4.08 and 12.88 respectively. The difference in the mean scores were tested for siginificance by using the t-test for correlated mean.

The computed t-value of 18.13 in module 1 ans 19.13 in module 2 is very much greater than the tabular t-value of 2.021 at .05 level of significance and 24 degree of freedom, thus the null hypothesis is rejected. The rejection of the null hypothesis implies that there is a significant difference between the mean score of the pretest and posttest. This shows that there is an increase in the mean scores of the posttest in the experimental group.

The increase or improvement in the score of the posttest in the experimental group can be interpreted that learning took place after the respondents were exposed to modularized instruction. The experimental group found modularized instruction more effective to attain the objectives of the lesson.

Table 6 presents the data and results of the t-test between the posttets mean score of the experimental and control group in module 1 and 2.

Hypothesis 4: There is no significant difference between the posttest mean score of the control and experimental groups.

Table 6 shows that the posttest mean score of the experimental group is greater than the posttest mean scores in the control group. The test revealed that the computed t-

Table 6

Result of the t-test Between the Posttes Mean Scores of the Experimental and Control Group

Module	Experimental Group X_1	S_1	N_1	Control Group X_2	S_2	N_2	Computed t	t value	Interpretation
1	14.28	2.60	25	12.48	2.72	25	2.4	2.064	R
2	12.88	2.55	25	10.08	3.05	25	3.54	2.064	R

Significant at 0.05 level of significant and 48 degree of freedom

Legend: A - accepted R - Rejected

value of 2.4 and 3.54 in module 1 and 2 respectively was greater than the tabular t-value of 2.064 at .05 level of significance and 48 degree of freedom. This led to the rejection of the null hypothesis. Consequently, there is a significant difference between the mean score of the posttest of the two groups. Quantitatively, it can be said that the experimental group which was exposed to the modular approach of teaching performed better than the control group which was exposed to the lecture-discussion method. This findings is also an indication of the validity of the contructed instructional materials.

Therefore, the significant difference between the results in the posttest of the control and experimental groups subtantiate the fact that modular instruction in teaching mathematics specifically exponents and radicals is more effective than the lecture method.

The findings of this study supported the findings of study conducted by Uy, Lacambra, and Perez, that modularized instruction is more effective than lecture method.

Table 7, presents the data and results of the t-test between the retest mean score of the experimental and control groups in module 1 and 2.

Hypothesis 5: There is no significant difference between the retention of the control and experimental groups.

It can be seen from table 7 that in module 1 the result of the re-test mean score of the experimental group was 13.80 which is very much greater that the result of the re-test mean score in the control group of 10.75. While in module 2 the result of the re-test mean score of the experimental group which was 11.4 is very much greater than the result of the re-test mean score in the control group of 8.64.

Table 7

Result of the t-test Between the Retest Mean Scores of the Experimental and Control Group

Module	Experimental Group X_1	S_1	N_1	Control Group X_2	S_2	N_2	Computed t	t value	Interpretation
1	13.80	3.67	25	10.75	2.58	25	3.51	2.064	R
2	11.40	2.49	25	8.64	3.64	25	3.14	2.064	R

Significant at 0.05 level of significant and 48 degree of freedom

Legend: A - accepted R - Rejected

To test the hypothesis, the t-test for uncorrelated mean was used to verify the significance of the difference between the re-test mean score of the control and experimental groups. The computed t-value in module 1 was 3.15 and 3.14 in module 2 which is very much greater than the tabular t-value of 2.064 at 0.05 level of significance and 48 degrees of freedom indicating the rejection of the null hypothesis.

The rejection of the null hypothesis shows that there is a significant difference between the retention of the control group and experimental group.

The interpretation is that the experimental group which was taught using the modularized instruction have better retention than the control group which was taught using the lecture-discussion method.

Chapter 5

SUMMARY, CONCLUSION AND RECOMMENDATION

This chapter presents the summary, conclusion, and recommendations based on the study.

Summary of Findings

This study is focused on the development and validation of the instructional materials in the form of module on exponent and radicals based on the identified difficulties in high school Mathematics III. The findings of the study are herein presented vis-a vis the specific questions and null hypotheses already stated.

1. Based on the result of diagnostic test, Exponent and Radicals were two sub-topics found to be difficult by the students in Mathematics III.

2. Based on the result of the Reading Ease Score (RES) and Human Interest Score (HIS) of the module and on the interpretation of readability level of the module, it shows that the readability level of the materials was fairly difficult but appropriate and interesting for the third year high school students.

3. The respondents belonging to the experimental and control groups have more or less the same entering behavior before taking up the lessons on exponents and radicals.

4. The computed t-value of 6.69. in module 1 and 2.49 in module 2 were very much greater than the tabular t-value of 2.021 indicating the rejection of number 2 null hypothesis. This can be interpreted that there is a significant difference between the mean score of the pretest and posttest results in the control group. Students with high score in the pretest got high score in the posttest. This means that learning took place when the students of the control group were subjected to the lecture-discussion method.

5. The computed t-value of 18.13 in module 1 and 19.13 in module 2 is a very much greater than the tabular t-value of 2.021 at 0.05 level of significance and 24 degrees of freedom. The null hypothesis number 3 is rejected. The rejection of null hypothesis implies that there is a significant difference between the mean score of the pretest and posttest. It shows that there is an increase in the mean scores of the posttest in the experimental group. So, learning took place after the respondents were exposed to modularized instruction.

6. The computed t-value of 2.4 and 3.54 in module 1 and 2 respectively was greater than the tabular t-value of 2.064 at 0.05 level of significance and 48 degrees of freedom. This led to the rejection of the number 4 null

hypothesis. It can be said that the experimental group which was exposed to the modular approach of teaching performed better than the control group which was exposed to the lecture-discussion method. This finding indicates the validity of the constructed instructional materials.

7. The computed t-value in module 1 is 3.15 and 3.14 in module 2 were very much greater than the tabular t-value of 2.064 at 0.05 level of significance and 48 degree of freedom, indicating the rejection of number 5 null hypothesis.

The rejection of the null hypothesis shows that there is a significant difference between the retention of the control group and the experimental group. This finding shows that the experimental group whose members were taught using the modularized instruction has better retention than the control group which was taught using the lecture-discussion method.

Conclusion

In the light of the finding of this study, the following conclusions were drawn.

1. The third year high school students of La Milagrosa Academy encountered difficulties in two sub-topics; the exponents and radicals

2. The experimental and control groups have more or less the same level of entry behavior or mathematical knowledge and experience in mathematics.

3. Students in control group gained knowledge on exponents and radicals with the use of lecture discussion method as reflected in their pretest and posttest results. This improvement in the performance of the students are attributed to the lecture, discussion, modelling, seatwork, exercises, assignments and quizzes.

4. There is a significant improvement in the performance of the students in the experimental group as reflected in their pretest and posttest results. This improvement was brought about by modular instruction.

5. The modular approach of teaching is more effective than the lecture-discussion method in so far as the sub-topics exponent and radicals are concerned. The fact is that students can go through the module even outside the classroom,they can repeat some sections of the work if needed, discover the lessons by themselves and progresses at his own rate until the feeling of self-satisfaction and accomplishment are attained.

6. The modules are appropraite and interesting to the third year high school students in terms of readability level.

7. Students in the experimentalgroup who were taught using the modularized instruction have better retention than the students in the control group who were taught using the lecture-discussion method.

Recommendations

On the basis of the conclusions made, the researcher recomended the following:

1. Slow learners and those who have encountered difficulties in Mathematics should be given learning materials like module to give them time to study even outside the classroom and to catch up with their lessons which are not clear to them.

2. Developed modules in mathematics specially exponents and radicals should be used as instructional materials in teaching-learning process.

3. Teachers should be motivated to prepare instructional materials in accordance with their field of specialization.

4. School administration should provide adequate financial support to their classroom teachers in the developement and production of instructional materials.

5. Modules should be used as teaching aids to supplement the teacher - centered method most especially the difficult mathematical concepts.

Chapter 6

THE MODULE

This chapter deals with the developed modules on Exponents and Radicals.

Rationale

Modular instruction as classroom technique of delivering lesson to students has gained popularity in its use. It has a number of advantages over other technique. It effectiveness as an instructional materialhas been established on other subjects and courses. It is along this line of thought that the researcher attempted to find the effectiveness of modules on the teaching of mathematics. Furthermore the researcher undertook the test of establishing the validity of the module as an instructional materials.

Objective of the Module

At the end of this module, you should be able to:

1. Apply the law of exponents for multiplication
2. Apply the law of exponents for division
3. Solve exercises involving zero, negative, and fractional exponents
4. Simplify expression with fractional exponents

5. Solve an equation when either or both members of the equation are in exponential form.
6. Visualize the nature of radicals and their properties.
7. Determine which radicals are rational and which are irrational.
8. Express radicals in their simplest form.
9. Perform fundamental operations on radicals
10. Solve simple radical equations.

<u>Content of the Module</u>

This module is divided into two main parts, Namely: 1) the exponents, 2) the radicals. The exponents is divided into 6 lessons and the radicals is divided into 5 lessons. Each lesson is presented in a separate booklet and it contains the following.

Overview

Pretest

Posttest

The title of these lessons are as follow:

Part 1 Exponents

Lesson 1 Concept of Exponents

Lesson 2 Laws of Exponents

Lesson 3 Zero and Negative Exponents

Lesson 4 Simplifying and evaluating expressions with exponents

Lesson 5 Equations Involving Exponents

Lesson 6 Scientific Notation

Part II Radicals

Lesson 1 Roots of Numbers

Lesson 2 Fractional Exponents

Lesson 3 Changing Radicals to Simplest Form

Lesson 4 Addition and Subtraction of Radicals

Lesson 5 Multiplication and Division of Radicals

Lesson 6 Equation involving radicals

<u>How to Use the Module</u>

In order to gain maximum benefit from this module, you should follow all the instruction carefully.

To help you use the module properly, the key points you need to be familiar with are summarized overpage.

1. Take the pretest
2. This module is divided into 2 parts, the first part contain 6 lessons, and the second part contains 5 lessons.
3. Pages are number and according to the lesson.
4. On the first page of each lesson, you will find the specific objectives for the lesson. Read them carefully.
5. Each lesson has a sequence of activities which are as follows:

OBJECTIVE These are the specific objectives of the lesson

INPUT. This contains new information and example of illustrations for you to learn.

PRACTICE TASK. This presents a series of task (Based on the input) which you must complete.

FEEDBACK TO THE PRACTICE TASK. This contains the correct answer to the practice task.

6. You must work through each lesson in the sequence it is presented. After going through the INPUT do the PRACTICE TASK. Look at the FEEDBACK TO PRACTICE TASK page only after you have completed the practice task.
7. Work as a member of a group whenever possible. When you cannot work in group, work on your own
8. Begin working with the next lesson in the module only after you have completed the previous lessons, and you are confident that you have achieved the objectives of the lesson.
9. When you have successfully completed all the eleven lessons in the module, answer the posttest. After you have done the posttest, compare your answers with the feedback provided. You must score 75% or better before proceeding to work

through any more modules in this series. If your score is less than 75%, go through this module again.

10. DO NOT MARK THIS MODULE IN ANY WAY. USE SEPARATE SHEET FOR WRITING AND FOR YOUR COMPUTATION.

THE MODULE

Pretest/Posttest

Direction: Read each statement carefully. Encircle the letter corresponding to the correct answer.

1. In the expression 12^y, y is the:

   a. exponent c. index

   b. base d. radicand

2. Express 27 using exponent is:

   a. 2^3 c. 3^3

   b. 3^4 d. 3^2

3. Evaluate the expression 7^3.

   a. 3.3.3.3.3.3.3 c. 7.7.7

   b. 3.3.3 d. 7.7.7.7.7.7.7

4. For the power 2^4, what is the value of the power:

   a. 8 c. 32

   b. 16 d. 64

5. The product of x^4 and x^5 is:

   a. x^9 c. x

   b. x^{20} d. 1/x

6. The quotient of y^7 and y^2 is:

   a. y^5 c. y^{14}

   b. y^9 d. 5/y

7. The quotient of $\frac{3^6}{3^2}$ is equal to

a. 3^4 c. 3^{-4}

b. 3^8 d. 3^{-8}

8. The quotient of the expression $\frac{x^4}{x^2}$ is equal to

a. x^5 c. x^{12}

b. $\frac{x^2}{x^7}$ d. $\frac{x^2}{x^{12}}$

9. The numerical value of $9^{5/2}$ is:

a. -1/243 c. 1/243

b. -243 d. 243

10. The product of $9^{1/2} . 81^{1/2}$ is equal to

a. 27 c. - 81

b. -27 d. 81

11. The expression $2^3 + 3^2$ is equal to

a. 10 c. 17

b. 12 d. $2^3 / 3^2$

12. If $5^x = 125$, then x is?

a. 3 c. 120

b. 25 d. 625

13. The reduced form of $(4/5)^{-3}$ is:

a. 64/125 c. - 64/125

b. 125/64 d. - 125/64

14. The expression $(x^{-3}\ y^{-4})^0$ is equivalent to

a. x^3y^4 c. 0

b. $1/\ x^3y^4$ d. 1

15. If $\frac{4^y}{4^2} = 4^3$ then y is?

a. 1/5 c. 5

b. -1/5 d. -5

16. If $3^{2x + 1}$ then x is?

a. 2 c. 16

b. 16 d. 32

17. A number .000 000 000 71 in scientific notation is written as

a. 71×10^{10} b. 71×10^{-10}

b. 71×10^{-10} d. 71×10^{10}

18. The sum of $(2.56 \times 10^3) + (9.37 \times 10^3)$ is equal to

a. 1.193×10^5 c. 11.93×10^5

b. 1.193×10^4 d. 11.93×10^4

19. The product of $(2.56 \times 10^2)\ (\ 3.10 \times 10^3)$ is equal to

a. 80.6×10^6 c. 11.93×10^7

b. 80.6×10^8 d. 11.93×10^8

20. In the expression $\sqrt[3]{64}$, 64 is the

a. index
c. radical
b. exponent
d. coefficient

21. Which of the following is rational number?

a. $\sqrt{3}$
c. $\sqrt{5}$
c. $\sqrt{4}$
d. $\sqrt{6}$

22. The law of radicals are applicable to all cases except?

a. $\sqrt{2} \cdot \sqrt{3}$
c. $(\sqrt{2} + \sqrt{3y})^2$
b. $\dfrac{\sqrt{2}}{\sqrt{3}}$
d. $(\sqrt{2} \cdot \sqrt{3})^2$

23. The simplest form of $\sqrt{18}$ is:

a. $3\sqrt{2}$
c. $-3\sqrt{2}$
b. $2\sqrt{3}$
d. $-2\sqrt{3}$

24. The expression $(3\sqrt{3} + 2\sqrt{12} + \sqrt{75})$ expressed in simplest form is

a. $7\sqrt{3} + \sqrt{75}$
c. $8\sqrt{3} + 2\sqrt{2}$
b. $12\sqrt{3}$
d. $36\sqrt{3}$

25. Express the expression $(\sqrt{2}\ \sqrt[3]{4})$, in simplest form.

a. $1/\sqrt[3]{2}$

c. 1/2

b. $\sqrt[6]{1/2}$

d. $\sqrt[6]{32/2}$

26. The product of $\sqrt{2}$ and $\sqrt{7}$ is equal to

-

a. $\sqrt{14}$

c. $2\sqrt{7}$

b. $7\sqrt{2}$

d. $\sqrt{9}$

27. The difference of $\sqrt{18}$ and $\sqrt{2}$ is

a. $2\sqrt{2}$

c. $\sqrt{2}$

b. $\sqrt{18}$

d. $\sqrt{20}$

28. The simplest form of the expression $\frac{5}{7} - \frac{\sqrt{3}}{7}$ is equal to

a. $\frac{5 - \sqrt{3}}{7}$

b. $\frac{\sqrt{5} - \sqrt{3}}{7}$

b. $\frac{\sqrt{5} - 3}{7}$

d. 2 / 7

29. If $\sqrt{x} = 6$, then x is?

a. 6

c. 36

b. 25

d. 216

30. The value of x in the expression $2\sqrt{x} = 16$ is

a. 8 c. 64

b. 16 d. 256

Module 1

Lesson 1

EXPONENT

BASIC CONCEPTS OF EXPONENT

OBJECTIVES

1.1 To identify the base and exponent of a given exponential powers.

1.2 To express the numbers into exponential form.

1.3 To express exponential form to factored form.

Subject Matter: Exponents

Sub-topics : Concept of Exponent

Time Allotment: 40 min.

INPUT

CONCEPT OF EXPONENTS

When a number is factored repeatedly until the factors are all prime numbers, the numeral one and the prime numbers are usually repeated several times in the expression. For instance;

$$144 = 1 \,.\, 12 \,.\, 12$$
$$= 1\,(4 \,.\, 3)\,(4 \,.\, 3)$$
$$= 1\,(2 \,.\, 2 \,.\, 3)\,(2 \,.\, 2 \,.\, 3)$$
$$= 1 \,.\, 2 \,.\, 2 \,.\, 2 \,.\, 2 \,.\, 3 \,.\, 3$$

Due to the property of unity in relation to multiplication, 1 is no longer written as a factor, so our example becomes:

$$144 = 2 \,.\, 2 \,.\, 2 \,.\, 2 \,.\, 3 \,.\, 3$$

For convenience, we usually write such expression using exponents, Thus

$$144 = 2^4 \,.\, 3^2$$

The expression 3^2 means that the product of (3 . 3) and is read "the square of 3" or the second power of 3". In the expression 3^2, we call 3 the base, and we call 2 the exponent. The exponent tells how many times the base is used as a factor. The expression 3^2 is called an exponential form.

base---> 3^2 --> exponent

==

Definition of Positive Integral Exponent

In general if n is a positive integers and x is any real number, then $X^n = \underbrace{x \cdot x \cdot x \cdot \cdot \cdot x}_{\text{n factors of x}}$

The number x is called the base and n is called the exponent.

==

Let us consider the following examples.

Example 1. Identify the base and exponent for each of the given exponential expression.

	Power	base	Exponent
a.	9^2	9	2
b.	12^y	12	y
c.	$(x + 5)^3$	x + 5	3

Example 2. Express the following numbers into exponential form.

a. $9 = 3^2$

b. $16 = 4^2$

c. $-8 = -2^3$

Example 3. Express without exponent the following power as product of one or more factors written repeatedly.

a. $7^3 = 7 . 7 . 7$

b. $a^4 = a . a . a . a$

c. $(xy)^2 = xy . xy$

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$$(-2)^4 = -2^4$$

$$(-2)^4 = (-2)\ (-2)\ (-2)\ (-2) = +\ 16$$

$$-2^4 = -\ (\ 2 \times 2 \times 2 \times 2\ ) = -6$$

It is important to recognize the difference between exponential form like $(-2)^4$ and -2^4, the parentheses indicate that the power applies to the negative sign as well as 2, but in -2^4, the power applies only to 2. Similarly: in $(7x)^3$, the parentheses indicate that the power applies to 7 as well as to x, where as in $7x^3 = 7(x)^3$, the power applies only to x.

