EFFECTIVENESS OF WORKBOOK IN TEACHING MATHEMATICS I

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By

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APPROVAL SHEET

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DEDICATION

This humble piece of work is heartily dedicated to the very special people who stood by the researcher even during the most trying moments of this study:

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Ephine

ABSTRACT

This study attempted to establish the relationship among the performances of secondary school administrators, of secondary school teachers, and of fourth year students in the NSAT- based achievement test in selected national high schools in the Division of Samar during the school year 1995-1996. On the correlation between the PASKO ratings of secondary school administrators and the average ratings of the students in the achievement test in every school, the results were the following: Pearson r+ 0.66 resulted when the official PASKO ratings of secondary school heads and the students average ratings in the achievement test were tested for their correlation, which indicated a substantial but not significant correlation. Pearson r + 0.17 resulted when the combined performance ratings of the secondary school administrators and students average ratings in the achievement test were tested for their correlation, which indicated a low, insignificant correlation. There was no significant correlation between the performance of the secondary school administrators and the average ratings of the students in the NSAT- based achievement test in every school. But two sets of variables resulted in substantial correlation between the performances of school administrators and students, although their actual relationship was not direct, because the teachers were directly in charge of the students. The official PASKO ratings of high school administrators and the official PAST ratings of high school teachers under them were deliberately increased compared to the perception and actual experiences of teacher-raters and students-raters respectively; and these facts were also reflected in the actual, very low ratings of the students in the NSAT-based achievement test.

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The Problem: Its Background

Introduction

Everyone has the right to quality education. So every nation must provide quality education for its citizens. The kind of education that is needed today must prepare people to cope with rapid social change. It must provide them with appropriate skills, positive values that will lead to productive and meaningful lives as responsible citizens of any nation (Aquino, 1990: 3).

In connection with this, former President Aquino issued Proclamation No. 480 which declared the period 1990 - 1999 as the "Decade of Education for All" in keeping with the provision in the 1987 Philippine Constitution which provides that the state shall protect and promote the right of all citizens to quality education at all levels.

The Educational Commission of 1990 was created to survey the state of our educational system. The result of the survey conducted pointed toward improving our educational system. It identified one major problem of education in the Philippines today and that is attaining quality education in general and the classroom instruction in particular.

Furthermore, the concern for quality education was not the sole dream of President Corazon C. Aquino because President Fidel V. Ramos also joined the worldwide movement for "Education for All" by launching the program Education 2000, one of the strategic plans of the Philippine government toward achieving the goal.

Educational institutions are mandated to deliver adequate, relevant and effective education in Science, Mathematics and Technology to cope with the changes brought about by the rapid development of society.

In the United States, Coronel (1993: 1) pointed out, there was a sense of urgency in their desire for changes. As a mather of fact, three documents came out in 1989 and 1990 which embodied the new vision for mathematics that the Americans thought they would need to sustain their position in the international scene. These documents are "Everybody Counts", "Curriculum, and Evaluation Standards for School Mathematics" and "Professional Standard for Teaching Mathematics".

The Philippine Educational System has instituted some reforms for new goals will necessitate changes in the teaching styles, techniques and strategies in teaching the different subjects. In mathematics education, Coronel (1993: 116) pointed out that changes were reflected in the objectives, curriculum, strategies and approaches, preparation of teachers and extensive researches.

In the teaching of mathematics, the major shift is placed on the learning process. Understanding the learning process calls for new visions. Mathematics learning is now viewed as an active constructive process. Hence, teaching strategies which enable students to formulate new ideas based on their previous knowledge and experiences are recommended.

The particular solution to the problem must start in the classroom. Hence we must focus our attention to the primary elements present in every classroom: the learner, the teacher and the instruction. Of the three elements, the learner is the center of the educative process, hence provision for his growth and total development is considered the basic aim of education. The learners must be provided with means to progress at their own rate. This is only possible if opportunities can be provided to take care of individual differences in the form of independent work and individualized instruction. One such strategy is using developed materials in the classroom although there are others. Researches hinted that this strategy caters to the needs of both fast and slow learners.

The fundamental role of the teacher is helping students to develop and clarify their understanding of the concepts presented and taught. Hence, the teacher should present his lessons such that he will start with students' prior knowledge, experiences and beliefs and then attempt to make it attractive, enjoyable and challenging enough to entice students to make liking for the subject.

The selection of teaching strategy which will be used for a particular subject matter depends on a number of factors. To mention some: the teacher's objectives; the teacher's understanding on how their students learn; the teacher's desire to teach in a way which caters for the individual needs of students in the classroom.

Based on experiences and observations not one strategy is suitable in all situations. Notwithstanding with the many strategies available, the strategy that teachers usually adopt is one which matches its objectives.