PRACTICE TASK

1. Identify the base and the exponent in each of the following powers.

   a. 4^3

   b. 8^x

   c. $(y + 4)^4$

   d. $(5 - y)^{3m}$

2. Express the following numbers into exponential form.

   a. 16
   d. 81

   b. 27
   e. -36

   c. -32

3. Express without exponent the following powers as product of one or more factors written repeatedly.

   a. 8^3
   a. $2x^4$

   b. x^5
   b. $(2x)^4$

   c. $(ab)^2$

4. Fill in the missing descriptions.

Exponential Form	Base	Exponent	Factored Form
a. 4^3	______	______	______
b. $(x-2)^2$	______	______	______
c. ______	4	5	______
d. ______	______	______	(-3)(-3)(-3)(3)
e. $(3x)^4$	______	______	______

FEEDBACK TO THE PRACTICE TASK

1.

	base	exponent		base	exponent
a.	4	3	d.	5 -y	3m
b.	8	x	e.	2x	5
c.	y+4	4			

2. a. 4^2 d. 3^4
   b. 3^3 e. $2^2 . 5^2$
   c. -2^5

3. a. 4 . 4 . 4 d. 2 . x . x . x . x
   b. x . x . x . x e. 2x . 2x . 2x . 2x
   c. ab . ab

4.

	Exponential Form	Base	Exponent	Factorial Form
a.		4	3	(4)(4)(4)
b.		x-2	2	(2-x)(x-2)
c.	4^5			(4)(4)(4)(4)(4)
d.	$(-3)^4$	-3	4	
e.		3x	4	

If you have answered 75% of the Practice Task correctly, then you have achieved the objectives of this lesson.

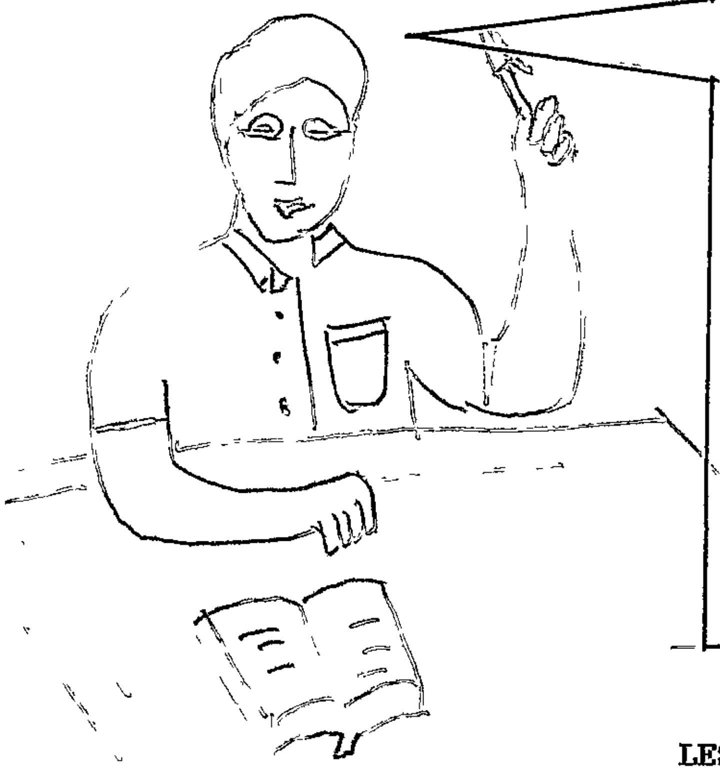
In this case, you are now ready to proceed lesson 2 of this module. Good Luck!

MODULE 1

LESSON 2

EXPONENTS

LAWS OF EXPONENTS



In the previous lesson of this module, you have learned to identify the base, power, and the exponent. The present lesson is on Laws of exponents. You are going to learn the operation using this laws of exponent. Okey, you may proceed to lesson 2 of this module. Good Luck!

LESSON 2

OBJECTIVES:

2.1 To explain the laws of exponents.

2.2 To simplify expressions using the laws of exponents.

Subject Matter : Exponent

Sub-topic : Laws of Exponent

Time Allotment : 40 minutes

INPUT

LAWS OF EXPONENT

Let us begin to develop the laws of exponents by considering the product of $2^3 \cdot 2^4$. Using the definition of exponent in the preceding section, we can write the first factor as

$$2^3 = 2 \cdot 2 \cdot 2$$

and the second factor as

$$2^4 = 2 \cdot 2 \cdot 2 \cdot 2$$

so that

$$(2)^3 (2)^4 = (2.2.2)(2.2.2.2)$$

$$= 2.2.2.2.2.2.2$$

$$= 2^7$$

We can see from the above illustration that when the exponential expressions with the same base are multiplied, their exponents are added. That is

$$(2^3)(2^4) = 2^{3+4}$$

$$= 2^7$$

Note that the first law of exponents or the product rule applies to exponential expressions with the same base only. A product of two powers with different bases, such as x^2y^3, cannot be simplified.

Moreover, the product of $(a^4)(a^5)$, where a is any real number, is obtained similarly as

$$(a^4)(a^5) = a^{4+5}$$

$$= a^9$$

We will generalize the above result for any real number [a] and positive integers [m] and [n] to exponential expressions with the same base.

$(a^m)(a^n) = (a.a.a...a)(a.a.a...a)$ --> by definition of power

m factor, n factor

$= a.a.a...a$ --> by associative property of multiplication of real number.

(m+n) factors

$= a^{m+n}$

These result give us the First Law of Exponent

To find the product of two powers with the same base copy the base and add the exponents.

In symbols,

$$a^m . a^n = a^{m+n}$$

Example I

a. $3^2 . 3^3 = 3^{2+3} = 3^5$ Read 3^2 as "three squared, 3^3 as "three cubed" and 3^5 as "three to the fifth power.

b. $x^4 . x^5 = x^{4+5} = x^9$

c. $x^2 . y^6$ cannot be simplified because the bases are not the same.

d. $x.y^4x^3.y^2 = x. x^3.y^4.y^2$

$= x^{1+3}. y^{4+2}$

$= x^4 y^6$

e. $x^4 . x^4 = x^{4+4} = x^8$

Note that $x^4 + x^4$ is not equal to x^{4+4}. In general, $a^m + a^n$ is not equal to a^{m+n}.

Next, let us consider the quotient of the two powers 2^7 and 2^4. Again, using the definition of exponents,

$2^7 = 2 . 2 . 2 . 2 . 2 . 2 . 2$

and

$2^4 = 2 . 2 . 2 . 2$

such that

$$\frac{2^7}{2^4} = \frac{2.2.2.2.2.2.2}{2.2.2.2}$$

$$= \frac{2.2.2.2.2.2.2}{2.2.2.2}$$ Applying the cancellation law

$$= 2.2.2$$

$$= 2^3$$

Observe that the four 2's in the denominator cancelled out with the same number of 2's in the numerator leaving three 2's in the numerator. The above illustration suggest that when powers of the same base are divided, the exponents are subtracted, thus

$$\frac{2^7}{2^4} = 2^{7-4}$$

$$= 2^3$$

Similarly, for any real number [a], where [a] is not zero

$$\frac{a^9}{a^5} = a^{9-5} = a^4$$

Notice that we consider here a case where the exponent is greater in the numerator than in the denominator. With this condition, the above results can be generalized as follows:

$$a^m \div a^n = \frac{a^m}{a^n} = \frac{\overbrace{a \cdot a \cdot a \cdots a}^{m \text{ factors}}}{\underbrace{a \cdot a \cdots a}_{n \text{ factors}}} = a^{m-n}$$

$= a.a.a...a$ ---> (m-n) factor

$= a^{m-n}$

Hence, the second law of exponent

To divide powers with the same base, subtract the exponent of the denominator from the exponent of the numerator to get the exponents of the quotient. In symbols, for any real number [a] where [a] is not zero, and for any positive integers [m] and [n], where [m] is greater than [n],

$$\frac{a^m}{a^n} = a^{m-n}, \quad m > n$$

Note that the second law of exponents or quotient rule applies to exponential expressions with the same base only. For quotient of two powers with different bases such as $\frac{x^2}{y^3}$ cannot be simplified.

Example II Assume that all variables represent nonzero numbers.

a. $\dfrac{x^2}{y^3} = x^{8-3} = x^5$

b. $\dfrac{10^6}{10^2} = 10^{6-2} = 10^4$

c. $\dfrac{3^5}{3^3} = 3^{5-3} = x^2$

Let us now consider a power raised to a power. For example, let us raise 2^2 to the third power.

$(2^2)^3 = (2^2)\ (2^2)\ (2^2)$, --> by definition of power

$= 2^{2+2+2}$ --> by the first law of exponents

$= 2^{2 \cdot 3}$ --> by multiplication

$= 2^6$

Take note that the exponent 3 indicates that 2^2 is to be used as a factor 3 times. Similarly,

$(a^3)^4 = (a^3)\ (a^3)\ (a^3)\ (a^3)$, --> by definition of power

$= a^{3+3+3+3}$ --> by the first law of exponents

$= a^{3.4}$ --> by multiplication

$= a^{12}$

Take note that the exponent 4 indicates that a^3 is to be used as a factor 4 times.

In general, for any real number [a] and any positive intergers [m] and [n]

$(a^m)^n = (a^m)(a^m)(a^m),$ ------> n factors

$= a^{m+m+m+...m}$ ------> n addends

$= a^{mn}$

Also

$(a^n)^m = (a^n)(a^n)(a^n),$ ----> m factors

$= a^{n+n+n+...n}$ ----> m addends

$= a^{mn}$

Thus, when an exponential expression is raised to a power, the exponents are multiplied.

Hence, the third law of exponents

The nth power of the power of [a] is the [mn]th power of [a]. In symbols, for any real number a and any positive integers [m] and [n],

$$[a^m]^n = [a^n]^m = a^{mn}$$

Example III

a.) $(3^2)^4 = 3^{(2)(4)} = 3^8$

b.) $(3^4)^2 = 3^{(4)(2)} = 3^8$

c.) $(x^4)^5 = x^{(4)(5)} = x^{20}$

Let us now consider raising a product to a certain power.

$(2.3)^4 = (2)(3)(2)(3)(2)(3),$ --> by definition of power

$= (2)(2)(2)(2)\ (3)(3)(3)(3)$ --> by associative law

$= 2^4 . 3^4$ --> by definition of power

Similarly, if [a] and [b] are real numbers,

$(ab)^3 = (ab)\ (ab)\ (ab)\ ,$

$= (a)(a)(a)\ (b)(b)(b)$

$= a^3 . b^3$

Take note that the exponent 3 indicates that the product ab is to be used as a factor 3 times. Again, we will generalize the above results for any real number [a]and [b] and any integers [n].

$(ab)^n = (ab)\,(ab)\,(ab)\,.\,.\,.\,(ab)$ ---> nth factor is ab. We can arrange the factors on the right side of the previous equations and express the result as follows.

$$= [(a)(a).\,.\,.\,(a)]\;[(b)(b)(b)...(b)]$$

$$= (a.a.a...a)\;(b.b.b...b)$$

(a.a.a...a) → n factors of a; (b.b.b...b) → n factors of b

$$= a^n \,.\, b^n$$

<u>This is the fourth law of exponents</u>

The nth power of a product is the product of the nth power of the factors. In symbols, for any real numbers [a] and [b] and any positive integers [n],

$$(ab)^n = a^n\, b^n$$

Example IV

a. $(2xy)^3 = 2^2\, x^3\, y^3 = 8x^3y^3$

b. $(x^3\, y)^5 = (x^3)^5\, y^5 = x^{15}y^5$

c. $[(3)(4)]^2 = (3^2)(4^2) = (9)(16) = 144$

d. $(3 + 4\,)^2 = 7^2 = 44$

Finally, let us consider the power of a quotient, say $\frac{2}{3}$, raised to the 5th power,

$$\left(\frac{2}{3}\right)^5 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) \quad \text{by definition of power}$$

$$= \frac{2.2.2.2.2}{3.3.3.3.3} \quad \text{by multiplication of factors}$$

$$= \frac{2^5}{3^5} \quad \text{by definition of power}$$

Similarly, for the quotient of any real numbers [a] and [b], where [b] is not zero, raised to a power, say the 4th power,

$$\left(\frac{a}{b}\right)^4 = \left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right) \quad \text{---> definition of power}$$

$$= \frac{a.a.a.a}{b.b.b.b} \quad \text{by multiplication of factors}$$

$$= \frac{a^4}{b^4}$$

==

The above results can be generalized for any positive integer exponents [n].

$$(a/b)^n = (a/b)\ (a/b)\ (a/b)\dots\ (a/b) \quad \text{n factors of}$$

$$= \frac{a.a.a.a\ \dots\ a}{b.b.b.b\ \dots\ b} \qquad b \neq 0$$

-------> n factors

$$= \frac{a^n}{b^n}$$

==

<u>We now state the fifth law of exponent</u>

The nth power of a quotient is the nth power of the dividend divided by the nth power of the divisor. In symbol, for any real numbers [a] and [b], where [b] is not zero. and any positive inetegers [n],

$$\left[\frac{a}{b}\right]^n = \frac{a^n}{b^n}$$

Let us look at some additional examples where the laws of exponents are applied.

Example 1. Find the product of $4x^2y$ and $3x^5y^4$

$$4x^2y\ .\ 3x^5y^4 = (4.3)\ (x^2.x^5)(y\ .y^4)$$

$$= 12\ x^7\ y^5$$

Note that after rearranging the factors the use of the first law is very clear in each of the grouped factors. Rearranging the factors such that similar bases are next to each other may be done mentally and the product may be written right away without writing the rearranged factors.

Example 2. Divide $15x^4y^5z^4$ by $3x^2y^2z^2$

This example may be considered as the product of several quotients, that is

$$\left(\frac{15}{3}\right)\left(\frac{x^4}{x^2}\right)\left(\frac{y^5}{y^2}\right)\left(\frac{z^4}{z^2}\right)$$

Where the second law can be applied to each of the quotients, like in example 1, this may be done mentally and the quotients can be written readily, as follows:

$$\left(\frac{15}{3}\right)\left(\frac{x^4}{x^2}\right)\left(\frac{y^5}{y^2}\right)\left(\frac{z^4}{z^2}\right) = 5\ x^2y^3z^2$$

Example 3. Raise the following numbers to its desired powers

a.) x^4 to the second power

$$(x^4)^2 = x^{4.2}$$

$$= x^8$$

b.) Raise 9 to the fourth power expressing the result as a power of 3. Fisrt, 9 has to be transformed to a power of three, that is to 3^2 and then apply the third law thus,

$$9^4 = (3^2)^4$$

$$= 3^{2.4} \qquad = 3^8$$

Example 4. Find 2y to the third power

$$(2y)^3 = 2^3y^3$$

$$= 8y^3$$

Example 5. Find the cube of $\frac{m^2}{n}$

$$\left[\frac{m^2)}{n}\right]^3 = \frac{(m^2)^3}{(n)^3} = \frac{m^6}{n^3}$$

Note that after using the fourth law, the third law can still be applied to the numerator to get the final result.

PRACTICE TASK

Perform the indicated operations using the laws of exponents

1. $x^4 \cdot x^5$

2. y^7 / y^2

3. $(m^4)^2$

4. $(hk)^7$

5. $(c/d)^5$

6. $8y^5 \cdot 4y^3$

7. $27x^5 / 9x^2$

8. $((9^2)^2)^2$

9. $(2r^2 s^3)^4$

10. $\left[\frac{c^2}{d^2}\right]^5$

FEEDBACK TO THE PRACTICE TASK

1. x^9

2. y^5

3. m^8

4. $h^7\ k^7$

5. c^5/d^5

6. $32y^2$

7. $3x^3$

8. p^8

9. $16\ r^8\ s^{12}$

10. c^{10}/d^{10}

If you have answered 75% of the Practice Task correctly, then you have achieved the objectives of this lesson. If not, please go over the lesson once more.

In this, you are now ready to proceed to lesson 3 of this module.

Good Luck!

MODULE 1

LESSON 3

EXPONENT

ZERO AND NEGATIVE EXPONETS

In the previous lesson of this module, you have learned to multiply and divide exponential expressions using the law of exponents. This time, you are going to learn other forms of exponents; like the zero & negative exponent This is easy if you understand and mastered the previous lesson.

Now, you can proceed to leson 3 of this module. Good Luck.

LESSON 3

OBJECTIVE:

3.1. To explain zero and negtaive exponents.

3.2. To simplify expressions involving zero and negative exponents

3.4. To evaluate expressions.

Subject Matter : Exponent

Sub-topic : Zero and Negative Exponents

Time allotment : 80 minutes

INPUT

ZERO AND NEGATIVE EXPONENTS

I. The zero Exponents

The laws of exponents for division are similar to the laws of exponents for multiplication.

For example:

$$\frac{3^6}{3^2} = \frac{3.3.3.3.3.3}{3.3} = 3.3.3.3 = 3^4$$

The difference of the exponents, 6-2, gives the exponent of the answer, 4.

If the exponents in the numerator and denominator are equal, we have, for example.

$$\frac{3^6}{3^6} = \frac{3.3.3.3.3.3}{3.3.3.3\ 3.3} = 1 \quad \text{--->} \quad \text{a number divided by itself is one (1).}$$

If, however, the exponents are subtracted, the resulting exponent reduces to 0.

$$\frac{3^6}{3^6} = 3^{6-6} = 3^0 = 1 \quad \text{--->} \quad \text{using quotient rule}$$

Therefore, this means that $3^0 = 1$. Based on this, for any real number [x], $x^0 = 1$ $(x = 0)$. As [x] represents any base, we conclude that any nonzero number, or algebraic expression raised to a zero exponent equals 1. No meaning is assigned to the expression 0^0.

Example

1. $5^0 = 1$
2. $(3mn^2)^0 = 1$
3. $(r/3)^0 = 1$
4. $(4a)^0 = 1$
5. $(-5)^0 = 1$
6. $-(5^0) = -(1) = -1$
7. $(5x^0) = 5(1) = 5$
8. $(5x)^0 = 1$

Take note of the difference between example 5 and 6 above

In example 5, the base is -5 and the exponent is 0. Any nonzero base raised to a zero exponent is 1. In example 6, the base is 5, since, $5^0 = 1$, therefore

$-(5^0) = -(1) = -1$

II. The Negative Exponent

Based on the previous discussion, we found that

$$\frac{3^6}{3^2} = 3^4$$

Where the exponent of the denominator is subtracted from the exponent of the numerator. Now, how about if the exponent of the denominator is larger than the exponent of the numerator? For example,

$$\frac{x^3}{x^5} = x^{3-5} = x^{-2}$$

If, however we allow exponents to be negative, then we subtract the larger exponent from the smaller. You would end up with a negative exponents. But if we subtract the smaller exponent from the larger exponent, then the resulting expression contains positive exponents. Thus,

$$\frac{x^3}{x^5} = \frac{1}{x^2}$$

Therefore,

$$x^{-2} = \frac{1}{x^2}$$

This lead us to the definition of negative exponents.

For any real number [x] and any integer [n]

$$x^{-n} = \frac{1}{x^n} \quad (x \neq 0)$$

This means that a quantity having a negative exponent is equal to 1 over that quantity with the corresponding positive exponents.

Examples:

1. a) $x^{-3} = 1/ x^3$

2. b) $3^{-2} = 1/ 3^2 = 1/9$

From the above example, we infer that any factor of the numerator may be transfered to the denominator, or vice versa, if the sign of its exponent is changed.

Thus;

$$\frac{rs^{-3}}{p^{-2}q} = \frac{p^2 r}{s^3 q}$$

Remember also that a negative exponent does not indicate a negative number; negative exponents lead to reciprocals

$$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

$$-5^{-2} = \frac{1}{-5^2} = \frac{1}{-25}$$

Using the definition of negative exponents, we can now give the laws of exponents for division.

Definition

For all positive integers [m] and [n], and for every nonzero real number x;

---

1. if m = n, then $\frac{x^m}{x^n} = 1$

Example:

$$\frac{3^5}{3^5} = 1 \quad \text{--->} \quad \frac{x^3}{x^3} = 1$$

2. If m > n, then $\frac{x^m}{x^n} = x^{m-n}$

Example:

$$\frac{2^8}{2^6} = 2^{8-6} = 2^2 = 4, \quad \frac{x^6}{x^2} = x^{6-2} = x^4$$

3. If m < n, then $\frac{x^m}{x^n} = \frac{1}{x^{n-m}}$

Example:

$$\frac{4^3}{4^5} = \frac{1}{4^{5-3}} = \frac{1}{4^2} = \frac{1}{16}, \quad \frac{x^3}{x^7} = \frac{1}{x^{7-3}} = \frac{1}{x^4}$$

PRACTICE TASK

Perform the indicated operations:

1. $(x + y)^0$

2. $(5x^2y)^0$

3. $(8x)^0$

4. $8x^0$

5. (-10^0)

6. $\dfrac{2^2}{2^5}$

7. $\dfrac{5^{-3}}{5^7}$

8. $\dfrac{r^{-3}}{r^{-3}}$

FEEDBACK TO THE PRACTICE TASK

1. 1
2. 1
3. 1
4. 8
5. -1
6. 1/125
7. 81
8. r^4

If you have answered 75% of the Practice Task correctly, then you have achieved the objective of this lesson. If not, please go over the lesson once more.

In this case, you are now ready to proceed to lesson 4 of this module. Good Luck!

MODULE 1

LESSON 4

EXPONENT

SIMPLIFYING AND EVALUATING EXPRESSION WITH EXPONENTS

LESSON 4

OBJECTIVE:

4.1. To simplify expressions containing exponents.

4.2. To evaluate algebraic expressions with exponents.

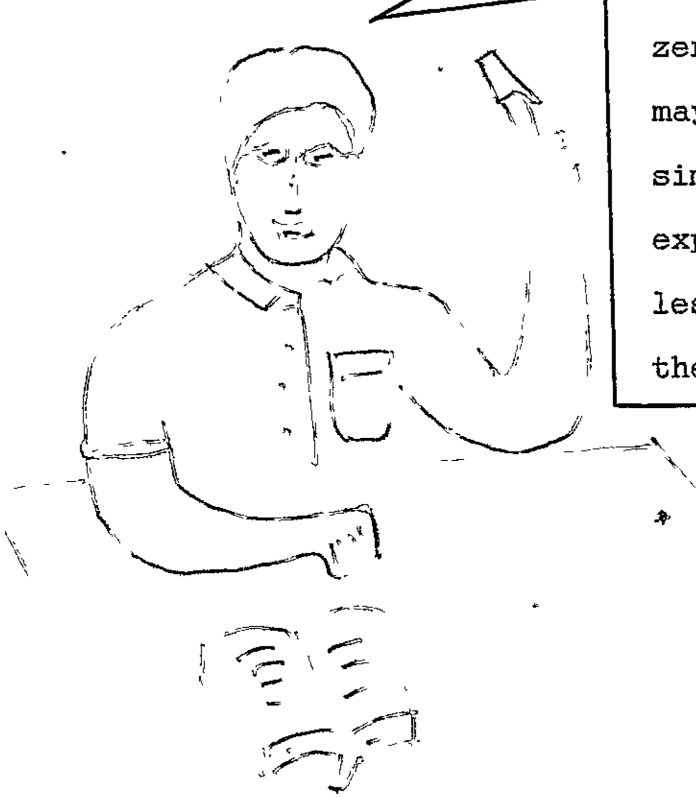
4.3. To simplify expression without negative exponents.

Subject Matter : Exponent

Sub-topic : Simplifying and evaluating expressions with exponents.

Time allotment : 40 mins

In the previous lesson of this module, you have learned to find and evaluate expressions containing zero & negative exponents. So, you may proceed to lesson 4 about simplifying and evaluating expressions with exponents. This lesson is very easy if you mastered the previous lessons.



INPUT

SIMPLIFYING EXPRESSION CONTAINING EXPONENTS

Let us agree on three characteristics of the simplest form of expressions containing integer with exponents:

1. a base appears only in the expression
2. all exponents are positive
3. the constant factors are in the lowest terms.

The first characteristic suggests that multiplication and division of power having the same bases have to be performed. The second characteristic requires the removal of zero and negative exponents. The law of exponents and the definitions of a^0 and a^{-n}, therefore, are important tools that we can use in simplifying expressions containing exponents.

Let us look at the following examples.

Example 1. Simplify $2x^3y^{-5}$

> The expression is not in simplest form because the exponent of y is negative: Hence the second characteristic is not present.

The factor y^{-5} was moved from the denominator by changing the sign of exponent

$$2x^3y^{-5} = 2x^3\,(1/y)^5$$

$$= \frac{2x^3}{y^5}$$

Example 2. Simplify $\dfrac{15\,x^{-2}y^4z^6}{3y^{-3}z^5}$

All the three characteristic are not present in the expression. To simplify,

The factor x^{-2} was moved from the numerator to the denominator by changing the sign of the exponent

divide the coefficient

$$\frac{15\,x^{-2}y^4z^6}{3y^{-3}z^5} = \frac{15}{3}\left(\frac{1}{x^2}\right)(y^{4+3})(z)$$

The factor y^{-3} was moved from the denominator to the numerator by changing the sign of its exponent.

$$= (5)\left(\frac{1}{x^2}\right)(y^7)(z)$$

$$= \frac{5y^7z}{x^2}$$

Evaluating Expressions with Exponents.

Evaluating an expression simply means finding its value. An expression with exponent can be evaluated only when the base and the exponents have numerical values. Again, the laws of exponents which we have seen applicable to positive, zero or negative exponents, as well as the definitions of a^0, a^n and a^{-n} are the useful tools in the evaluation process. Examine some examples.

Example 1. Find the value of 5^{-2}

$$5^{-2} = \frac{1}{5^2} \quad \longleftarrow \text{by definition of negative exponent}$$

$$= \frac{1}{25}$$

Example 2. Give an equivalent value for $\frac{2^3}{4^2}$ without using exponent.

$$\frac{2^3}{4^2} = \frac{8}{16} = \frac{1}{2}$$

Example 3. Find an equivalent value for $(7^2)^{-2}$ by removing the exponents

multiply exponents

$$(7^2)^{-2} = 7^{-4} \quad \longleftarrow \text{definition of negative exponent.}$$

$$= \frac{1}{7^4} = \frac{1}{2,401}$$

(7⁴) ------------> definition of positive exponent

Example 4. Evaluate $2x^4y^{-2}z^0$ when x = 2, y = 3, z = 5

$$2x^4y^{-2}z^0 = 2\,(-2^4)(3^{-2})\,(5)^0$$

(3^{-2}) ------ definition of negative number

$$= 2\,(-2^4)(1/3^{-2})\,(5^0)$$

(5^0) ----> The zero power of of 5 is 1

$$= 2\,(-16)\,(1/9)\,(1)$$

$$= \frac{32}{9}$$

Example 5. Evaluate $\dfrac{10\,a^4b^{-3}c^2}{5\,a^{-2}\,b^{-1}\,c^4}$

given a = 2, b = -1 c = 3.

divide coefficient

quotient rule

$$\frac{10\,a^4b^{-3}c^2}{5\,a^{-2}\,b^{-1}\,c^4} = \frac{10\,(2)^4\,(-1)^{-3}\,(3)^2}{5\,(2)^{-2}\,(-1)^{-1}\,(3)^4}$$

----------> quetient rule

$$= \frac{2(2)^{4+2}}{(-1)^{-1+3}(3)^{4-2}}$$

$$= \frac{2\ (2)^{6}}{(-1)^{2}(3)^{2}}$$ <------ definition of exponent

$$= \frac{2\ (64)}{1\ (9)} = \frac{128}{9}$$

Many mistakes are made when working with exponents because of misuses of the basic rules and definitions. These are some of the common mistakes that you should try to avoid.

Wrong	Right
$(3)^2\ (3)^4 = 3^8$ (Do not multiply exponents $(3)^2\ (3)^4 = 9^6$ (Do not multiply the base numbers)	$(3)^2\ (3)^4 = 3^6$ (Product Rule
$\frac{2^8}{2^2} = 2^4$ (Do not divide the exponents) $\frac{2^8}{2^2} = 1^6$ (Do not divide the base number	$\frac{2^8}{2^2} = 2^6$ (Quotient Rule)

Wrong	Right
$(3^2)^4 = 3^6$ (Do not add the exponent)	$(3^2)^4 = 3^8$ Power of a Power
$(-2)^4 = -2^4$ (Misreading the parentheses)	$(-2)^4 = (-1)^4(2)^4) = 2^4$ Power of a Product
$(-3)^0 = -1$ (Misreading definition of zero exponent)	$(-3)^0 = 1$ Definition of Zero exponents
$2^{-3} = -\frac{1}{2^3}$ (Misreading definition of negative exponent)	$2^{-3} = \frac{1}{2^3}$ Definition of negative exponents
$(x+y)^{-1} = x^{-1} + y^{-1}$ (Wrong use of definition of negative exponent)	$(x+y)^{-1} = \frac{1}{x + y}$ Definition of negative exponent

PRACTICE TASK

I. Tell whether the given expression is in simplest form.

1. $y^2 \cdot y^2$

2. y^2 / y^{-5}

3. $y^0 / x^{-1}y^2z^3$

II Simplify the following expressions without zero and negative exponents

4. $x^{-3}\ y^2$

5. $3m^2\ n^0$

6. $(a^{-2}b^{-6}y)^{0}$

III Evaluate the following expressions

7. 3^{-3}

8. $2^{-3} + 5^2$

IV Given x = 2, y = 3, and z = -2, find the value of the following expressions

9. $y^2 + y^3$

10. $3x^4 - 4y^2 + z$

FEEDBACK TO THE PRACTICE TASK

1. not in simplest form
2. not in simplest form
3. not in simplest form
4. y^2/x^3
5. $3m^2$
6. 1
7. $1/3^3$
8. $1/2^3 + 5^2$
9. 31
10. 10

If you have answered 75% of the Practice Task correctly, then you have achieved the objectives of this lesson. If not, please go over the lesson once more. In this case, you are now ready to proceed lesson 5 of this module. Good Luck!

MODULE 1

LESSON 5

EXPONENT

EQUATION INVOLVING EXPONENTS

LESSON 5

OBJECTIVE:

5.1. To Solve exponential equations.

Subject Matter	:	Exponent
Sub-topic	:	Equations involving exponents
Time allotment	:	40 mins

In the previous lesson of this module, you have learned to simplify and evaluate expressions with exponents. The next lesson is quite easy if you mastered the laws of exponents. So, you may proceed to the next page for your lesson 5.

Good luck.

INPUT

EQUATIONS INVOLVING EXPONENTS

An important property of equality for real numbers is that if two numbers that are equal are raised to the power, then the result are also equal. An exponential equation is one that contains the variable in an exponent.

For any real numbers a and b, and any integer n, if a = b, then $a^n = b^n$

Based on this, we will defined equal powers as follows: >

Two powers are equal if and only if their bases are equal and their exponents are equal, that is

$a^n = b^n$, if and only if a= b and n = m

Example 1. Solve the equation $a^{2x} = a^8$

Solution: Since the bases of the equal powers are equal, then the exponents are equal.

$$2x = 8$$

$$x = 4$$

Example 2. Solve the equation $2^x = \frac{1}{2^6}$

Solution: By definition of a^{-n},

$$2^x = \frac{1}{2^6} = 2^{-6}$$

therefore, by definition of equal powers,

$$x = -6$$

Example 3. Find the value of k: $y^k\ y^k = y^{-14}$

Solution: $y^k . y^k = y^{k+k} = y^{2k} = y^{-14}$

$$2k = -14$$

$$k = -7$$

Example 4. Find the value of x: $2^{x-3} - 2^3 = 24$

Solution: $2^{x-3} - 2^3 = 24$

$2^{x-3} = 24 + 8 = 32 = 2^5$ --> definition of equal powers

$X - 3 = 5$

$x = 5 + 3 = 8$

PRACTICE TASK

I. Solve the following equations.

1. $a^x = a^3$

2. $a^{-x} = a^{-2}$

3. $a^{2x} = a^{15}$

II Find the value of k

4. $3^k = 9$

5. $1/3^k = 27$

III. Solve for n.

6. $a^n . a = a^4$

7. $a^{2n} . a^n = a^{12}$

IV. Find r

8. $2^{r+1} - 2^2 = 60$

9. $2^{3r+5} - 3^2 = 7$

10. $5^{4r-2} + 2^4 = 41$

FEEDBACK TO THE PRACTICE TASK

I.

1. x = 3
2. x = 2
3. x = 7.5

II

4. k = 2
5. k = -3

III

6. n = 3
7. n = 4

IV

8. r = 5
9. r = -1/3
10. r = 1

If you have answered 75% of the Practice Task correctly, then you have achieved the objectives of this lesson. If not, please go over the lesson once more. In this case, you are now ready to proceed lesson 5 of this module. Good Luck!

MODULE 1

LESSON 6

EXPONENT

SCIENTIFIC NOTATION

In the previous lesson of this module, you have learned how to solve equations involving exponents.

Now, the lesson that you are going to learn is very much related to the previous lessons and you will learn how to add, subtract, multiply, and divide a scientific notation. As you go on, you will find out that you will enjoy learning especially doing the practice task which is a challenge to everyone.

Now, you may proceed to lesson 6,

LESSON 6

Objectives:

6.1. To write the big or small numbers in scientific notation.

6.2. To add or subtract scientific notation

6.3. To multiply or divide scientific notation

Subject Matter: Exponents

Sub-topic : Scientific Notation

Time allotment: 80 mins

INPUT

SCIENTIFIC NOTATION

Scientific notation is a convenient way of writing the decimal representation of very large or small numbers used in science and mathematics. It is helpful in simplifying certain kinds of computations.

The following are examples of numbers in scientific notation.

1. The speed of light $= 2.998 \times 10^{8}$ m/s
2. The mean distance of the earth from the sun

   $= 1.495 \times 10^{11}$ m

3. The mass of the earth $= 5.98 \times 10^{24}$ kg.

4. The mass of the electron $= 9.109 \times 10^{-3}$ kg.

5. The thickness of a sheet of paper $= 9.6154 \times 10^{-5}$ m
6. The population of the Philippines in 1990 $= 6.5 \times$

   10^{7} person

The preceding examples indicate a scientific notation consist of the <u>product of a rational number in decimal form and a power of 10.</u> Note also that the decimal part is less than 10 and more than or equal to 1.

If we let the decimal part be represented by k, then the scientific notation for a given number can be defined as follows

---

Definition : A number is written in scientific notation if it is written as a product of a number between 1 (including 1) and 10, and a power of 10.

In symbols, $k \times 10^n$ where 1 $k < 10$ and n is an integer.

In actual practice, the decimal part is rounded off to two, three or four decimal places. The scientific notation in such case is just an approximation and not an exact representation of the number.

To change from ordinary notation to scientific notation, the decimal point is moved so that only one nonzero digit is to its left. The number of places moved is the value of the exponent [n] of the power of 10. The sign n is <u>positive</u> if the decimal point is moved to the left; otherwise, it is <u>negative.</u>

Example 1. Write 40,533,000 in scientific notation.

Solution: The decimal point is understood to be at the right end of the number. Moved the decimal point seven places to the left.

$$40{,}533{,}000 = 4.0522 \times 10^{7}$$

<--------
seven places
left

Example 2. Write .0000000235 in scientific notation

Solution: The number is very small. Hence the exponent is negative. Move the decimal point in 2.35 eight places to the right.

$$.0000000235 = 2.35 \times 10^{-8}$$

-------->

Eight place
right

Numbers in scientific notation can be added only if the exponents of the power are the same. If different, one can be changed to be the same as the other but with the corresponding movement of the decimal point. If necessary, adjustments have to be made in the result to have an answer which is also in scientific notation.

Example 3. $2.56 \times 10^{3} + 9.37 \times 10^{3} = (2.56 + 9.37) \times 10^{3}$

$$= 11.93 \times 10^{3}$$

$$= 1.193 \times 10^4$$

Example 4. $3.16 \times 10^4 + 2.43 \times 10^5 = (.316 \times 10^5 + 2.43 \times 10^5$

$$= (.316 + 2.43) \times 10^5$$

$$= 2.746 \times 10^5$$

In example 4, it is better to adjust the smaller power to equal the exponent of the greater power than to adjust the greater power to equal the exponent of the smaller power. The former gives an answer already in scientific notation so that no adjustment is needed anymore.

It is not necessary for number in scientific notation to have the same power of 10 to be able to do multiplication and division. The laws of exponents are useful tools in doing these operation. If necessary, the decimal part of the result has to be changed to satisfy the conditions for k as stated in the definition.

Example 5.

$(2.59 \times 10^2)(3.19 \times 10^5 = (2.96 \times 3.19)\text{x } (10)^2 \times 10^5$

(use the associative law.)

$$= 8.261 \times 10^7$$

$$= 8.261 \times 10^7$$

Example 6.