It is a known fact that students learn in a variety of ways and that they come to our mathematics classroom with varied entry knowledge. Hence, teachers should use a variety of teaching strategies and resources. Quality teaching (and quality learning) often demands risk taking by the teacher (and the learner). Nowhere is this risk more obvious than for teachers exploring new and different teaching strategies. Many a current trend of mathematics education explicitly calls for such new teaching strategies

(Grant, 1990: iii).

In response to the call for quality mathematics education through quality instruction and modern instructional material, the researcher wants to help improve mathematics teaching by using a developed instructional material that could provide the students the chance to improve their optimum potential capabilities. This is possible through an instructional material which can be used independently by the students themselvves and thereby help them dirrect their own learnng capabilities. The use of teacher-made wokkbook came into the mnd of the researcher. So with this developed instructional material, the researcher has high hopes that it will cater to the varied needs and abilities of students in mathematics.

Statement of the Problem

This study determined the effectiveness of a developed workbook in Mathemmatics I on tthe academic performance of first year high school students of Calapi National High School, Calapi, Motiong, Samar. Specifically, it sought answers to the following questions:

1. What is the profile of students comprising the experimental group and the control group, according to:

1.1 Age;

1.2 Sex;

1.3 Average Rating of the Firrst and Second Grading Periods in Mathematics I;

1.4 Pretest Results;

1.5 Posttest Results;

2. Is there a significant difference between the pretest mean scores of the experimental group and the control group?

3. Is there a significant difference between the pretest and the posttest mean scores of the control group?

4. Is there a significant difference between the pretest and the posttest mean scores of the experimental group?

5. Is there a significant difference between the posttest mean scores of the experimental group and the control group?

6. What implications for instructional redirections may be derived from the results of the study?

Null Hypotheses

The following null hypotheses were formulated and tested based on the above specific questions.

1. There is no significant difference between the pretest mean scores of the experimental group and the

control group.

2. There is no significant difference between the pretest and posttest mean scores of the control group.

3. There is no significant difference between the pretest mean scores and posttest mean scores of the experimental group.

4. There is no significant difference between the posttest mean scores of the experimental group and the control group.

Theoretical Framework

The theoretical anchorage of the study hinges along the belief that an individual can be developed to the limits of his capacity (Gregorio, 1967: 213). In the case of students, they must be helped towards maximizing their achievement in the classroom. This can be accomplished if teachers will provide students opportunities to attain mastery of concepts. This means that teachers should consider reviewing and structuring their teaching procedures so that students will develop the right attitudes and acquire the basic knowledge and skills necessary for survival in our present world of expanding technology.

Conceptual Framework

The conduct of this study is guided by the research paradigm shown in Figure 1. At the base of this paradigm is the research environment and the subjects of the study, the first year high school students composed of two intact classes of Calapi National High School, Calapi, Motiong, Samar, during the SY 1997-1998. The upper frame demonstrates the experimentation process designed to compare the two approaches of teaching Mathematics I, namely: the traditional approach used with the control group, First Year - Section I and the individualized instruction with the use of the developed workbook adapted to the experimental group, First Year Section II.

The experimentation considered the teaching approach as the treatment, the pretest as the independent variable and the posttest as the dependent variable of the study. The pretest was considered as the independent variable since it was supposed not to be affected by the treatment because it was based on students' prior knowledge of the content topic. The posttest was considered as the dependent variable since it was supposed to be affected by the treatment.

The two groups of students were categorized as the experimental group and the control group. Each group was given the validated pretest. The pretest results were the input used to proceed to the next step which was the actual teaching done to the two groups using the approach. The