$(8.32 \times 10^4)(3.46 \times 10^8 = (8.32 \times 3.46) x (10^{-5} \times 10^4$

$= 28.7872 \times 10^{12}$ ---> add exponent

$= (2.87872 \times 10^1)\ 10^{12}$ --> definition of scientific notation

$= 2.88 \times 10^4$

Example 7. $(9.54 \times 10^4) \div (2.14 \times 10^{-2} = \dfrac{9.54 \times 10^4}{2.14 \times 10^{-2}}$

$= 4.46 \times 10^6$ ----> subtract exponents

Example 9. $(1.62 \times 10^5) \div (8.24 \times 10^{10} = \dfrac{1.62 \times 10^5}{8.24 \times 10^{10}}$

$= .197 \times 10^{-5}$ ----> definition of scientific notation

$= (1.97 \times 10^{-1}) \times 10^{-5}$ ---> add exponents

$= 1.97 \times 10^{-6}$

PRACTICE TASK

Write the following numbers in scientific notation

1. 145 000 000 =

2. .000 000 236 =

3. 324×10^{6} =

Add the following numbers:

4. $(3.67 \times 10^{8} + (2.12 \times 10^{8})$ =

5. $(2.35 \times 10^{3} + (9.74 \times 10^{3})$ =

Perform the multiplication. Express the product in scientific notation

6. $(3.16 \times 10^{3})\ (2.15 \times 10^{2})$ =

7. $(4.25 \times 10^{-8})\ (1.34 \times 10^{-3})$ =

8. $(5.96 \times 10^{5})\ (1.25 \times 10^{-6})$ =

Perform the following division

9. $(9.63 \times 10^{5}) \div (2.47 \times 10^{3})$ =

10. $(8.36 \times 10^{-4}) \div (5.48 \times 10^{-2})$ =

FEEDBACK TO THE PRACTICE TASK

1. $1,45 \times 10^{8}$
2. 2.36×10^{8}
3. 3.24×10^{8}
4. 5.79×10^{8}
5. 1.209×10^{5}
6. 6.794×10^{5}
7. 5.695×10^{-4}
8. 7.45×10^{-2}
9. 3.89×10^{2}
10. 1.52×10^{-2}

If you have answered 75% of the practice task correctly, then you have achieved the objective of this lesson. If not, please go over the lesson once more.

In this case, you are now ready to proceed lesson 7 of this module. Good Luck!

MODULE 2

LESSON 1

RADICALS

ROOTS OF A NUMBER

In the previous lesson of this module, you have learned the concept of scientific notations.

Now, the lesson that you are going to learn is very much related to the previous lessons. You will learn how to find the square root and cube root of a number. As you go on, you will find out that you will enjoy learning especially doing the practice task which is a challenge to everyone.

Now, you may proceed to lesson 1.

LESSON 1

Objectives:

1.1. To identify whether the radicals is rational or irrational

1.2. Give the principal square root and cube root of a radical.

1.3. To find the root of a radicand

Subject Matter: Radicals

Sub-topic : Roots of numbers

Time allotment: 40 mins

ROOT OF A NUMBER

Do you recall how to find the values of these expressions?

1. 3^2, 3^3, 3^4

2. $(-2)^2$, $(-2)^3$

3. $\left[\frac{1}{5}\right]^2$, $\left[\frac{-1}{5}\right]^2$

To square a number, multiply it by itself:

If x = 6, then $x^2 = (6)(6) = 36$

If x = -6, then $x^2 = (-6)(-6) = 36$

If x = 1/3, then $x^2 = (1/3)\ (1/3) = 1/9$

If x = -1/3, then $x^2 = (-1/3)\ (-1/3) = 1/9$

Many problems in science, business and economics involve equations with exponential form like.

$x^2 = 36$, $x^2 = \frac{1}{9}$, or $y^3 = 125$

Solution to such equations involved the finding of roots of numbers. For example we say:

If $x^2 = 36$, then x a square root of 49

If $x^2 = 1/9$, then x is the square root of $\frac{1}{9}$

If $x^3 = 125$, then x is the cube root of 125

Since $6^2 = 36$ and $(-6)^2 = 36$, we say that [-6] are square roots of 36. Therefore, 36 has two square roots, 6 and -6. Similarly, both 1/3 and -1/3 are square roots of 1/9. These examples suggest that each positive real number x has distinct square roots, one is negative and the other positive.

Definition

The positive square root of x is denoted by the symbol $\sqrt{x}$. The positive square root of $\sqrt{100}$ is 10, written as $\sqrt{100} = 10$

The negative square root of x is denoted by the $-\sqrt{x}$. For example, the negative square root of 100 is -10, written $-\sqrt{100} = -10$

The positive square root, $\sqrt{x}$, is also called the <u>principal square root of x</u>. When no plus or minus sign precedes the radical sign, the principal root is always implied. For example, the number 4 has two square roots,

± 2. Here +2 is the principal square root of 4. Also $\sqrt{4}$ mean +2 (but not −2), while $-\sqrt{4}$ means −2 (but not +2).

Examples: 1. $\sqrt{64} = 8$ becuase $64 = 8^2$

2. $-\sqrt{64} = -8$ because $(-8)^2 = 64$

In case a number has only one root for a given index, that root is the principal root. Consider $\sqrt[3]{27}$. We need a number that can be cubed to give 27. Since $3^3 = 3 . 3 . 3 = 27$, we have $\sqrt[3]{-27} = -3$

Based on the above example, we can say the following;

1. The cube root of a number is positive
2. The cube root of a negative number is negative.
3. There is only one real number cube root for each real number.
4. When the root is even (square root, fourth root, and so on), the radicand must be nonnegative to get a real number root. Also for even roots, the symbol $\sqrt{}$, $\sqrt[4]{}$, $\sqrt[6]{}$ and so on are used for the nonnegative roots.

The symbol $\sqrt{}$, called radical sign, is often used to indicate that a root is to be found and, used alone, always represents the nonnegative sqaure root. A small figure, written in the angle of the radical sign, is called the <u>index of the root</u>. It shows what kind of root is desired. The radical sign without an index universally represents the square root. The numerical or expression inside the radical sign is called the <u>radicand</u> and the entire expression, radicand sign and radical, is called a <u>radical</u>. A radical is an indicated root of any number or algebraic expression. The order of a radical is determined by its index.

```
     ---------->  index of the root
     !
     n____
    \/  x ---->   Radicand
    !
    ----------->  Radical sign
```

Example:

1. a.) $\sqrt[3]{5}$ is a radical of the third order.

   b.) $\sqrt[4]{35}$ is a radical of the fourth order.

   c.) $\sqrt{81}$ is a radical of the second order.

2. $\sqrt{5}$ and $\sqrt[4]{xy}$, $\sqrt[4]{x}$, 5 and xy are radicands

4. $\sqrt[4]{x}$, 4 is the index of the radical.

The number 0 has just one square root, 0, since 0 is the only number whose square is 0. So you write $\sqrt{0} = 0$.

A number that is not a perfect square has a square root that is not rational number. For instance, $\sqrt{7}$ is not a rational number, because it cannot be written as the ratio of two integers. However, $\sqrt{7}$ is a real number and corresponds to a point on the number line. A real number that is not rational is called an irrational number. Therefore, $\sqrt{7}$ is irrational. Remember, if x is a positive integer that is not a perfect sqaure, then $\sqrt{x}$ is irrational.

Example of Rational Numbers

$\frac{3}{4}$ A fractions are rational numbers

$\sqrt{25}$ some square roots are rational numbers

Examples of irrational numbers are $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$,

$\frac{1}{\sqrt{2}}$, $\overline{\text{II}}$, etc.

Keep in mind that not every number has a real number square root. For example, there is no real number that can be squared to get -49. (The square of a real number can never be negative.) Because of this $\sqrt{-49}$ is not a real number. So, if x is a negative number then $\sqrt{x}$ is a not a real number. Thus, such expressions as $\sqrt{-2}$, $\sqrt{-5}$ and so on, do not name real numbers.

Now, tell whether each square root is rational or irrational

Examples:

a. $\sqrt{17}$ since 17 is not a perfect square, hence an irrational number.

b. $\sqrt{36}$ the number 36 is a perfect square, 6^2 so $\sqrt{36} = 6$, a rational number.

c. $\sqrt{35}$ is irrational because 35 is not a perfect square

d. $\sqrt{49}$ is rational because 49 is a perfect square

e. $\sqrt{1/4}$ is rational because 1 and 4 are perfect squares

PRACTICE TASK

A. Which of the following expressions are rational numbers?

1. $\sqrt{9}$

2. $\sqrt{2}$

3. $\sqrt[3]{9}$

B. Find is the principal square root of the following?

4. 36 =

5. 1 =

6. 100 =

C. Find is the principal cube root of the following.

7. $\sqrt[3]{64}$

8. $\sqrt[3]{-27}$

D. Find each root that exist

9. $\sqrt{100}$

10. $\sqrt{169/49}$

FEEDBACK TO THE PRACTICE TASK

1. Rational
2. Irrational
3. Irrational
4. 6
5. 1
6. 100
7. 4
8. -3
9. 10
10. $\frac{13}{7}$

If you have answered 75% of the practice task correctly, then you have achieved the objectives of this lesson. If not, please go over the lesson once more.

In this case, you are now ready to proceed lesson 2 of module 2. Good Luck!

MODULE 2

LESSON 2

RADICALS

FRACTIONAL EXPONENTS

Lesson 2

Objectives

2.1 To write the fractional expression in radical form.

2.2 To write the radical form in exponential form.

2.3 To simplify expressions.

Subject Matter: Radicals

Sub-topic : Roots of Numbers

Time Allotment: 40 Minutes

INPUT

Fractional Exponents

It is possible to raise numbers to fractional powers. For example we consider the symbol $5^{1/2}$. If fractional exponents are to obey the same rule as integral exponents, then $(5^{1/2})^2$ can be evaluated by multiplying exponents:

$$(5^{1/2})^2 = 5^{2/2} = 5^1 = 5$$

However, $\sqrt{5}$ is a number whose square is 5: $(\sqrt{5})^2 = 5$ Because $(\sqrt{5})^2$ and $(5^{1/2})^2$ are both equal to 5, we define $5^{1/2}$ to be $\sqrt{5}$. In a similar manner, $5^{1/3} = \sqrt[3]{5}$, and so on.

Definition:

If n is any positive integer, and $x \geq 0$ when n is even then $x^{1/n} = \sqrt[n]{x}$ and if $x \neq 0$, then $x^{-1/n} = \dfrac{1}{\sqrt[n]{x}}$

Example:

$$9^{1/2} = \sqrt{9} = 3$$

$$8^{1/3} = \sqrt[3]{8} = 2$$

$$9^{1/2} = \frac{1}{9^{1/2}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

The definition can be extended to cover exponential expressions such as $8^{2/3}$ that have fractional exponent with numerators diffent 5 from 1, Thus $8^{2/3}$ may be interpreted either as $\sqrt[3]{8^2}$ or $(\sqrt[3]{8})^2$. you can use either of the solution below and you will get the same result which is 4.

Solution:

$$\sqrt[3]{8^2} = \sqrt[3]{64} = 4$$

or

$$(\sqrt[3]{8})^2 = (2)^2 = 4$$

The generalization of this example leads to the following definitions.

Definition:

If m and n are positive integers, and x is a real number, then

$$x^{m/n} = \sqrt[n]{x^m} \quad \textbf{and if } x \neq 0,\ x^{-m/n} = \frac{1}{x^{m/n}} \quad \textbf{provided}$$

$\sqrt[n]{x}$ **is a real number**

Bear in mind that radicals and fractional exponents are just two different forms of notation for the same idea.

Now, suppose you are given a negative fractional exponent like $8^{-2/3}$. Take note that this exponent tells you to do three things to the base:

1. The negative sign tells you to invert.
2. The 3 tells you to take the cube root.
3. The 2 tells you to square.

It will be helpful to observe the analogy that exists between a power with fractional exponent and its corresponding radicals.

Elements	Power $x^{m/n}$	Radicand $(\sqrt[n]{x})^m$
x	Base	Radicand
m	Numerator of exponent	Exponent
n	Denominator of exponent	Index or Order

Solution:

$$8^{-2/3} = \frac{1}{8^{2/3}} = \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{(2)^2} = \frac{1}{4}$$

Example 1. Find the numerical value of the following:

a.) $25^{5/2}$

Solution:

$$25^{5/2} = (\sqrt{25})^3 \quad \text{or} \quad 25^{5/2} = (5^2)^{3/2}$$

$$= 5^3 \qquad\qquad = 5^3$$

$$= 125 \qquad\qquad = 125$$

The first approach is less work. It is easier to take the nth root first and then raise to the nth power, rather than the reverse.

b. $4^{-5/2}$

Solution

$$4^{-5/2} = \frac{1}{4^{5/2}}$$ ----> definition of power with fractional exponents

$$= \frac{1}{(\sqrt{4})^5}$$

$$= \frac{1}{(2)^5} = \frac{1}{32}$$ ---> definition of exponent.

c. Simplify $-16^{5/2}$

Solution: Change to radical form

$$-16^{5/2} = -(\sqrt{16})^5$$ Take square root of 16

$$= -(4)^5$$ Take 5th root of 4

$$= -[(4)(4)(4)(4)(4)]$$

$$= -1024$$

Practice Task

A. Write each of the following in radical form

1. $36^{1/2}$
2. $25^{-1/2}$
3. $8^{4/3}$
4. $8^{-4/3}$
5. $-81^{1/2}$

2. Write each of the following in exponential form

6. $\sqrt{49}$
7. $\sqrt[4]{16}$
8. $\sqrt[3]{27}$
9. $(\sqrt{16})^3$
10. $(\sqrt[3]{27})^4$

3. Find the numeral value on each of the following expressions

11. $100^{1/2}$
12. $64^{-1/2}$
13. $64^{1/3}$
14. $25^{3/2}$
15. $1000^{2/3}$

FEEBBACK TO THE PRACTICE TASK

1. $\sqrt{36}$
2. $\dfrac{1}{\sqrt{36}}$
3. $(\sqrt[3]{8})^4$
4. $\dfrac{1}{(\sqrt[3]{8})^4}$
5. $\dfrac{1}{\sqrt{81}}$
6. $49^{1/2}$
7. $16^{1/4}$
8. $27^{1/3}$
9. $16^{2/3}$
10. $27^{4/3}$
11. 10
12. $\dfrac{1}{8}$
13. 4
14. 125
15. 100

> If you have answered 75% of the practice task correctly, the you have achieved the objectives of this lesson. If not, please go over the lesson once more.
>
> In this case, you are now ready to proceed lesson 3 of module 2. Good Luck

MODULE 2

LESSON 3

RADICALS

CHANGING RADICALS TO SIMPLEST FORM

In the previous lesson of this module, you have learned how to simplify powers with fractional exponents.

Now, the lesson that you are going to learn is changing radicals to its simplest form.

You may now proceed to the next page for your lesson 3, and good luck.

LESSON 3

Objectives:

3.1 To apply the properties of radicals in finding the indicated root.

3.2 To express radicals to its simplest form.

3.3 To simplify radicals with fractions in the radicand.

Subject Matter: Radicals

Sub-topic : Properties of Radicals

Time allotment: 120 mins

INPUT

CHANGING RADICALS TO SIMPLEST FORM

When the index of a radicals is n, and the radicand is a perfect n^{th} power, then there is usually no difficulty in computing with the radical. For example,

$\sqrt{36} = 6$ because $6^2 = 36$

$\sqrt[3]{64} = 4$ because $4^3 = 64$

However, when the radicand is not a perfect n^{th} power, we have to rewrite the radicand so that no perfect square appears as a factor under the radical sign. Thus,

$$\sqrt{20} = \sqrt{4.5} = \sqrt{4}.\sqrt{5} = 2\sqrt{5}$$

We say that $2\sqrt{5}$ is the simplified form of $\sqrt{20}$.

NOTE: That the Fundamental Property used above is that

$$\sqrt[n]{ab} = \sqrt[n]{a}\ \sqrt[n]{b} \qquad \textbf{(Property I)}$$

When simplifying the square root of a number, find the largest perfect square that is a factor of the number, for example,

$4 = 2^2$ is a perfect square:

$$\sqrt{28} = \sqrt{4.7} = \sqrt{4}.\sqrt{3} = \sqrt{2^2}\sqrt{3} = 2\sqrt{3}$$

$9 = 3^2$ is a perfect square:

$$\sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = \sqrt{3^2}\,\sqrt{3} = 3\sqrt{5}$$

$121 = 11^2$ is a perfect square:

$$\sqrt{242} = \sqrt{121 \cdot 2} = \sqrt{121} \cdot \sqrt{2} = \sqrt{11^2}\,\sqrt{2} = 11\sqrt{2}$$

Similarly, in order to simplify the n^{th} root of a number [q], find the largest number [p] such that $q = p^n \cdot r$, and then take the n^{th} root as illustrated next.

$8 = 2^3$ is a perfect square:

$$\sqrt[3]{56} = \sqrt[3]{8 \cdot 7} = \sqrt[3]{8} \cdot \sqrt[3]{7} = \sqrt[3]{2^3}\,\sqrt[3]{7} = 2\sqrt[3]{7}$$

$64 = 4^3$ is a perfect square:

$$\sqrt[3]{192} = \sqrt[3]{64 \cdot 3} = \sqrt[3]{64} \cdot \sqrt[3]{3} = \sqrt[3]{4^3}\,\sqrt[3]{3} = 4\sqrt[3]{3}$$

You may find the following list of squares and cubes helpful when simplying radicals.

Number	**Perfect Squares**	**Perfect Cubes**
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000

Example 1. Simplify $\sqrt{12x^4y^3}$

Solution: To start with, ask yourself the question "are there factors of the radicand that are perfect squares? If there are, the radicand can be written as a product of two factors, one of which is a perfect square.

$$\sqrt{12x^4y^3} = \sqrt{2^2 \cdot 3x^4 \cdot y^2 \cdot y}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 2 \,\cdot & x^2 & y \end{array}$$

$= 2x^2y\sqrt{3y}$ <---- simplified form

or $\sqrt{12x^4y^3} = \sqrt{4 \cdot x^4 \cdot y^2 \cdot 3y}$

- x^4, y^2 ----------> exponents of x & y divisible by the index (2).
- 4 ---------------> perfect squares

$= \sqrt{4x^4y^2}\ \sqrt{3y}$

$= 2x^2y\sqrt{3y}$ <---- simplified form

Example 2. Simplify $\sqrt[3]{54\,x^4y^5}$

Solution: Here we look for perfect cubes which are factors of the radicand. Hence, or $\sqrt[3]{54x^4y^5} = \sqrt[3]{27x^3y^3 \cdot 2xy^2}$

- 27 → perfect cubes
- exponents of x & y divisible by the index (3).

$$= \sqrt[3]{27x^3y^3}\ \sqrt[3]{2xy^2}$$

$$= 3xy\sqrt[3]{2xy^2}$$ <---- simplified form

Some radical expresions contain fractions in the radicand, yet the denominator is rational. For example,

$$\sqrt{\frac{25}{36}} = \frac{\sqrt{25}}{\sqrt{36}} = \frac{5}{6}$$ ---> denominator is rational

$$\sqrt{\frac{x^4}{y^6}} = \frac{\sqrt{x^4}}{\sqrt{y^6}} = \frac{x^2}{y^3}$$ ---> denominator is rational

---

NOTE: that the Fundamental Property used in this previous examples is that

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$ **Property II**

We now consider radicals such as $\sqrt{\frac{5}{3}}$ whose radicand are fractions. To simplify radicals with fractions in the radicand, we see to it that no radicals appears in the denominator.