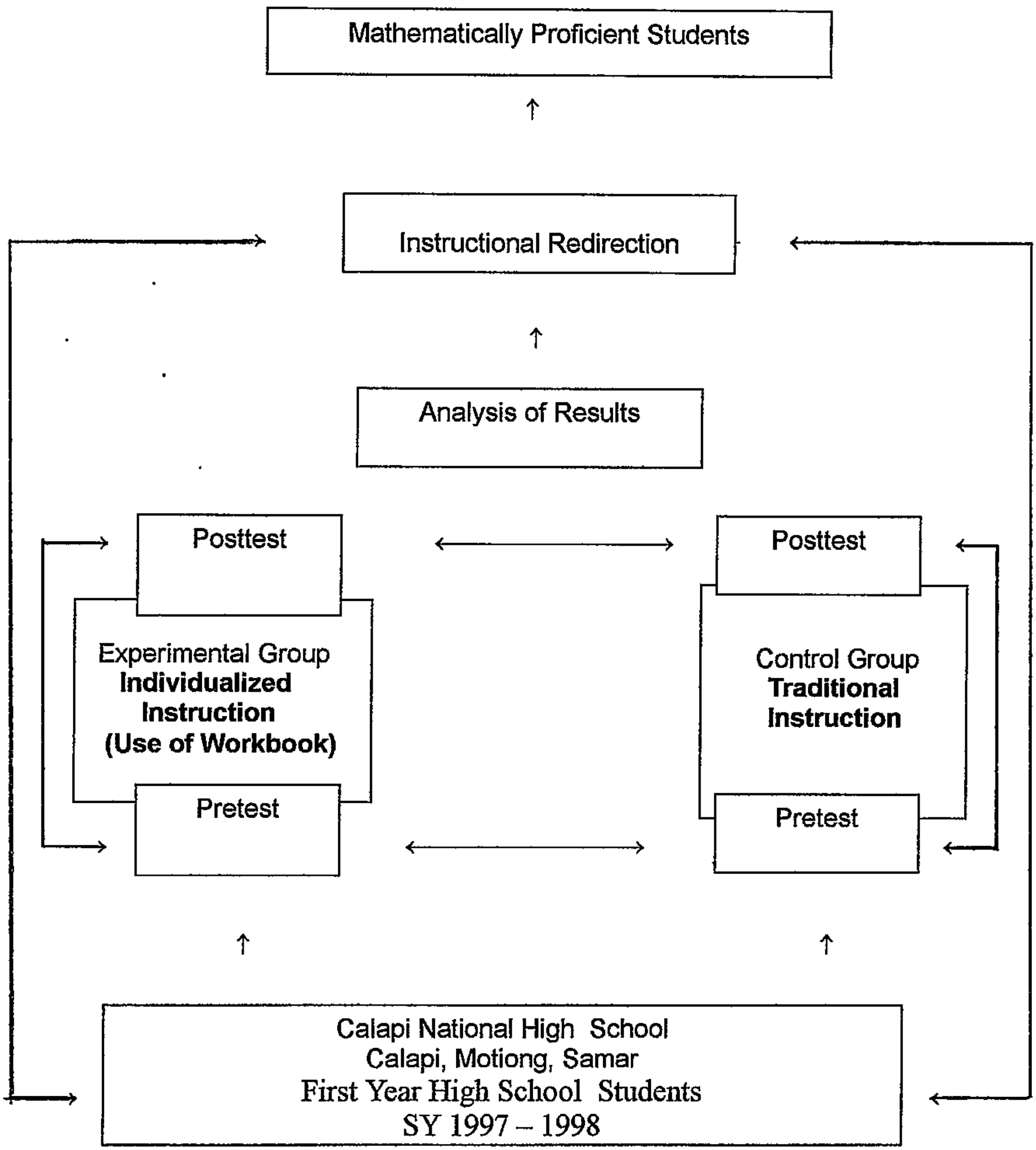


Figure 1. The Conceptual Paradigm of the Study Showing the Research Environment, the Experimentation Process and the Expected Research Outputs.

control group was taught using the traditional lecture - discussion method of the mathematics concepts on the topic being investigated. The experimental group was given the developed workbook on the topic investigated. After all the topics had been taught a posttest was given to both groups using the same test instrument to determine the performance of each group as a result of the applied approaches in this study.

The results of the pretest and posttest were analyzed. Pretest and posttest results for both groups were compared. The pretest results gave the researcher's an idea of the nature of the grouping and the performance of the two groups on the topic investigated at the start of the experiment. The posttest results gave her the gains in performance of the two groups as a result of the approach applied. Also comparison of the posttest scores with the pretest scores of both groups was made to determine which of the two approaches considered is a much better approach - lecture discussion or the individualized instruction with the use of the prepared workbook.

The findings of the study provided the researcher inputs for the formulation of instructional redirection as feedback to the research environment which is expected to provide avenues for the attainment of the ultimate objective of the study and that is to produce mathematically gifted students.

Significance of the Study

This study was conducted because the development and use of effective teaching materials in the form of workbook is a learner as well as a teacher function and is therefore significant to school administrators, teachers, parents, students, and future researchers.

To the Administrators. This study provides valuable inputs or information to the school administrators in terms of what policies or strategies can be adopted to improve and pursue quality instruction in mathematics. A training program on teaching strategies and workbook writing may be conducted to improve teaching competencies and enhance their knowledge on the development of teaching devices.

To the Mathematics Teachers. The result of this study can provide mathematics teachers with information about effective methods in teaching mathematics. Also, it can help identify topics/areas which are best taught using instructional centered-materials, hence, they can reinforce classroom learning activities through the most appropriate and applicable strategy. Moreover, with workbooks teachers can attend seminars/workshops to improve their teaching skills with the hope that students can direct their own learning in mathematics classes. Also, with the workbook a teacher can have a multitude of activities going on simultaneously. He may have some students reading silently, some working on their assignments while he gives assistance to slow learners. The teacher can monitor the progress of his students in and out of the classroom and he can use the workbook with both bright and slow learners in the same class by providing them the differences in the use of the workbook.

To the Students. They serve as the major motivating factor in the conduct of this study. Policies and development programs formulated and implemented by administrators are basically intended to improve the quality of instruction that students are exposed to. Moreover, their understanding and learning of mathematics will be further improved with their teacher being knowledgeable of the different concepts. Also, with available material resources their teacher can monitor their progress in and out of the classroom. With the workbook the students will have masterry of the lessons taught in class because they will have ample time to review using the said workbook. Furthermore, the students can ask the assistance of their parents, siblings and friends regarding difficult topics.

To the Parents. The primary concern of parents in sending their children to school is for them to learn. Financial support from parents, usually in the form of rental or payment for the workbook, can easily be solicited. Moreover, if parents know that efforts are being done to improve the quality of teaching given to their children, they will be motivated to participate actively in school activities. In the process, parents will be closer to the school. Hence, parent-teacher relationship and parents-school relationship will be futher enhanced. By using the workbook, parents too can learn the lessons thereby enabling them to reinforce what their children have gained in school. More importantly, parents can evaluate their children's strong and weak points in Mathematics. Thus, they will be able to do their share in giving corrective measures.

To the Future Researchers. The results of this study maybe utilized by researchers in conceptualizing a researchable problem of similar nature or in conducting a research study to further test the effectiveness of the developed workbook.

<u>Scope and Delimitation of the Study</u>.

This is an experimental study which compared two approaches of teaching Mathematics I in Calapi National High School, Calapi, Motiong, Samar. These two approaches are: (1) individualized instruction with the use of workbook; and (2) lecture-discussion method.

The study was confined to two intact classes in the first year. Forty first year high school students, twenty coming from each intact class were included in the control group and the experimental group by matching their average grades in the first and second grading periods in Mathematics I.

Two tests were administered to the two groups, namely: the pretest and the posttest. The lessons covered the following topics in Mathematics I: (1) Constants and Variables; (2) Mathematical Phrases and Sentences; (3) Coefficients, Exponents, Base and Powers; (4) Evaluation of Numerical and Algebraic Expressions; (5) Algebraic Expressions and Polynomials; (6) Addition and Subtraction of Monomials and Polynomials; (7) Multiplication of Polynomials; and (8) Division of Polynomials. The lessons were all reflected in the prepared table of specifications before making the draft of the pretest/posttest.

The study period was school year 1997-1998. The preparation of the table of specifications, pretest/ posttest, documentary analysis of age, sex, grade were done during the early part of the school year.

The experimentation period which covered a span of five weeks including administration and correction of pretest and posttest was conducted from January 19, 1998 to February 20, 1998.

Definition of Terms

For better comprehension and understanding of the study, the following terms are defined:

Calapi National High School. This is the former Calapi Barangay High School which was converted to a newly nationalized high school by virtue of Section 7, R.A. 6655, s. 1998. It offers a complete secondary curriculum.

Control group. This refers to the group in the experiment which is not exposed to the approach/project/ technique in question (Herrin, 1987: 39). In this study, this refers to the group of twenty students coming from First Year - Section I in Calapi National High School who were taught using the lecture-discussion method.

Effectiveness. This term refers to the productive result of using the teacher-made text/workbook in teaching basic mathematics (Webster, 1986: 787). In this study, the term refers to the significant gains in the posttest results as compared with the pretest results.

Exercises. Conceptually, this term means performing or practicing in order to develop or improve specific power or skill (Webster, 1986: 797). Operationally, this refers to the learning task in each lesson used to supplement or augment learning of mathematical concepts and skills.

Experimental group. Generally, this term refers to the group in the experiment which is exposed to the approach/ project/technique in question (Herrin, 1987:39). In this study, this refers to the group of twenty first year high school students in Calapi National High School who were taught using the workbook.

Instructional redirection. This refers to the policy or specific area in a certain institution to enrich, modify, establish or maintain, as the case may be, toward achieving a common goal (Pino, 1992: 13). In this study, this refers to the development, implementation or adoption of innovative strategies, approaches and techniques in teaching mathematics.

Intact class. This refers to a body or group that is physically and functionally complete (Webster, 1976: 1173). In this study, this refers to the two first year sections of Calapi National High School, Calapi, Motiong, Samar.

Learning. This refers to the psychological activity in development, such as the process of acquisition and extinction of symbolic modifications in existing knowledge, skills, habits and motor skill (Webster Dictionary, 1976: 1286).

Mathematics. The science that deals with the relationship of symbolism of numbers and magnitude and includes quantitative operations and the solutions of quantitative problem (Webster Dictionary, 1976: 1893).

Mathematical performance. This term refers to the capacity to achieve a desired outcome/result in Mathematics (Webster, 1986: 1679). In this study, this refers to the scores obtained by the first year high school student respondents of both groups in the pretest and the posttest.