Example 3. Simplify $\sqrt{\frac{5}{3}}$

Solution: Multiply both numerator and denominator of the radicand by the same expression of the lowest power that will convert the denominator into a perfect square.

$$\sqrt{\frac{5}{3}} = \sqrt{\frac{5}{3}\cdot\frac{3}{3}} = \sqrt{\frac{15}{9}}$$

$$= \frac{\sqrt{15}}{\sqrt{9}}$$

$$= \frac{\sqrt{15}}{3}$$

In example 3, we eliminate the radical from the denominator of a fraction. This process is known as <u>rationalizing the denominator</u>.

Example 4. Rationalize the denominator of $\frac{2}{\sqrt{5}}$

Solution: Multiply both numerator and denominator by $\sqrt{5}$ since $\sqrt{5} \cdot \sqrt{5} = 5$ Hence

$$\frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{5}\ \sqrt{5}} = \frac{2\sqrt{5}}{5}$$

- $\frac{2}{\sqrt{5}}$: The denominator is NOT a rational number.
- $\frac{\sqrt{5}}{\sqrt{5}}$: The fraction is one (1)
- $\frac{2\sqrt{5}}{5}$: The denominator is now a rational number.

A radical is in its simplest form when the following conditions are satisfied.

1. The radicand does not contain factors whose indicated root can still be taken;
2. The radicand does not contain fractions.
3. There are no radicals in the denominator of the fraction
4. The index of the radical is in its lowest form possible.

PRACTICE TASK

A. Simplify if possible. Assume variable are positive real numbers

1. $\sqrt{75}$
2. $\sqrt{1080}$
3. $\sqrt{x^4 y}$
4. $\sqrt{a^2 b^5}$
5. $\sqrt{125 b^2}$
6. $\sqrt{20x^4 y^5}$
7. $\sqrt{75 x^2 y}$
8. $\sqrt{48x^2 y^4}$

B. Rationalize the denominator of each of the following expressions.

9. $\sqrt{1/2}$
10. $\sqrt{3/20}$
11. $\sqrt{5/24}$
12. $\dfrac{2}{\sqrt{2}}$
13. $\dfrac{\sqrt{3/6}}{\sqrt{5}}$
14. $\sqrt{3/5}$
15. $\dfrac{3}{\sqrt{8/9}}$

FEEDBACK TO THE PRACTICE TASK

1. $5\sqrt{3}$
2. $6\sqrt{30}$
3. $x^2\sqrt{y}$
4. $ab^2\sqrt{b}$
5. $5b\sqrt{5}$
6. $2a^2b\sqrt{5}$
7. $5x\sqrt{3y}$
8. $2y\sqrt[3]{6x^2y}$
9. $\frac{\sqrt{2}}{2}$
10. $\frac{\sqrt{15}}{10}$
11. $\frac{\sqrt{30}}{12}$
12. $\sqrt{2}$
13. $\frac{\sqrt{15}}{3}$
14. $\frac{\sqrt{15a}}{5a}$
15. $\frac{\sqrt[3]{24}}{3}$

If you have answered 75% of the Practice Task correctly, you have done it well and you may proceed to Lesson 4 of this module 2. Otherwise, you have to go over Lesson 3 again.

Now, you can proceed to the next lesson. Good Luck.

MODULE 2

LESSON 4

RADICALS

ADDING AND SUBTRACTING RADICALS

LESSON 3

OBJECTIVE:

4.1 To differentiate like from unlike radicals.

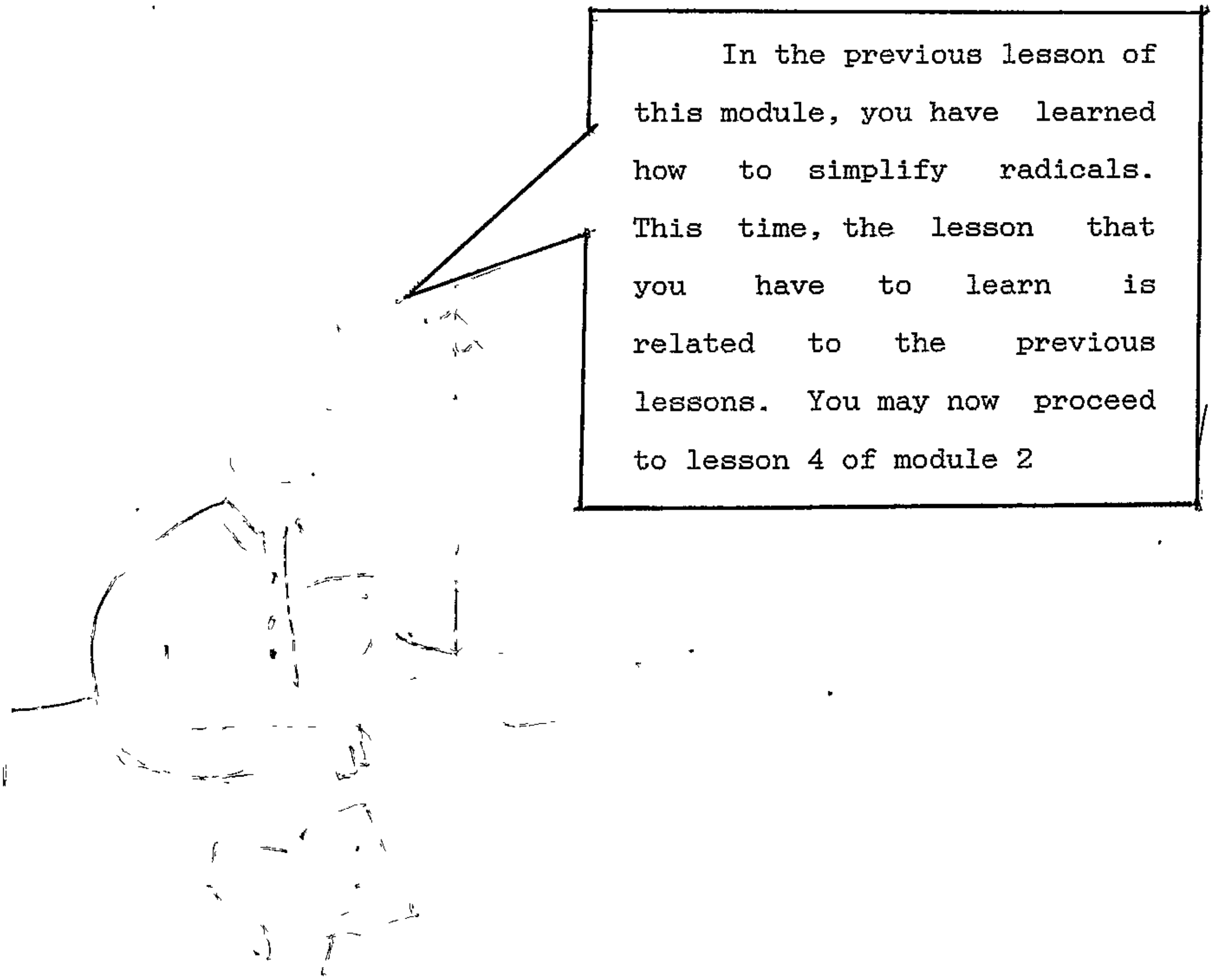
4.2 To add radicals of the same order.

4.3 To subtract radicals of the same order.

Subject Matter : Radicals

Sub-topic : Adding and subtracting radicals

Time allotment : 120 minutes



INPUT

ADDING AND SUBTRACTING RADICALS

Adding or subtracting radicals is similar to adding and subtracting algebraic expressions. To find the sum of monomials with similar terms like 3a and 7a, we apply the distributive property. So $3a + 7a = (3+7)a$ or $10a$. In a similar manner radical can only be added to a single radical when they are of the same order/index and their radicands are identical.

Examples of like radicals are:

a.) $\sqrt{3}$, $2\sqrt{3}$, $-5\sqrt{3}$

b.) $\sqrt{x}$, $-3\sqrt{x}$, $4\sqrt{x}$

c.) $\sqrt[3]{5}$, $3\sqrt[3]{5}$, $-7\sqrt[3]{7}$

From the above examples, it is evident that like radicals have the same order and the same radicands.

For example, the sum of $3\sqrt{5}$ and $2\sqrt{5}$ is $3\sqrt{5} + 2\sqrt{5}$. The sum of $3\sqrt{5}$ and $2\sqrt{5}$ is $(3+2)\sqrt{5}$ or $5\sqrt{5}$.

As in monomials like x in which the coefficient is understood to be 1, $\sqrt{5}$ means $1\sqrt{5}$.

So $6\sqrt{5} + \sqrt{5} = (6+1)\sqrt{5}$ or $7\sqrt{5}$

$$6\sqrt{5} - \sqrt{5} = (6-1)\sqrt{5} \text{ or } 5\sqrt{5}$$

In adding algebraic expressions, the constants are added separately from those with literal coefficients. Similarly, constants are added separately from radical expressions.

For example,

$$3\sqrt{3} - 2\sqrt{3} + \sqrt{3} = (3-2+1)\sqrt{3}$$

$$= 2\sqrt{3}$$

==

Note that when we combine radicals only their coefficients are combined and the common radical is annexed to the result.

==

Sometimes, we may encounter unlike radicals or radicals having different radicands.

Examples of unlike radicals are:

a.) $2\sqrt{7}$, $-3\sqrt{5}$, $-8\sqrt{6}$

b.) $5\sqrt{x}$, $3\sqrt{y}$, $-11\sqrt{6}$

Unlike radicals may become like radicals when they are simplified by applying the properties of radicals.

Example 1. $\sqrt{8} + \sqrt{18}$

$= \sqrt{4 \cdot 2} + \sqrt{9 \cdot 2}$

(4 and 9) ------------------ Perfect squares

$= 2\sqrt{2} + 3\sqrt{2}$

$= (2+3)\sqrt{2}$ <------ combine like radicals

$= 5\sqrt{2}$ <------ distributive law of addition

Example 2. $8\sqrt{10} + \sqrt{40}$

$= 8\sqrt{10} + \sqrt{4 \cdot 10}$

(4) ---------> Perfect squares

$= 8\sqrt{10} + 2\sqrt{10}$

$= (8+2)\sqrt{10}$ <------ combine like radicals

$= 10\sqrt{10}$ <------ distributive law of addition

Example 3. $\sqrt{12} - \sqrt{27} + 5\sqrt{3}$

$= \sqrt{4 \cdot 3} - \sqrt{9 \cdot 3} + 5\sqrt{3}$

(4 and 9) -------------------------> Perfect squares

$$= 2\sqrt{3} - 3\sqrt{3} + 5\sqrt{3}$$

$$= (2-3+5)\sqrt{3}$$ <------ combine like radicals

$$= 4\sqrt{3}$$ <------ distributive law of addition

To add Radicals:

1. Reduce the given radicals, whenever feasible, to similar radicals by any of the procedures of simplication discussed in the properties of radicals.

2. Combine similar radicals according to the distributive law.

WARNING

$\sqrt{16+9}$ ← is the square root of a sum

$$\sqrt{16+9} \neq \sqrt{16} + \sqrt{9}$$

$\sqrt{16} + \sqrt{9}$ ------> is the sum of square roots

The square root of a sum, $\sqrt{16+9} = \sqrt{25} = 5$

The sum of the square roots, $\sqrt{16} + \sqrt{9}$

$$= 4 + 3$$

$$= 7$$

Certainly, $5 \neq 7$

PRACTICE TASK

A. Tell whether the given expressions are like or unlike radicals.

1. $\sqrt{5}$, $2\sqrt{5}$, $-3\sqrt{5}$

2. $\sqrt[3]{7}$, $-\sqrt{7}$, $2\sqrt{7}$

3. $\sqrt{3x}$, $\sqrt{5x}$ $-$ $\sqrt{3x^2}$

4. $\sqrt{x^2}$, $-3\sqrt{x^2}$ $+ 5\sqrt{5}$

5. $\sqrt[3]{5x}$, $5\sqrt[3]{5x}$, $-3\sqrt[3]{5x}$

B. Simplify the radicals and combine all like terms.

6. $\frac{5}{7} - \frac{\sqrt{3}}{7}$

7. $2\sqrt{7} + 3\sqrt{7} - \sqrt{7} =$

8. $\frac{\sqrt{2}}{3} - \frac{\sqrt{3}}{3} =$

9. $\sqrt{20} + 4\sqrt{20} =$

10. $\frac{3\sqrt{4}}{5} - \frac{\sqrt{9}}{5} =$

11. $\sqrt{75} - 2\sqrt{27} =$

12. $2\sqrt{20} - \sqrt{2} - \sqrt{50} =$

13. $\sqrt{20} + \sqrt{45} + \sqrt{80} =$

14. $\sqrt{xy^2} + \sqrt{xz^2} - \sqrt{x^3} =$

15. $5\sqrt[3]{2x} + 2\sqrt[3]{16x} =$

FEEDBACK TO THE PRACTICE TASK

1. like radicals
2. unlike radicals
3. unlike radical
4. like radical
5. like radicals
6. $\dfrac{5 - \sqrt{3}}{7}$
7. $4\sqrt{7}$
8. $\dfrac{\sqrt{2} + \sqrt{3}}{3}$
9. $10\sqrt{3}$
10. 3/5
11. $-1\sqrt{3}$ or $-\sqrt{3}$
12. $2\sqrt{2}$
13. $9\sqrt{5}$
14. $(y + z - x)\sqrt{x}$
15. $9\sqrt[3]{2x}$

If you have answered 75% of the Practice Task correctly, then you have done achieved the objectives of this lesson. If not, please go over the lesson once more.

In this case, you are now ready to proceed to lesson 5 of this module.

MODULE 2

LESSON 5

RADICALS

MULTIPLICATION AND DIVISION OF RADICALS

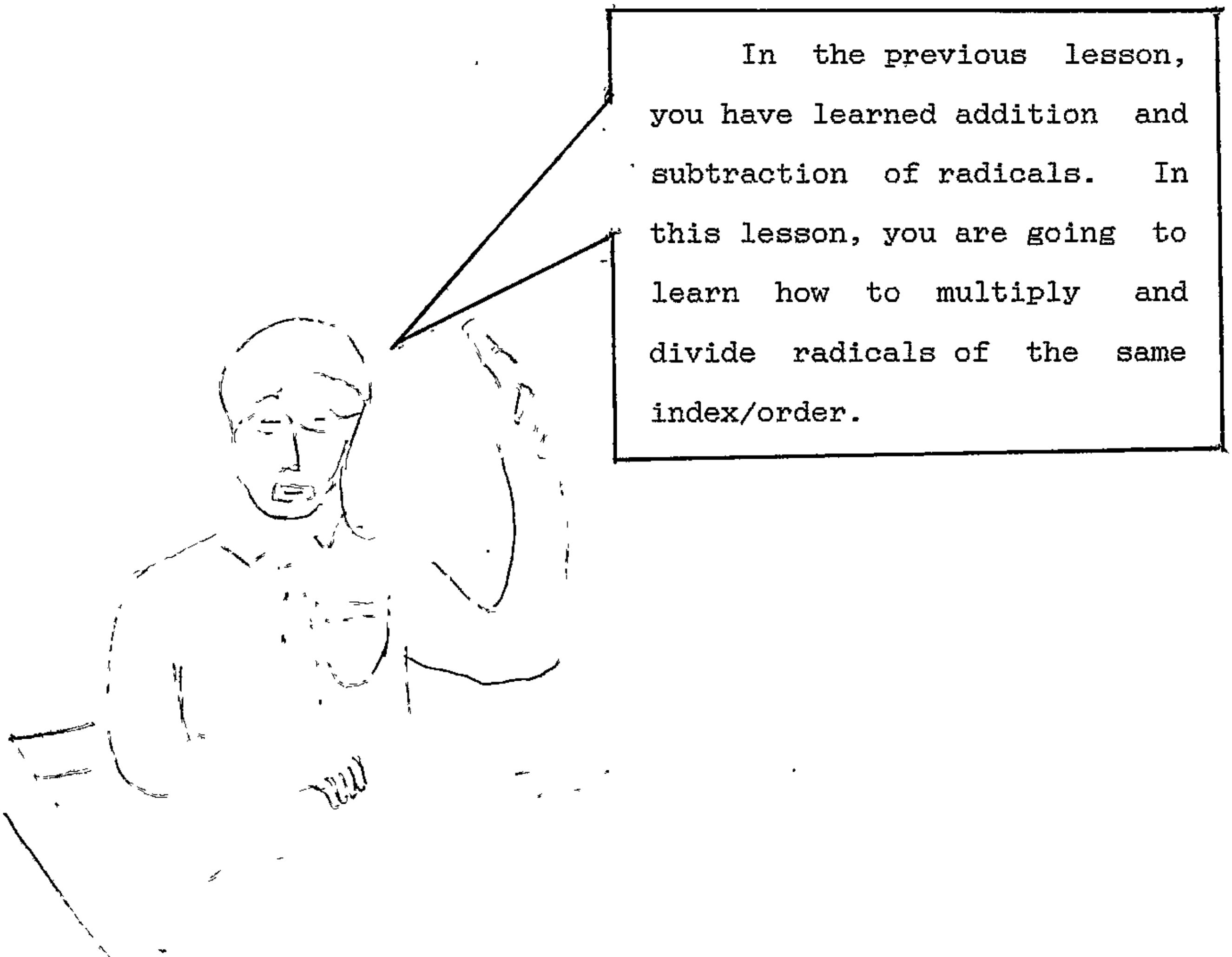
LESSON 5

OBJECTIVES:

5.1. To multiply radicals of the same order.

5.2. To divide radicals of the same order.

Subject Matter	:	Radicals
Sub-topic	:	Multiplication and division of radicals
Time allotment	:	40 minutes



INPUT

Multiplication and Division of Radical

We shall consider multiplication of radicals of the same order. Radicals of the same order are multiplied according to the property.