Pretest. This refers to the preliminary test which serves to explore rather than evaluate (Webster, 1986: 1972). In this study, this refers to the 30-item test in Mathematics I administered to both the experimental and control groups prior to the experimental activity to determine the initial knowledge of the students on the topic under study.

Posttest. This term refers to a test given after a period of time (Webster, 1986: 1801). In this study, posttest contains the same test items found in the pretest which was also administered to both the experimental and control groups designed to compare individualized instruction using the workbook and lecture discussion approach in teaching mathematics.

Rating. A marked indicator of one's study in relation to a perceived criteria for the evaluation of achievement (Webster Dictionary, 1976: 1185).

Respondents. This term refers to the forty first year students of First Year - Sections I and II who were taken as samples of the two groups under study and were officially enrolled in Calapi National High School, Calapi, Motiong, Samar, for the School Year 1997-1998.

Skill. Generally, this term means knowledge of the means or method of accomplishing a task (Webster, 1986: 2133). As used in this study, the term refers to the ability of the student to perform the required mathematics operation in a given problem to arrive at the required result.

Strategies. This term means the art of devising or employing careful plans or methods toward a goal (Webster, 1986: 2256). In this study, this refers to the mathematics teacher's way of doing his task at hand according to the plans, specifications, or objectives associated with it.

Teaching. In a classroom situation, it deals with the process of stimulating, directing, guiding, and encouraging learning activities (Gregorio, 1967: 9).

Technique. This is one's ability to use several methods and procedures in performing a particular task (Webster, 1976: 2349). In this study, this refers to the mathematics teacher's way of doing his task at hand according to the plans, specifications, or objectives associated with it.

Traditional approach. Generally, this term means the use of inherited or established way of doing (Webster, 1986: 2422). As used in this study, this refers to the combined lecture-discussion method, question and answer method, and conventional textbook method used by the teacher in teaching the content of Mathematics I.

Validity. This refers to the degree to which a test or measuring instrument measures what it intended to measure (Calmorin, 1994: 63). As used in this study, validity refers to the ability of pretest and the posttest to evaluate the performance of the student in Mathematics I.

Workbook. This refers to a book outlining a suggested course of study in some subject or field. It contains the student's individual exercises or a book containing a progressive series of problems to be solved directly on the pages and often supplementing the texbook (Webster, 1976:

2634). In this study, this refers to a self instructional textbook in which the subject matter has been broken into minute details of the learning sequence. The program is written containing exercises to be solve directly on the pages which was given to forty first year high school students of Calapi National High School.

Chapter 2

REVIEW OF RELATED LITERATURE AND STUDIES

This chapter contains the relevant information in the form of conceptual literature obtained from books, periodicals and documents and research studies including unpublished works like theses, dissertations, and other research papers. It includes a brief explanation on how the items of information relate to or differ from the present study.

Related Literature

The use of material - centered strategy in instruction like the use of modules, workbooks, and self-learning kits is supported by the following literature reviewed.

Cunningham (1985: 113-120) said that workbooks can provide students with: (1) a means of practicing details of what has been taught, (2) extra practice for students who need it, (3) intermittent reviews of what has been taught, (4) ways for students to apply new learning with examples, (5) practice in following directions, (6) practice in variety of formats that they will experience when they take tests, and (7) opportunity for students to work independently at their own pace.

The workbook developed in this study is intended to provide all of these advantages as cited by Cunningham. It is so constructed that students can work independently at their own pace so that they will attain a certain level of mastery.

Ornstein (1990: 348 - 352) cited the merits of workbooks. He pointed out that workbook have merit for students who need to learn knowledge base or the low achieving students especially students with whom learning to read is difficult. He gives the following guidelines for teachers using workbooks. The workbook must be appropriate for their specific teaching and learning situations.

Osborne (1990: 45 - 111) said that in the elementary level, workbooks are used separately or independently to provide exercises for practice and drill in language, arts, reading, and mathematics along with the textbooks. The students spend as much time or more time alone with the workbooks than they do with other teacher - student activities. In the secondary grade levels workbooks are used to supplement the textbooks for purposes of practice or they are used to reinforce new learning.

Osborne considers the workbook as a supplement of the textbook for students in the secondary level. In this study it will not only supplement but it will take the place of the textbook among others and at the same time it will provide the secondary high school students the much needed drill or practice.

Allington and McGill (1989: 529 -542) pointed out the value of a workbook to teachers as a form of (1) a "busy work" to keep students occupied or worse as a substitute for teaching when the teacher is to grade papers, perform clerical functions or confer with an individual student or group of students (2) to facilitate seatwork activities or management mentality.

The present study will use workbook not for the purpose of keeping students occupied or to help facilitate seatwork activities but rather as a self-instructional material wherein students will move on to mastery at their own pace.

The workbook is but one of the self - instructional materials or programmed materials which the teacher can use to enhance learning in the classroom.

Lardizabal, et. al. (1991: 186 -189) said that in the programmed textbooks the programmed exercises are presented not through a machine but by requiring the pupil to read a specially prepared book. In the programmed textbook, the pupil is required to perform the steps of a learning experience all at the same time: (1) presentation, (2) response, and (3) reinforcement.

In this study the developed workbook is a programmed textbook. It presents the learning material to the pupil, test him on his mastery of the material, and provide for the correction of his wrong responses.

Calderon (1998: 265 - 269) considers the following methods as self-pacing methods: modular learning technique, programmed instruction, self-learning kits and distance education. These methods have different names but they are practically the same in many respects such as:

1. They have the same purpose, and that is, to make each student progress at his own rate because instruction is highly individualized.

2. They enable each student to study the learning materials as thoroughly as he can so that the expected learning skills are acquired.

3. The whole course is divided into logically sequenced topics, each topic being a learning unit by itself.

4. Each topic is divided into logically arranged lessons which are preceded by questions to be answered by the learner. If he is able to answer the questions correctly, then he has learned what is expected of him. He has to check his answers using the answer key which is provided at the end of the lesson.

5. Because the instruction is individualized, the

tests are the criterion - referenced test. This is knowing how much he has learned in comparison with those of others.

6. All the students cover the whole course thoroughly, unlike in group learning where the slow learners who cannot cope up with the fast learning tempo of the bright student have to satisfy themselves with the little learning they are able to pick up here and there without learning the essential part of the lesson.

7. There is a minimum interaction between the teacher and the students.

8. The distance education or instruction by correspondence is exactly the same as the other self-pacing methods except that the learning materials are sent through mail or television.

The workbook developed in this study is a self-pacing tutor since the learner can go over the materials many times until he has attained a certain level of mastery.

Shipley et. al. (1972, 260) commented that three techniques of teaching are found in a well- designed program. They are spiral, discovery, and heuristic techniques. The spiral technique operates when the frame of a program is built upon previously acquired learning, and the student moves progressively from learning simple facts and concepts to more difficult levels of learning. The discovery technique presents self-evident information which lead the student to discover new facts on his own. The heuristic technique, calling for a response, teaches by involving the student in responding to each frame as he moves through a program.

The three techniques of teaching which were noted by Shipley et. al. present in programmed materials are all present in the workbook which makes it possible for the material to provide for individual learning.

Shipley commented that individualized instruction is possible where programmed materials such as workbooks are used as medium of instruction. This approach to teaching and learning allows each student to move through the program at his own individual rate. He has two tutors: the workbook as the personal tutor in the written program and the teacher as his personal tutor in the classroom. The workbook teaches him the fundamentals of the course; the latter explains and elaborates the subject matter when he needs assistance, as well as guiding him toward enriching his learning experiences.

Hughes and McNamara (1961: 225 - 231) made added observations as to the application of programmed instruction in industry: (1) The number of days employees need to spend at central training centers learning a given course can be greatly reduced, and (2) The feasibility of preparing educational packages that can be sent to local centers rather than bringing students many miles for study or decentralization of training makes use of previously prepared programed instruction to train personnel very effectively in less time and expense.

These observations of Hughes and McNamara are not new to us teachers and educators alike. The principles underlying auto-instructional method which are (1) small steps, (2) self-pacing, (3) active participation, (4) immediate feedback, and (5) testing are some of the principles of good teaching.

Hughes and McNamara's observations were based on industrial settings but their observations can also be applied to high school students who can be effectively taught using a programmed workbook.

Teachers' satisfaction under the traditional method as opposed to programmed method was noted by Tobias (1969: 299-306).

Tobias observed that teachers tended to have a negative attitude toward anything automated and job replacing and this attitude has direct bearing on student's under-achieving when given a programmed lesson. They do not like to be dispensable when this new teaching medium is used.

He advised teachers using this method to strive for the middle of the road attitude. He said that programmed materials are not designed to replace teachers anymore than textbooks did centuries ago; rather; its purpose becomes a challenge to the teachers' flexibility and inventiveness in using this new medium as a tool in helping to find more time to discuss, investigate, create, and do these things with more students.

Grant (1990: iii) remarked that quality teaching (and quality learning) often demands risk taking by the teacher (and the learners).

The use of workbook in teaching Mathematics can be explored by mathematics teachers as an alternative strategy to attain quality learning in some situations.

The selection of strategies according to Grant will depend on quite a number of factors: the teacher's objectives; the teacher's understanding of how students learn; and the teacher's desire to teach in such a way that the individual needs of students in the classroom are met. As teachers, we are aware that there are certain topics in Mathematics that are better taught using a particular method. This suggests that strategies of teaching must be varied according to topics to keep students interested and motivated to learn.

The workbook can be used by the teacher to meet the needs of the students in the classroom. If the objective is to attain a certain level of mastery, the workbook could be used to drill students on some concepts and at the same time let them progress and develop at their own pace.