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$$

Example 1. Multiply:

Product of square roots → left side; square root of product → right side

a.) $\sqrt{2} \quad \sqrt{8} = \sqrt{2 \cdot 8} = \sqrt{16} = 4$

b.) $\sqrt{5} \quad \sqrt{5} = \sqrt{5 \cdot 5} = \sqrt{25} = 5$

c.) $\sqrt{4x} \quad \sqrt{x} = \sqrt{4x^2} = 2x$

d.) $\sqrt{3x} \quad \sqrt{12x^3} = \sqrt{3x \cdot 12x^3} = \sqrt{36x^4} = 6x^2$

e.) $\sqrt{3} \quad \sqrt{6} \quad \sqrt{2} = \sqrt{3.6.2} = \sqrt{36} = 6$

f.) $\sqrt{2} \quad \sqrt{7} = \sqrt{2 \cdot 7} = \sqrt{14}$

g.) $\sqrt{3} \quad \sqrt{x} = \sqrt{3 \cdot x} = \sqrt{3x}$

From the illustrations above, it is evident that the product of two or more radicals of the same order is equal to a radical of same order whose <u>radicand</u> is equal to the <u>product</u> of the <u>radicands.</u>

Example 2 Multiply:

a.) $\sqrt{3x} \quad \sqrt{6x^2} = \sqrt{3x \cdot 6x^2}$ ---> multiply the square roots.

$= \sqrt{18\, x^3}$

$= \sqrt{2.9.\ x^2 \cdot x}$ --> simplifying

$= \quad 3 \quad x \sqrt{2x}$

b.) $3\sqrt{2x} \;.2 \sqrt{5x} = (2 \cdot 3) \sqrt{2x \cdot 5x}$ ---> multiply the square roots.

$= 6 \sqrt{10\, x^2}$

$= 6x \sqrt{10}$ -----------> simplifying

When multipliying radical expressions, the following formulas may be used wherever applicable.

1. Product of sum and difference

   $(x+y)\ (x-y) = x^2-y^2$

2. Square of sum or difference

   $(x+y)^2 = x^2+2xy+y^2$

   $(x-y)^2 = x^2-2xy+y^2$

Example 3. Multiply $(\sqrt{13} + \sqrt{7})(\sqrt{13} - \sqrt{7})$

Solution: Apply the product of sum and difference.

$$(\sqrt{13} + \sqrt{7})(\sqrt{13} - \sqrt{7}) = (\sqrt{13})^2 - (\sqrt{7})^2$$
$$= 13 - 7$$
$$= 6$$

Example 4 Multiply $(\sqrt{3} + \sqrt{2})^2$

Solution: Apply the square of sum:

$$(\sqrt{3} + \sqrt{2})^2 = (\sqrt{3})^2 + 2(\sqrt{3})(\sqrt{2}) + (\sqrt{2})^2$$
$$= \sqrt{9} + \sqrt{3 . 2} + \sqrt{4}$$
$$= 3 + 2\sqrt{6} + 2$$
$$= (3 + 2) + 2\sqrt{6}$$
$$= 5 + 2\sqrt{6}$$

II Dividing Radicals

Since division is the inverse of multiplication, we can see that if $\sqrt{5}\ \sqrt{6} = \sqrt{30}$, then

$$\frac{\sqrt{30}}{\sqrt{5}} = \sqrt{6} \quad \text{and} \quad \frac{\sqrt{30}}{\sqrt{6}} = \sqrt{5}$$

In general, we say that:

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Example 5. Divide the following radicals

a. $$\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$$

b. $$\frac{\sqrt{x^5}}{\sqrt{x^3}} = \sqrt{\frac{x^5}{x^3}} = \sqrt{x^2} = x$$

c. $$\frac{12\sqrt{8}}{6\sqrt{2}} = \frac{12}{6}\sqrt{\frac{8}{2}} = 2\sqrt{2}$$

d. $$\frac{\sqrt{28xy^3}}{\sqrt{7xy}} = \sqrt{\frac{28xy^3}{7xy}} = \sqrt{4y^2} = 2y$$

e. $$\frac{\sqrt{10x}}{\sqrt{20x^2}} = \sqrt{\frac{10x}{20x^2}}$$

$$= \sqrt{\frac{1}{2x}}$$ <---- rationalize the denominator

$$= \frac{\sqrt{1}}{\sqrt{2x}} \cdot \frac{\sqrt{2x}}{\sqrt{2x}}$$

<--- multiply the numerator and denominator by $\sqrt{2x}$

$$= \frac{\sqrt{2x}}{2x}$$

PRACTICE TASK

Find each product and simplify.

1. $\sqrt{3} \cdot \sqrt{3}$
2. $\sqrt{2} \cdot \sqrt{32}$
3. $\sqrt{25} \ \sqrt{x}$
4. $\sqrt{2} \ \sqrt{5} \ \sqrt{10}$
5. $\sqrt{7} \cdot \sqrt{5}$
6. $\sqrt{16x} \cdot \sqrt{x^4}$
7. $\sqrt{3x^2y} \cdot \sqrt{27xy}$
8. $\sqrt{x} \ \sqrt{6} \ \sqrt{3x}$
9. $\sqrt{5x^2y} \ \sqrt{15x}$
10. $(\sqrt{11} + \sqrt{3})(\sqrt{11} - \sqrt{3})$
11. $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$
12. $(\sqrt{11} + \sqrt{3})^2$
13. $(\sqrt{7} + \sqrt{2})^2$

Perform the division and simplify.

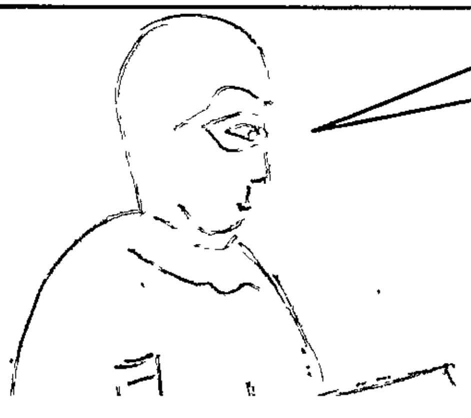
14. $\dfrac{\sqrt{20}}{\sqrt{5}}$
15. $\dfrac{\sqrt{7}}{\sqrt{28}}$
16. $\dfrac{\sqrt{27}}{\sqrt{3}}$
17. $\dfrac{\sqrt{98}}{\sqrt{2}}$
18. $\dfrac{\sqrt{4}}{\sqrt{5}}$
19. $\dfrac{\sqrt{15x}}{\sqrt{5x^2}}$
20. $\dfrac{\sqrt{27x^2y^3}}{\sqrt{3x^2y}}$
21. $\dfrac{\sqrt{x^6y}}{\sqrt{3y}}$
22. $\dfrac{\sqrt{x} \ \sqrt{3x}}{\sqrt{3y}}$

FEEDBACK TO THE PRACTICE TASK

1. 3
2. 8
3. $5x$
4. 10
5. $\sqrt{35}$
6. $4x^2 \sqrt{x}$
7. $9xy \sqrt{x}$
8. $3x \sqrt{2}$
9. $5x \sqrt{3y}$
10. 8
11. 1
12. $2 - 2 \sqrt{5}$
13. $9 + 2 \sqrt{14}$
14. 2
15. 1/2
16. 3
17. 7
18. $\frac{2 \sqrt{5}}{5}$
19. $\frac{\sqrt{3x}}{x}$
20. $3y$
21. $\frac{x^3 \sqrt{5}}{3}$
22. x

If you have answered 75% of the practice task correctly, then you have achieved the objectives of this lesson. If not, please go over the lesson once more.

In this case, you are now ready to proceed to lesson 6 of this module. Good Luck!



MODULE 2

LESSON 6

RADICALS

EQUATIONS INVOLVING RADICALS

LESSON 6

OBJECTIVE:

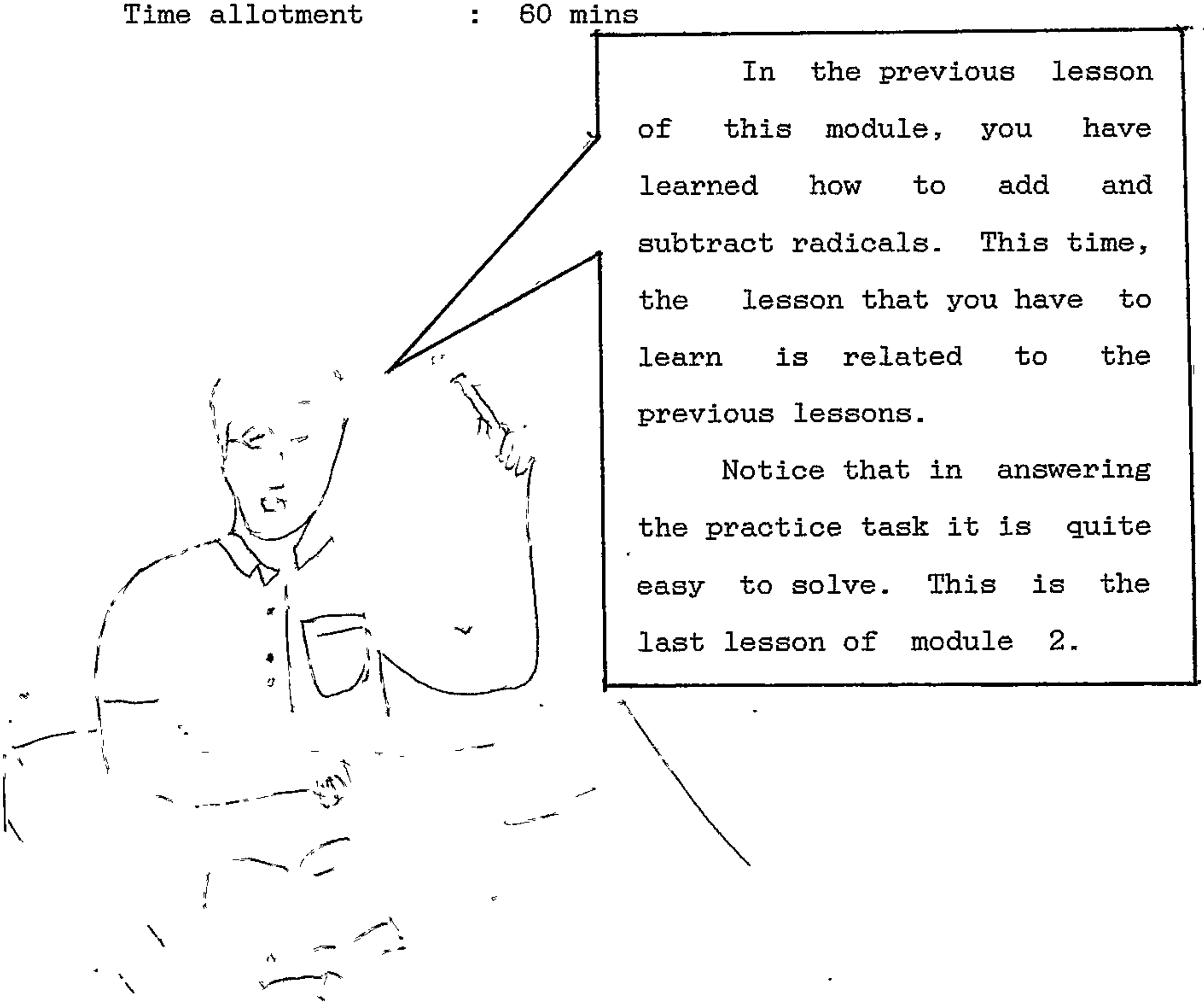
6.1 To recognize radical equation.

6.2 To solve equations involving radicals.

Subject Matter : Radicals

Sub-topic : Equations involving radicals

Time allotment : 60 mins



INPUT

EQUATION INVOLVING RADICALS

An equation in which the variable occurs under a radical sign is called radical equation. For example,

$\sqrt{3x-2} = 9$, $\sqrt{2x+7} = 3$, and $\sqrt{2x-3} = \sqrt{x} + 5$

are radical equations. To solve radical equations, we use the following rule.

==

If $x = y$ then $x^n = y^n$ for any positive integers n. This statement says that every solution of $x = y$ will also be a solution of $x^n = y^n$

==

For all nonzero numbers [x] and [y] and integer [n] if $x = y$ then $x^n = y^n$

We will use this equation as a basis for solving equations involving radicals or fractional exponents. >>

Example 1. Solve for x if $\sqrt{x} = 2$

Square both members --> $(\sqrt{x})^2 = 2^2$

$x = 4$

Check by using substitution in the given equation --> $\sqrt{4} \overset{?}{=} 2$

$2 = 2$

Suppose the given equation is $\sqrt{x} = -2$

then $(\sqrt{x})^2 = (-2)^2$

$x = 4$

> Since we have, defined $x^{1/2}$ to be the principal root, >> the right side should not be negative. 4 is said to be an extraneous root. The equation $x^{1/2} = -2$ has a solution.

Example 2. Find x if $-\sqrt{2x - 1} = -2$

$\sqrt{2x - 1} = +2$

$(\sqrt{2x - 1})^2 = +2$ square both side

$2x - 1 = 4$

$2x = 4 + 1$

$x = 5/2$

To check

$$\sqrt{2x - 1} = 2$$

$$\sqrt{5 - 2} = 2$$

$$\sqrt{4} = 2$$

$$2 = 2$$

So 5/2 is a solution

Example 3. Solve $\sqrt{2x + 7} - x = 2$

Solution: It is best to isolate the radicals before squaring both sides.

$\sqrt{2x - 7} - x = 2$	Given
$\sqrt{2x - 7} = x + 2$	Isolate radical
$(\sqrt{2x - 7})^2 = (x + 2)^2$	Square both sides
$2x + 7 = x^2 + 4x + 4$	Combine similar terms
$0 = x^2 + 2x - 3$	Standard form
$0 = (x + 3)(x - 1)$	Factor
$x + 3 = 0$ ¦ $x - 1 = 0$	Set factors to zero
$x = -3$ ¦ $x = 1$	Possible solution

Check: $\sqrt{2(-3) + 7} - (-3) \stackrel{?}{=} 2$ — Check: $x = -3$ is not a solution

$$\sqrt{1} + 3 = 2$$

$$1 + 3 \neq 2$$

Check: $\sqrt{2(1) + 7} - 1 \overset{?}{=} 2$ Check: x = 1 is a solution

$\sqrt{9} - 1 = 2$

$3 - 1 = 2$

Take note that values that would not satisfy the equation are called **extraneous roots**

Example 4. Solve $\sqrt[3]{2x + 7} = 3$

Solution: Because the equation contains a cubic root, we cube both sides.

$\sqrt[3]{2x - 7} = 3$	Given
$(\sqrt{2x - 7})^3 = 3^3$	Cube both side
$2x + 7 = 27$	Combine similar terms
$x = 10$	Divide by 2

Check: $\sqrt[3]{2(10) + 7} \overset{?}{=} 3$ Check: x = 10 is a solution

$\sqrt[3]{27} = 3$

$3 = 3$

To solve a radical equation:

1. Relocate radicals if necessary, sothat one radical alone on one side of the equal sign.
2. Raise each side of the equation to a power so that at least one radical sign disappears.
3. Repeat steps 1 and 2, if necessary, until all radical signs disappear.
4. All possible solutions must be checked in the original equation.

PRACTICE TASK

Solve the following radical equations.

1. $x^{1/3} = 8$

2. $(-x)^{1/3} = -4$

3. $\sqrt{x - 2} = 0$

4. $\sqrt{x - 2} = 3$

5. $\sqrt{x} = 5$

6. $\sqrt{x + 7} = 3$

7. $\sqrt{-x} = -4$

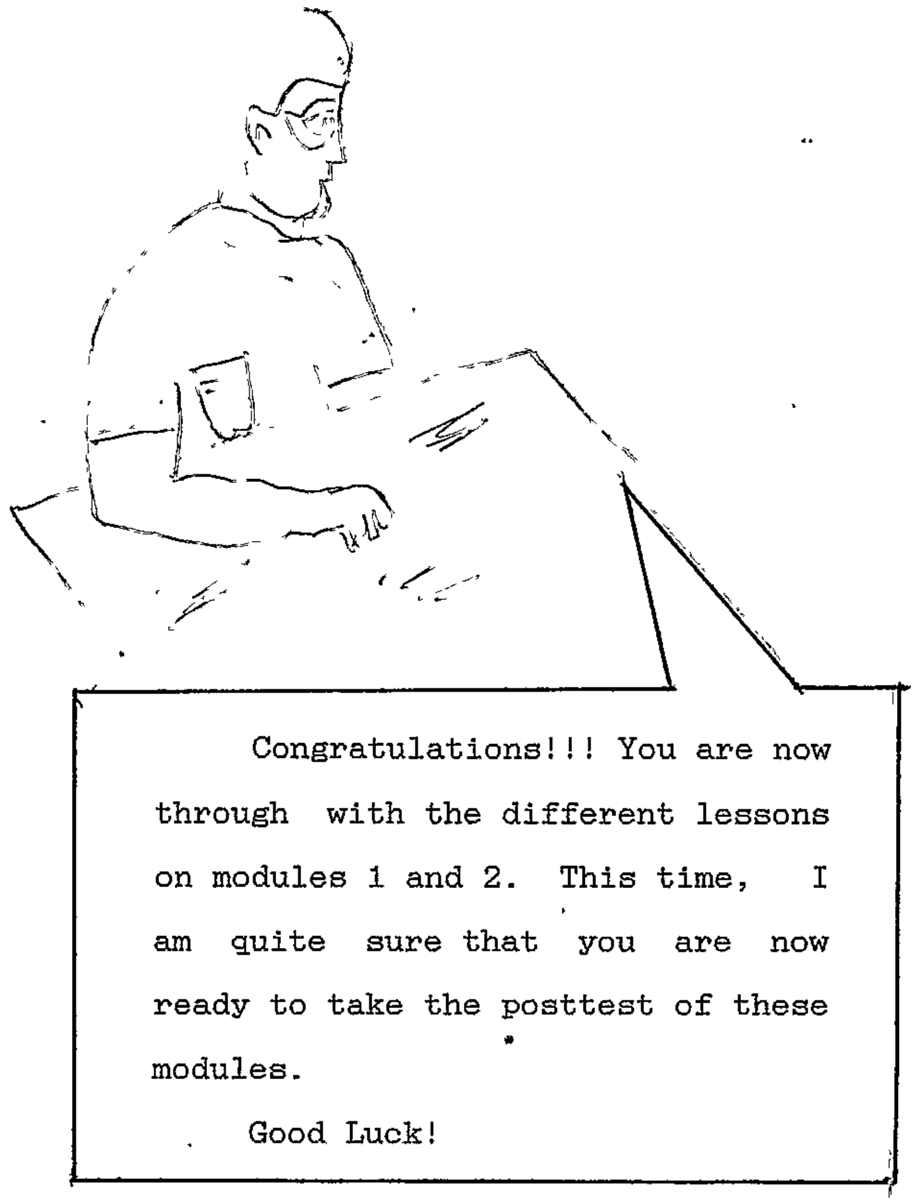
8. $\sqrt{2x - 11} = \sqrt{x + 1}$

9. $-\sqrt{x} = -4$

10. $\sqrt{x + 1} = \sqrt{2x + 6}$

FEEDBACK TO THE PRACTICE TASK

1. 512
2. 64
3. 2
4. 11
5. 125
6. 20
7. 64
8. 12
9. 16
10. -5



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A P P E N D I C E S

APPENDIX A

Republic of the Philippines
SAMAR STATE POLYTECHNC COLLEGE
Catbalogan, Samar

March 22, 1995

The Dean of Graduate Studies
Samar State Polytechnic College
Catbalogan, Samar
Thru Channel

M a d a m :

In my desire to start writing my thesis proposal. I have the honor to submit for your approval one of the following research problems, preferably problem No. 1

1. DEVELOPMENT AND VALIDATION OF A MODULE ON EXPONENT AND RADICALS IN MATHEMATICS III

2. TWO WAY APPROACHES IN TEACHING FACTORING AND SPECIAL PRODUCT

3. THE STATISTICAL STUDY OF THE ENROLMENT OF LA MILAGROSA ACADEMY, CALBAYOG CITY FROM THE SCHOOL YEAR 1975 TO THE PRESENT

I hope for your early and favorable action on this request.