Carlos (1988: 3) quoted Sutaria saying that the quality of learning is determined by the process of making the children learn and that the quality of learning outcomes can be improved by the process of learning.

The learning outcomes can be improved with the use of programmed materials in the learning process. The teacher can act as private tutor in the teaching - learning situation in the classroom with the programmed workbook as the personal tutor of the students. Also, the workbook can be used separately or independently to provide exercises for practice and drill in mathematics along with the textbook. In time, mastery learning will lead to quality learning.

Carin and Sund (1970: 221) in a brief review of research program which considered individualized instruction have this to say: (1) Children do more collateral reading, (2) Problem on discipline are likely to decrease, (3) Gifted pupils achieve to a greater extent than with traditional group instruction, (4) Pupils achieve well in individualized classes, (5) Children prefer the individualized instruction over the traditional approach.

Carin and Sund's review favors the use of individualized instruction in teaching especially for gifted pupils. In this study the students are to use the developed workbook in Mathematics I with high hopes that students' mathematics achievement will be improved so that quality learning will be attained.

Related Studies

In the past years, a large number of studies were concerned on the development of instructional materials. Most studies attribute this to increasing demand of relevant instructional materials. The related studies reviewed provided materials for this study.

Cachero (1994) generated the development and evaluation of modules for enhancing problem-solving skills in Math for second year high school students of the University of Santo Tomas, SY 1994-1995.

The study utilized two intact classes of high school sophomores. From a population of 98 students, only 60 students were chosen. They were matched and equated according to sex, age, grade in Mathematics I, pretest scores, and results of Otis-Lennon Mental Ability Test. Thirty students from randomly chosen intact class composed the experimental group that was exposed to modular instruction. Thirty students from the other intact class, the control group, were exposed to the traditional lecture-discussion method.

The descriptive-developmental method of research was used in the development of the modules. The quasi-experimental method with the pretest-posttest design was used in the evaluation of the modules.

An aptitude test, an achievement test, and two questionnaires for the evaluation of the modules were the instruments used in the study. The data gathered in the study were tabulated and analyzed using the mean, standard deviation and t-test.

The following findings were arrived at:

1. The second year high school students encountered difficulties in (a) translating word phrases/sentences to algebraic expressions/equations, (b) transforming and solving equations, and (c) acquiring the necessary technique for solving word problems. They could relate to problems involving percent/discount, number relations, grade computation, financial budgeting, and time management;

2. The teachers gave a highly favorable rating for the modules. The students rated the modules to be favorable; and,

3. Although the students learned after being exposed to either the modular instruction or the lecture-discussion method, more learning took place when the students were exposed to the modular approach.

The following conclusions were made:

1. Majority of the second year high school students recognized the need for additional learning aids in acquiring skills in problem-solving;

2. The teacher and the students found the prepared modules readable and possessing a highly favorable degree of content validity and reliability; and,

3. Students who used the modules performed better than those who were exposed to the traditional lecture-discussion method of instruction.

The study of Cachero is similar to the present study in the following aspects: (1) both studies utilized intact classes of high school students, (2) both studies were concerned in testing the effectiveness of a developed material, and (3) both studies tried to control the effect of other variables so that change in performance can be attributed only to the treatment.

The two studies are different in certain aspects. In Cachero's study the design is quasi-experimental pretest-posttest while the present study is of experimental pretest-posttest design. The study developed a workbook in algebraic expressions and operations while Cachero's developed modules concentrated on problem solving in Algebra.

Saclot's (1994) study on "Development and Evaluation of a Modular Approach in Teaching Integrated Mathematics IV" developed and evaluated a module for teaching fourth year high school Mathematics. The module was called Math-Pack. This was evaluated using a questionnaire and the non-equivalent control group design experiment.

Through a questionnaire, the teachers perceived the Math-Pack module as having the necessary characteristics of an acceptable self-instructional material. They found it acceptable in terms of (1) objectives, (2) subject matter, (3) organization, (4) language approach, (5) style, (6) adaptability, and (7) evaluation.

The students found the module to be interesting. The presentation of the lessons was deemed easy and there were adequate examples to facilitate comprehension.

The experimental setting was the University of Southern Mindanao in Kabacan, Cotabato. The study used two experimental classes and two control classes. Since modules were generally studied by groups of students, two types of groupings were considered - the forced and the unforced groups.

One class in the experimental classes composed the unforced group while the other class composed the forced group. The same technique was employed in the control classes.

A 50-item achievement test was constructed by the researcher. It was used both as pretest and posttest. The analysis of covariance was used to test the pretest as covariance and the posttest as dependent variable. The results revealed that the experimental classes performed better than the control classes, both in the forced and unforced grouping schemes. The unforced group performed better than the forced group in both the experimental and control classes. There was no significant interaction effect between methods of teaching and types of grouping in terms of students' achievement.