Very truly yours,

(SGD) DANILO R. ALANDINO
Researcher

APPROVED:

(SGD) RIZALINA M. URBIZTONDO
Dean, Graduate & Post Graduate Studies

APPENDIX B

Republic of the Philippines
SAMAR STATE POLYTECHNIC COLLEGE
Catbalogan, Samar

SCHOOL OF GRADUATE STUDIES

APPLICATION FOR ASSIGNMENT OF ADVISER

NAME: ALANDINO (Surname) Danilo (First Name) R. (Middle Name)

CANDIDATE FOR DEGREE: Master of Arts in Teaching

AREA OF SPECIALIZATION: Mathematics

TITLE OF PROPOSED THESIS/DISSERTATION: DEVELOPMENT AND VALIDATION OF MODULES IN EXPONENTS AND RADICAL IN MATHEMATICS III

(SGD.) DANILO R. ALANDINO
Applicant

(SGD.) EUSEBIO T. PACOLOR, Ph. D.
Name of Designated Adviser

APPROVED:

RIZALINA M. URBIZTONDO, Ed. D.
Dean, Graduate Studies

APPENDIX C

Table of Specification for Test Construction of the Diagnostic Test for High School Mathematics III

Sub- Topics	Number of Test Items Know- ledge	Compre- sion	Appli- cation	To- tal
1. Exponent (laws, zero, negative, fractional, simplifying, evaluating, equation	5	5	5	15
2. Radicals (Roots of a number, fracttiton, changing to simlest form, multiplication, division, addition, subttruction, equation)	5	5	5	15
3. Special Product and Factors (Multiplication of Polynomials, special products, factoring polynomials)	5	5	5	15
4. Rational Expression (Simplifying, adding, subtracting, dividing, multiplying, complex fractions, solving equation.	5	5	5	15
5. Quadratic Equation (Standard form, solving quadratic equation, discriminant, geometric meaning of solution of quadratic, forming quadratic.	5	5	5	15
Total	25	25	25	75

APPENDIX D

SAMAR STATE POLYTECHNIC COLLEGE
Catbalogan, Samar

March 06, 1995

Dr. Senecio D. Ayong
President
Tiburcio Tancinco Memorial Institute
of Science and Technology
Calbayog City

S.i.r :

In connection with my masteral thesis entitled "Development and Validation of Module in Mathematics III" for third year high school students at La Milagrosa Academy, I have the honor to request permission to conduct a diagnostic test to the third year-Temperance and third year-Humility on February 28, 1995 at 2:00 to 3:30 P.M.

Request further the availability of rooms and the class advisers of said sections including all practice teachers to assist the test.

The above mentioned test is a vital component of my thesis.

Your approval on this request is highly appreciated.

Very truly yours,

(SGD.) Mr. DANILO R. ALANDINO
(Researcher)

NOTED:

(SGD.) PAQUITA M. BASCO
Principal. Sec. Lab. Sch. Dept.

APPROVED:

(SGD.) Dr. SENECIO D. AYONG
President

APPENDIX E

Number of Retained, Improved, Revised and Rejected Item per Topics

Sub- Topics	Number of Test Items: Re- tained	Im- proved	Re- vised	Rejec- ted
1. Exponent (laws, zero, negative, fractional, simplifying, evaluating, equation	8	5	0	2
2. Radicals (Roots of a number, fracttiton, changing to simlest form, multiplication, division, addition, subttruction, equation)	2	5	6	2
3. Special Product and Factors (Multiplication of Polynomials, special products, factoring polynomials)	4	5	4	2
4. Rational Expression (Simplifying, adding, subtructing, dividing, multiplying, complex fractions, solving equation.	6	4	0	5
5. Quadratic Equation (Standard form, solving quadratic equation, discriminant, geometric meaning of solution of quadratic, forming quadratic.	6	1	4	4
Total	26	20	14	15

APPENDIX F

Diagnostic Test

Name: Section:

Direction: Read each statement carefully. Encircle the letter corresponding to the correct answer.

1. $(x^{-3}\ y^{-2})^0$ is equivalent to

   a. x^3y^4 c. 0

   b. $\frac{1}{x^3y^4}$ d. 1

2. The expression $(2x^3y)^{-2}$ written with positive exponent is equal to:

   a. $\frac{1}{x^3y^4}$ c. $\frac{2}{x^6y^2}$

   b. $\frac{x^6}{4y^2}$ d. $\frac{10\ y^2}{x^4}$

3. The numerical value of $36^{3/2}$ is

   a. 1/216 c. 18

   b. 1/18 d. 216

4. $2^{7/2}\ .\ 2^{1/2}$ is equal to

   a. 2^4 c. 4^4

   b. 2^3 d. 4^3

5. The numerical value of $(1/4)^3$ is

   a. 3/4 c. 4/3

   b. 64 d. 1/64

6. $\dfrac{3^{-3}}{3^{7}}$ is equal to

a. 3^{-10} c. 3^{-4}

b. 3^{10} d. 3^{4}

7. The reduced form of $(4/5)^{-1}$ is;

a. 4/5 c. -4/5

b. 5/4 d. -5/4

8. $-\left(\dfrac{-4x^2}{3y}\right)^2$ is equal to

a. $\left(\dfrac{16x^4}{9y^2}\right)$ c. $\left(\dfrac{-4x^4}{3y^2}\right)$.

b. $\left(\dfrac{16x^4}{9y^2}\right)$ d. $\left(\dfrac{4x^4}{3y^2}\right)$

9. $-(-3x^2y)^3$ is equal to

a. $27x^6y^3$ c. $9x^5y^4$

b. $-27x^6y^3$ d. $-9x^5y^4$

10. $(-2xy)^3\ (-3x^3y)^2$ equals:

a. $72x^9y^5$ c. $-24x^9y^5$

b. $-\ 72x^9y^5$ d. $-18x^9y^5$

11. If $5^x = 125$ then x = ?

a. 3 c. 120

b. 25 d. 625

12. If $\frac{4^x}{4^2} = 4^3$ then x = ?

a. 1/5 c. 5

b. - 1/5 d. -5

13. If $2^{3y+1} = 64$ then y = ?

a. 7/3 c. 5/3

b. 3/7 d. 3/5

14. The law of radicals are applicable to all cases excepts:

a. $\sqrt{x} \cdot \sqrt{y}$ c. $(\sqrt{x} + \sqrt{y})^2$

b. $\frac{\sqrt{x}}{\sqrt{y}}$ d. $(\sqrt{x} \cdot \sqrt{y})^2$

15. The example $\sqrt[3]{64}$, 64 is the:

a. Index c. Radicand

b. Exponent d. Coefficient

16. $\sqrt[4]{3} + \sqrt[4]{3}$ expressed in simplest form is:

a. $2\sqrt[4]{3}$ c. $3\sqrt[4]{3}$

b. $\sqrt[4]{3}$ d. $2\sqrt[3]{4}$

17. The product of $(\sqrt{3} - \sqrt{2})$ and $(\sqrt{3} + \sqrt{2})$ is equal to:

a. 1 c. 3

b. 2 d. 5

18. The sum of $(8 + \sqrt{3})$ and $(8 - \sqrt{3})$ is equal to

a. 0

b. 10

c. 13

d. 16

19. $\sqrt[3]{27} + \sqrt[4]{81}$ is equal to:

a. 0

b. 1

c. 3

d. 6

20. The simplified form of $\sqrt{\frac{5}{16}}$ is

a. $\frac{\sqrt{20}}{4}$

b. $\frac{\sqrt{20}}{2}$

c. $\frac{\sqrt{5}}{4}$

d. $\frac{\sqrt{5}}{2}$

21. The simplest form for $\sqrt[3]{\frac{250}{2}}$ is:

a. $\sqrt[3]{25}$

b. $\sqrt[3]{5}$

c. 5

d. 125

22. The greatest common monomial factor of $2x^2 - 4x$ is:

a. 2

b. 2x

c. $4x^2$

d. $x - 2$

23. The expression $(x + 5)^2$ is an example of a

a. sum of 2 squares
c. sum of 2 cubes
b. difference between 2 perfect squares
d. square of a binomial

24. The factored form of the algebraic expression $6x^2 - 13x - 5$

a. $(3x - 1)(2x + 5)$
c. $(3x - 1)(2x - 5)$
b. $(3x + 1)(2x - 5)$
d. $(3x + 1)(3x + 1)$

25. The factored form of $4a^2 - 20ab + 9b^2$ is

a. $(2a - b)(2a - 9b)$
c. $(2a + b)(2a - 9b)$
b. $(2a - b)(2a + 9b)$
d. $(2a + b)(2a + 9b)$

26. The factored form of $9x^2 - 25y^2$ is:

a. $(9x - y)(x + 25y)$
c. $(9x + 5y)(x - 5y)$
b. $(3x - 5y)(3x + 25y)$
d. $(3x + 5y)(3x - 5y)$

27. The factored form of $y(2x - 3) + 4(2x - 3)$ is:

a. $(y + 4)(2x - 3)^2$
c. $4y(2x - 3)^2$
b. $(y + 4)(2x-3)$
d. $4y(2x - 3)$

28. The product of $(5x + 7y)$ and $(5x - 7y)$ is:

a. $25x^2 - 70xy - 49y^2$
c. $25x^2 + 49y^2$
b. $25x^2 + 70xy - 49y^2$
d. $25x^2 - 49y^2$

29. The product of $(x - 1)$ and $(x + 8)$ is:

a. $x^2 - 8$
c. $x^2 + 7x - 8$
b. $x^2 - 9x - 8$
d. $x^2 - 7x - 8$

30. The expression $(x - 3)^2$ is the same as:

a. $x^2 - 9$ c. $x^2 - 6x + 9$

b. $x^2 + 9$ c. $x^2 + 6x + 9$

31. The factored form of $1 - 0.49x^2$ is:

a. $(1 + 0.7x)(1 - 0.7x)$ c. $(1 + 0.49)(1 - x)$

b. $-(1 + 0.07x)(1 - 0.07x)$ d. $(1 + 0.49)(1 + x)$

32. $2x^2y(-3xy^3 + 7xy^2)$ is equal to

a. $-6x^2 + 14x^2y^2$ c. $-6x^3y^4 + 14x^3y^3$

b. $-5x^2y^3 + 9x^2y^2$ d. $-5x^3y^4 + 9x^3y^3$

33. The sides of a rectangle with an area of $x^2 + 3x + 2$ are:

a. $(x + 1)(x + 2)$ c. $(x - 1)(x + 2)$

b. $(x - 1)(x - 2)$ d. $(x + 1)(x - 2)$

34. The length of a rectangle with an area of $(x^2 + 10x)$ square unit if the width is a unit is:

a. x^2 c. $x + 10$

b. $x^2 + 10$ d. $(x + 10)^2$

35. Reduce $\frac{15a^2b}{48ab^2}$ to the lowest termm is

a. $\frac{15\ a}{48\ b}$ c. $\frac{5\ a}{16\ b}$

b. $\frac{15\ a^3}{16\ b^3}$ d. $\frac{5\ a^3}{16\ b^3}$

36. The reduce form of $\frac{15x - 5}{5x + 25}$ is

a. $\frac{3x}{5}$

c. $\frac{3x}{-5}$

b. $\frac{3x - 1}{x + 5}$

d. $\frac{3x + 1}{x - 5}$

37. The reduced form of $\frac{3x^2 - 27}{x - 3}$ is:

a. $3(x - 3)$

c. $3(x + 3)$

b. $-3(x - 3)$

d. $-3(x + 3)$

38. When simplified $\frac{4(x - y)^2}{3(x - y)}$ equals:

a. $\frac{4(x - y)}{3}$

c. $\frac{4(x - y)^3}{3}$

b. $\frac{4}{3(x - y)}$

d. $\frac{4}{3(x - y)^3}$

39. The simplest form of $\frac{4 - 3y}{8} \cdot \frac{2}{3y - 4}$ equals

a. $\frac{4(x - y)}{4}$

c. $\frac{1}{4}$

b. $\frac{1}{4(xy - 4)}$

d. $\frac{-1}{4}$

40. The simplest form of $\frac{1 - c}{d - d^2} - \frac{1 - c^2}{1 - d^2}$ is

a. $\frac{1 - d}{d\,(1 + c)}$

b. $\frac{1 + c}{1 - d}$

c. $\frac{1 + c}{d}$

d. $\frac{1 + d}{d\,(1 + c)}$

41. $\frac{16}{15x} - \frac{1}{15\,x}$ equals

a. x^2

b. 1

c. x

d. 1/x

42. The product of $\frac{4}{1 - x^2} \cdot \frac{1 + x}{4}$ expressed in simplest form.

a. 1 - x

b. $\frac{1}{1 - x}$

c. x - 1

d. $\frac{1}{x - 1}$

43. $6x + \frac{3x}{4} = ?$

a. $\frac{18\,x^2}{4}$

b. 9/2X

c. 7X

d. 1/8X

44. $\dfrac{9x^2}{75} + 3x = ?$

a. $\dfrac{27x^3}{5}$

c. $\dfrac{3x^3}{5}$

b. $\dfrac{25}{x}$

d. $\dfrac{x}{25}$

45. $\dfrac{x - 10}{x} + \dfrac{10}{x}$ equals

a. 1/x

c. -1

b. -1/x

d. 1

46. If 2.5 kg. of dressed chicken cost P105.00, what will be the cost of 3.75 kgs?

a. P393.75

c. P157.50

b. P262.50

d. P147.00

47. Which one of the following is not an example of quadratic equation?

a. $1 - 4x + 3x^2$

c. $4x^3 + 2x(x - 3x^2) = 8 - 2x^3$

b. $5x^2 - 3 = 7x$

d. $\dfrac{7y - 3}{6} = \dfrac{3y^2 - 2y}{3}$

48. Identify a, b, c, when the equation $x^2 = 4x - 8$ is written in the standard form.

a. (1, 4, 8)

c. (-1, 4, -8)

b. (1, -4, 8)

d. (1, -4, -8)

49. All example below are quadratic except:

a. $37 - 46x = 2x^2$ c. $5x^3 + 2x(x - 4x^2) = 16 - 3x^3$

b. $3x - 7= 0$ d. $1 + x^2 = x$

50. The quadratic equation $x^2 - 9x + 20 = 0$ will have roots to :

a. 4, 5 c. 4, -5

b. -4, -5 d. -4, 5

51. The solution set of the equation $(x - 3)^2 - 49 = 0$ is:

a. 10, -4 c. 4, -5

b. -10, 4 d. 10, 4

52. The discriminant of $x^2 + 6x + 9 = 0$ is:

a. 0 c. 6

b. 1 d. 9

53. The quadratic equation whose roots are 2 and -3 is:

a. $x^2 - x = 0$ c. $x^2 + x - 6 = 0$

b. $x^2 - x + 6 = 0$ d. $x^2 - x + 6 = 0$

54. In the equation $2x^2 - 3x - 10 = 0$, the sum of the roots is;

a. 3/2 c. 1/3

b. -3/2 d. -1/3

55. In the equation $3x^2 + 5x + 4 = 0$, the product of the roots is equal to:

a. 4/3 c. -5/3

b. 4/3 d. 5/4

56. Given the roots $2 + \sqrt{3}$ and $2 - \sqrt{3}$, the equation formed is:

a. $x^2 + 4x + 1 = 0$ c. $x^2 + 4x - 1 = 0$

b. $x^2 - 4x - 1 = 0$ d. $x^2 - 4x + 1 = 0$

57. The solution set of the equation $(m - 3)(m + 4) = 8$ is:

a. (4, 5) c. (-4, -5)

b. (4, -5) d. (-4, 5)

58. To complete the square in $x^2 - 5x + k$, you would replace k with:

a. -5/2 c. -25/4

b. 5/2 d. 25/4

59. Using the quadratic formula, the solution set of $3x^2 - 7x + 2 = 0$ is:

a. (-1/3, -2) c. (6, 1)

b. (1/3, 2) d. (-6, -1)

60. If $y^2 - 4y = 8$, then $(y - 2)^2 = ?$

a. 4 c. 12

b. 8 d. 64

Feedback to the Revised Diagnostic Test

1. d	21. c	41. d
2. a	22. b	42. b
3. d	23. d	43. a
4. a	24. b	44. a
5. d	25. a	45. d
6. a	26. d	46. c
7. b	27. b	47. c
8. b	28. d	48. b
9. a	29. c	49. c
10. b	30. c	50. a
11. a	31. a	51. a
12. c	32. c	52. a
13. c	33. a	53. c
14. c	34. c	54. a
15. c	35. c	55. a
16. a	36. b	56. d
17. a	37. c	57. b
18. d	38. a	58. d
19. d	39. d	59. b
20. c	40. d	60. c

APPENDIX G

SAMAR STATE POLYTECHNIC COLLEGE
Catbalogan, Samar

March 06, 1995

REV. FR. ANTON VERZOSA
Director
La Milagrosa Academy
Calbayog City

Father:

In connection with my masteral thesis entitled "Development and Validation of Module in Mathematics III" for third year high school students at La Milagrosa Academy, I have the honor to request permission to conduct a diagnostic test to the third year-Temperance and third year-Humility on March 8, 1995 at 7:30 to 9:00 A.M.

Request further the availability of rooms and the class advisers of said sections including all practice teachers to assist the test.

The above mentioned test is a vital component of my thesis.

Your approval on this request is highly appreciated.

Very truly yours,

(SGD.) Mr. DANILO R. ALANDINO
(Researcher)

APPROVED:

(SGD.) REV. FR. ANTON VERZOSA
Director

APPENDIX H

Tally of Wrong Responses per Item in the Diagnostic Test (Mathematics III)

Item No.	Wrong res.	Item No.	Wrong res.	Item No.	Wrong res.
1	19	21	18	41	18
2	12	22	12	42	15
3	18	23	16	43	19
4	20	24	13	44	8
5	19	25	11	45	19
6	16	26	16	46	20
7	14	27	15	47	18
8	17	28	17	48	23
9	18	29	11	49	14
10	19	30	9	50	19
11	16	31	9	51	8
12	13	32	17	52	18
13	20	33	19	53	17
14	15	34	17	54	11
15	10	35	19	55	14
16	21	36	21	56	10
17	16	37	16	57	10
18	16	38	12	58	14
19	21	39	12	59	17
20	17	40	18	60	15

APPENDIX I

Ranking and Computation of Percentage of Wrong Response Per Sub-Topics on the Diagnostic Test Result

Sub-Topic	Total No of wrong Responses (TWR)	Maximum possible wrong response (MPW)	Percentage of wrong response (PWR)	Rank
1. Exponents	221	377	59%	2
2. Radicals	144	232	62%	1
3. Factoring	182	377	48%	5
4. Rational Ex-pression	197	346	57%	3
5. Quadratic Equation	208	406	51%	4

Formula Used to find percentage of wrong responses:

$$PWS = \frac{TWR}{MPW}$$

Where:

PWS - percentage of wromg response

TWR - total no. of wrong response per sub-topics

MPW - maximum possible wrong response

APPENDIX J

Computation of Mean and Standard Deviation

Scores	Frequency (f)	Md. Pts (x)	fx	d	fd	$(fd)^2$
41 - 43	5	42	210	4	20	80
38 - 40	2	39	78	3	6	18
35 - 37	4	36	144	2	8	16
32 - 34	1	33	33	1	1	1
29 - 31	7	30	210	0	0	0
26 - 28	0	27	0	-1	0	0
23 - 25	4	24	48	-2	-8	16
20 - 23	6	21	126	-3	-18	54
	29		fx = 849		fd = 9	$(fd)^2$ = 185

Mean

$$\text{Mean} = \frac{fx}{n} = \frac{849}{29} = 29.27$$

Standard Deviation

$$SD = \sqrt{\frac{fd^2}{n} - \frac{(fd)^2}{n}}$$

$$= 3 \quad \frac{185}{29} - \left(\frac{9}{29}\right)^2$$

$$= 3\sqrt{6.38 - (.31)^2}$$

$$= 3\ (2.51)$$

$$= 7.52$$

APPENDIX K

Computation of the Reliability Coefficient of the Diagnostic Test

Score	Frequency	Score	Frequency
20	1	32	0
21	3	33	1
22	2	34	0
23	0	35	2
24	2	36	0
25	2	37	2
26	0	38	0
27	0	39	0
28	0	40	2
29	0	41	3
30	2	42	1
31	5	43	1

N = 60 (No. of Items)

S = 7.52 (Standard Deviation)

X = 29.27 (Mean)

$$r_{11} = \left(\frac{n}{n-1}\right)\left(\frac{s^2 - npq}{s^2}\right)$$

where:

n = 60 (no. of item of the test

s = 7.52 (standard deviation of the test)

p = $\overline{X}/n$ = proportion of the group passing an item; x is the mean of the test.

$$p = \frac{29.27}{60} = 0.49$$

q = 1- p = proportion of the group failing an item

$$q = 1-0.49 = 0.51$$

r^{11} = reliability coefficient of the test.

$$r_{11} = \left(\frac{60}{60 - 1}\right)\left(\frac{(7.72)^2 - 60\ (.49)(.51)}{(7.51)^2}\right)$$

$$r_{11} = \left(\frac{60}{59}\right)\left(\frac{56.55 - 15}{56.55}\right)$$

$$r_{11} = \left(\frac{60}{59}\right)\left(\frac{41.55}{56.55}\right)$$

$$r_{11} = \left(\frac{2,493.00}{3,336}\right)$$

$$r_{11} = 0.75$$

APPENDIX L

Interpretation of the Coefficient of Reliability

Reliability Coefficient	Degree of Reliability
0.95 - 1.00	Very high, rarely found among teacher made tests
0.90 - 0.94	High, equalled by few test
0.80 - 0.89	Fairly hig, adequate for individual measurement
0.79 - 0.79	Rather low, adequate for group measurement but not very satisfactory for individual measurement.
below - 0.70	Low, entirely inadequate for individual measurement although useful for group average and school survey.

APPENDIX M

Table of Specification for Test Construction of the Pretest/posttest in Exponent and Radicals

Lessons/Sub-topics	Number of Items: Knowledge	Comprehension	Application	Total
1. Concept of exponent	1	2	1	4
2. Laws of exponent	1	2	1	4
3. Forms of exponent	1	2	1	4
4. Simplifying and evaluation expressions with exponents	1	2	1	4
5. Equation involving exponents	1	2	1	4
6. Scientific Notation	1	2	1	4
7. Roots of numbers	1	2	1	4
8. Changing radicals to simplest form	1	2	1	4
9. Multiplication and Division of radicals	2	4	2	8
10. Addition and subtraction of radicals	1	2	1	4
11. Equation involving radicals.	1	2	1	4
Total	13	26	13	52

APPENDIX N

SAMAR STATE POLYTECHNIC COLLEGE
Catbalogan, Samar

March 9, 1995

THE VICE-DIRECTOR
Christ the King College
Calbayog City

Madam:

In connection with my masteral thesis entitled "DEVELOPMENT AND VALIDATION OF MODULE IN MATHEMATICS III" for third year high school students at La Milagrosa Academy, I have the honor to request permission to conduct a tryout of my pretest/posttest to the first year BSE, BSEED, AB students on March 10, 1995 at 10:00 to 11:30 AM.

The above mentioned test is a vital component of my masteral thesis paper.

Your approval on this request is highly appreciated.

Very truly yours,

Mr. DANILO R. ALANDINO
(Researcher)

NOTED:

Mr. ALIAS BRONCANO
Dean of College

APPROVED

Dr. FRANCISCA SANTOS
Vice-Director of Academic Affair

APPENDIX O

Data of Math Grade, Pretest, Posttest and Retest of Experimental Group

Respon-dent	Math Grade	Pretest M-1	Pretest M-2	Posttest M-1	Posttest M-2	Retest M-1	Retest M-2
1	80	9	5	10	11	10	10
2	80	6	2	16	10	15	9
3	80	6	7	13	10	10	10
4	80	3	0	15	17	11	13
5	80	10	10	18	15	8	11
6	80	12	2	15	15	12	16
7	81	2	0	16	10	16	16
8	81	2	3	16	10	16	8
9	81	9	2	11	10	12	14
10	81	9	4	14	11	19	15
11	81	8	7	15	13	17	13
12	82	5	1	17	10	8	9
13	82	5	8	10	13	16	15
14	82	6	8	17	15	16	12
15	82	6	8	14	14	17	9
16	82	6	1	15	16	13	13
17	83	5	4	10	12	15	10
18	83	8	7	15	12	16	12
19	83	7	2	15	10	14	9
20	83	7	2	10	18	15	13
21	84	3	1	17	14	13	8
22	84	7	2	14	16	11	10
23	85	4	5	11	15	14	9
24	85	2	5	10	10	9	11
25	85	6	6	17	15	10	13
Total		153	102	357	322	345	286
Mean		6.12	4.08	14.28	12.88	13.8	11.4
SD		2.55	2.8	2.6	2.55	3.67	2.49

APPENDIX P

Data of Math Grade, Pretest, Posttest and Retest of Control Group

Respondent	Math Grade	Pretest M-1	Pretest M-2	Posttest M-1	Posttest M-2	Retest M-1	Retest M-2
1	80	4	5	17	16	8	14
2	80	4	3	13	6	6	14
3	80	9	4	10	14	13	4
4	80	7	0	11	12	12	8
5	80	2	4	16	8	7	5
6	80	6	5	15	8	10	12
7	81	2	7	13	7	8	14
8	81	8	7	16	9	8	5
9	81	4	6	10	11	14	10
10	81	7	4	14	16	9	7
11	81	6	3	10	13	10	6
12	82	8	4	15	13	10	5
13	82	5	2	8	13	12	13
14	82	9	7	12	7	14	7
15	82	7	7	16	8	13	7
16	82	6	2	14	8	10	14
17	83	6	3	8	13	11	7
18	83	3	1	12	7	12	10
19	83	3	3	8	8	8	6
20	83	7	5	10	6	12	4
21	84	9	5	12	8	17	10
22	84	4	0	10	8	9	5
23	85	6	0	10	9	10	6
24	85	9	4	16	14	14	7
25	85	9	5	16	10	12	16
Total		150	96	312	252	269	216
Mean		6.00	3.84	12.48	10.08	10.75	8.64
SD		2.27	2.14	2.72	3.05	2.58	3.64

APPENDIX Q

Computation of t-test for Hypothesis 1, for Module 1 and Module 2

Module 1

$$t = \frac{\bar{x} - \bar{x}}{\sqrt{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_1}}}$$

$$t = \frac{6.12 - 6.00}{\sqrt{\frac{(2.55)^2}{25} + \frac{(2.27)^2}{25}}}$$

$$t = \frac{0.12}{\sqrt{\frac{6.50}{25} + \frac{5.15}{25}}}$$

$$t = \frac{0.12}{\sqrt{.466}}$$

$$t = \frac{0.12}{0.68}$$

$$t = 0.18$$

Module 1

$$t = \frac{\bar{x} - \bar{x}}{\sqrt{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_1}}}$$

$$t = \frac{4.08 - 3.84}{\sqrt{\frac{(2.8)^2}{25} + \frac{(2.14)^2}{25}}}$$

$$t = \frac{0.24}{\sqrt{0.31 + 0.18}}$$

$$t = \frac{0.24}{0.7}$$

$$t = 0.34$$

APPENDIX Q-A

Computation of t-test for Hypothesis 2 for Module 1 and Module 2

Module 1

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1 + s_2}{N_1} - (2r_{12})\left(\frac{s_1}{N_1}\right)\left(\frac{s_2}{N_2}\right)}}$$

$$t = \frac{6.00 - 12.48}{\sqrt{\frac{2.25) + 10.08}{25} - 2\,(0.49)\left(\frac{2.22}{25}\right)\left(\frac{10.08}{25}\right)}}$$

$$t = \frac{-6.48}{\sqrt{0.49 - 0.98\,(0.09)(0.40)}}$$

$$t = \frac{-6.48}{\sqrt{0.49 - 0.035}}$$

$$t = \frac{-6.48}{0.67} = 9.67$$

Module 2

$$t = \frac{3.84 - 2.72}{\sqrt{\frac{2.14 + 3.05}{25} - 2\,(0.50)\left(\frac{2.14}{25}\right)\left(\frac{3.05}{25}\right)}}$$

$$t = \frac{1.12}{\sqrt{0.121 - 1\,(0.08)(0.12)}}$$

$$t = \frac{1.12}{\sqrt{0.21 - 0.0096}}$$

$$t = \frac{1.12}{0.45} = 2.49$$

APPENDIX Q-B

Computation of t-test for Hypothesis 4, for Module 1 and Module 2

Module 1

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1 + s_2}{N_1} - (2r_{12}) \left(\frac{s_1}{N_1}\right)\left(\frac{s_2}{N_2}\right)}}$$

$$t = \frac{6.12 - 14.28}{\sqrt{\frac{(2.25) + 2.60}{25} - 2\,(0.49) \left(\frac{2.55}{25}\right)\left(\frac{2.60}{25}\right)}}$$

$$t = \frac{8.16}{\sqrt{0.21 - 0.98\,(0.10)(0.10)}}$$

$$t = \frac{8.16}{\sqrt{0.21 - 0.010}}$$

$$t = \frac{8.16}{0.47} = 18.13$$

Module 2

$$t = \frac{4.08 - 12.88}{\sqrt{\frac{2.80 + 2.55}{25} - 2\,(0.50) \left(\frac{2.80}{25}\right)\left(\frac{2.55}{25}\right)}}$$

$$t = \frac{8.8}{\sqrt{0.21 \quad - 1\ (0.11)(0.10)}}$$

$$t = \frac{8.8}{\sqrt{0.21 - 0.011}}$$

$$t = \frac{8.8}{0.45} \quad = \quad 19.56$$

APPENDIX Q-C

Computation of t-test for Hypothesis 4, for Module 1 and Module 2

Module 1

$$t = \frac{x - x}{\sqrt{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_1}}}$$

$$t = \frac{14.28 - 12.48}{\sqrt{\frac{(2.6)^2}{25} + \frac{(2.72)^2}{25}}}$$

$$t = \frac{1.8}{\sqrt{0.27 + 0.30}}$$

$$t = \frac{1.8}{0.75}$$

$$t = 2.4$$

Module 2

$$t = \frac{x - x}{\sqrt{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_1}}}$$

$$t = \frac{12.88 - 10.08}{\sqrt{\frac{(2.55)^2}{25} + \frac{(3.05)^2}{25}}}$$

$$t = \frac{2.8}{\sqrt{0.26 + 0.37}}$$

$$t = \frac{2.8}{0.79}$$

$$t = 3.54$$

APPENDIX Q-D

Computation of t-test for Hypothesis 5, for Module 1 and Module 2

Module 1

$$t = \frac{x - x}{\sqrt{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_1}}}$$

$$t = \frac{13.8 - 10.75}{\sqrt{\frac{(3.09)^2}{25} + \frac{(2.58)^2}{25}}}$$

$$t = \frac{3.05}{\sqrt{0.38 + 0.37}}$$

$$t = \frac{3.05}{0.87}$$

$$t = 3.5$$

Module 2

$$t = \frac{x - x}{\sqrt{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_1}}}$$

$$t = \frac{11.4 - 8.64}{\sqrt{\frac{(2.49)^2}{25} + \frac{(3.64)^2}{25}}}$$

$$t = \frac{2.76}{\sqrt{0.25 + 0.53}}$$

$$t = \frac{2.76}{0.88}$$

$$t = 3.14$$

APPENDIX R

Computation for Determining the Coeffecient of Correlation in the Control Group for Module 1 and 2

Module 1

Respondent	Pre-test	Post-test	d	d^2
1	4	17	13	169
2	4	13	9	81
3	7	10	1	1
4	7	11	4	16
5	2	16	14	196
6	6	15	9	81
7	2	13	11	121
8	8	16	8	64
9	4	10	6	36
10	7	14	7	49
11	6	10	4	16
12	8	15	7	49
13	5	8	3	9
14	9	12	3	9
15	7	16	9	81
16	6	14	8	64
17	6	8	2	4
18	3	12	9	81
19	3	8	5	25
20	7	19	3	9
21	9	12	3	9
22	4	10	6	36
23	6	10	4	16
24	9	16	7	49
25	9	16	7	49
			E d^2=	1320

$$r_{12} = 1 - \frac{6\,(E\,d^2)}{N\,(N^2 - 1)}$$

$$r_{12} = 1 - \frac{6\,(1\ 320\ )}{25\ \ (25^2 - 1)}$$

$$r_{12} = 1 - \frac{7920}{25\ (624)}$$

$$r_{12} = 1 - 0.51$$

$$= 0.49$$

Module 2

Respondent	Pre-test	Post-test	d	d^2
1	5	16	9	81
2	3	6	3	9
3	4	14	10	100
4	0	12	12	144
5	4	8	4	16
6	5	8	3	9
7	7	7	0	0
8	7	9	2	4
9	6	11	5	25
10	4	16	12	144
11	3	13	10	100
12	4	13	9	81
13	2	13	11	121
14	7	7	0	0
15	7	8	1	1
16	2	8	6	36
17	3	13	10	100
18	1	7	6	36
19	3	8	5	25
20	5	6	1	1
21	5	8	3	9
22	0	8	8	64
23	0	9	9	81
24	4	14	10	100
25	5	10	5	25
			E d^2=	1312

$$r_{12} = 1 - \frac{6\ (E\ d^2)}{N\ (N^2 - 1)}$$

$$r_{12} = 1 - \frac{6\ (1\ 312\ )}{25\ \ (25^2 - 1)}$$

$$r_{12} = 1 - \frac{7872}{25\ (624)}$$

$$r_{12} = 1 - 0.50$$

$$= 0.50$$

APPENDIX R-A

Computation for Determining the Coeffecient of Correlation in the Experimental Group for Module 1 and 2

Module 1

Respon- dent	Pre- test	Post- test	d	d^2
1	9	10	1	1
2	6	16	10	100
3	6	13	7	49
4	3	15	12	144
5	10	18	8	64
6	12	15	3	9
7	2	16	4	16
8	2	16	14	196
9	9	17	8	64
10	9	14	5	25
11	8	15	5	25
12	5	17	12	144
13	5	10	10	100
14	6	17	11	121
15	6	14	8	64
16	6	15	9	81
17	5	10	5	25
18	8	15	7	49
19	7	15	6	36
20	7	10	3	9
21	3	17	14	196
22	7	14	7	49
23	4	11	7	49
24	2	10	8	64
25	6	17	11	121
			$E\ d^2 =$	1,801

$$r_{12} = 1 - \frac{6\ (E\ d^2)}{N\ (N^2\ -1)}$$

$$r_{12} = 1 - \frac{6\ (1\ 801\ )}{25\ \ (25^2\ -1)}$$

$$r_{12} = 1 - \frac{10,806}{25\ (624)}$$

$$r_{12} = 1 - 0.69$$

$$= 0.31$$

Module 2

Respondent	Pre-test	Post-test	d	d^2
1	5	11	6	36
2	2	10	8	64
3	7	10	3	9
4	0	17	17	289
5	10	15	5	25
6	2	15	3	9
7	0	10	10	100
8	3	10	7	49
9	2	10	8	64
10	4	11	7	49
11	7	13	8	64
12	1	10	9	81
13	8	13	5	25
14	8	15	7	49
15	8	14	6	36
16	1	16	15	225
17	4	12	8	64
18	7	12	4	16
19	2	10	8	64
20	2	18	16	256
21	1	14	13	169
22	2	16	14	196
23	5	15	10	100
24	5	10	5	25
25	6	15	9	81
			E d^2=	1,801

$$r_{12} = 1 - \frac{6\ (E\ d^2)}{N\ (N^2 - 1)}$$

$$r_{12} = 1 - \frac{6\ (2305\ )}{25\ \ (25^2 - 1)}$$

$$r_{12} = 1 - \frac{13,830}{25\ (624)}$$

$$r_{12} = 1 - 0.89$$

$$= 0.11$$

HONORS AND AWARDS RECEIVED

First Honorable Mention. .	Fourth year high school
Recognition Award.	Fourth year high school
Leadership Award	Fourth year high school
Loyalty Award	Fourth year high school

POSITION HELD

Office Helper	Bureau of Lands Calbayog City 1985 - 1987
Office Helper	City Treasurer Office Calbayog City 1986-1987
Payroll Master	Marjocris Manufacturing Novaliches, Manila 1987 - 1989
Secondary School Teacher .	La Milagrosa Academy Calbayog City 1988 - to the present

TRAINING/SEMINAR ATTENDED

Secondary Education Development . . Program (SEDP) in Integrated Mathematics II	Palo, Leyte April 1 to 15, 1991

LIST OF TABLES

LIST OF FIGURES