The findings led to the conclusion that the module was an effective self-instructional material for teaching Mathematics in the fourth year high school.

The study of Saclot is similar to the present study in research design and target respondents. Both studies developed instructional materials intended for high school students. They differ in the grade level used. While the present study utilized only two groups of respondents, Saclot's used four different groupings. The developed workbook in this study was not tested for effectiveness by gathering opinions of the teachers and student-users of the workbook. This phase of the study was done in Saclot's.

Pahila's (1994) study entitled "Differential Effectiveness of the Modular Approach in the Teaching of Integrated Science" used three methods of teaching (1) modular individualized instruction, (2) modular cooperative learning, and (3) traditional method. The effect of these different methods was measured in terms of the posttest scores obtained by third year high school students. The students were grouped according to the method of teaching they were subjected to. The scores of these students in the posttest were compared. Significant differences in the achievement of students in the different methods of teaching were observed and assessed.

The reseacher employed the non-equivalent control group design. Intact classes were used. An achievement test of 50 items was constructed and was used both in the pretest and posttest. The data collected were then subjected to an analysis of covariance (ANCOVA). The pretest score, grade in Biology, and the entrance test score were the covariates. The posttest score was made the dependent variable.

There was no interaction effect between the method of teaching and the educational attainment of the fathers. However, when the differences between means for the different methods of teaching where compared, the Scheffe method indicated that students taught by the modular individualized instruction performed better than those taught by the modular cooperative learning. Students who were taught using the traditional method performed better than those who were taught using the modular cooperative learning. But students who were taught by the modular individualized instruction did not show better achievement over those who were subjected to the traditional method.

It could be concluded that using modular individualized instruction in teaching Integrated Science further enhanced student achievement. The traditional method of teaching promoted comparative performance of students with those taught using individualized instruction. The cooperative learning modular approach was not as effective as the (1) modular individualized instruction and (2) the traditional method in enhancing achievement in Integrated Science (Chemistry).

The study of Pahila is similar to the present study in its intention to compare the effect of a method of teaching subjected to lecture-discussion method, which is considered a traditional method. Her study compared modular cooperative learning with lecture discussion method and modular individualized with lecture-discussion method. The present study is concerned with comparing the effect of individualized instruction with workbook to the effect of traditional method of teaching. The research design used differs. Pahila's study used non-equivalent control design while the present study utilized pretest-posttest design using equal number of samples.

Palmes' (1994) study examined the effects of the three teaching methods on the achievement of second year college students in Chemistry.

The three methods of teaching were (1) the traditional lecture approach, (2) the individualized instruction method, and (3) the cooperative learning approach. The cooperative learning approach was a method where students worked in small, mixed-ability groups for a common task activity. Individual instruction was a method in which students performed the activity individually at their own pace. The traditional approach was the lecture method where the teacher was in command of the whole learning process. Worksheets, as self-instructional packets, were designed for both individualized instruction and cooperative learning groups.

The effectiveness of these three teaching methodologies was measured in terms of the posttest scores obtained by the students in the achievement test developed for the study.

The researcher used the non-equivalent control group design where intact classes were used. A 30-item achievement test was constructed and validated. This was used in the pretest and posttest. A 2 X 3 factorial design was used. Analysis of covariance was used as data analysis procedure with the pretest as covariate.

The F-test revealed significant differences in the achievement of students exposed to the three methods of teaching. When the differences between means for the different methods of teaching were compared using the Scheffe method of multiple comparison, the traditional method group performed better than the individualized instruction group. No other pair of means was found to be significantly different.

The analysis of data also showed that the group means for the introverts was found not significantly different from that of the extroverts. The analysis further revealed no significant interaction between the treatment, the methods of teaching, and the extrovert/introvert type of students. Thus, regardless of the method used in teaching, its effect on the achievement of introverts and extroverts was the same.

Results of the study further showed that there were certain topics better taught using a particular method. This suggested that teaching strategy should be varied appropriate to topics taught to keep students interested and motivated to learn.

The study of Palmes is similar to the present study in purpose: to develop and determine the effectiveness of using instructional materials in teaching mathematics. The difference is in the target users. Palmes developed materials intended for college students while this study is for secondary students. Palmes' study was in Chemistry while the present study is in Mathematics. However, both studies used intact classes so as not to disrupt schedule of classes.

Espano's (1994) study on the "Effectiveness of Teacher Made Text/Workbook in Teaching Basic Mathematics" employed the experimental method using the pretest - posttest control design. Fifty BSE/IE freshmen students from Samar State Polytechnic College were the subjects of the study selected on the basis of their NCEE mathematics ratings. The samples were grouped into two: 25 students for the control group and another 25 for the experimental group. The t-test for dependent means was used to determine the students' gains in understanding the concepts of basic mathematics. The t-test for independent means was used to compare the pretest and posttest scores of the experimental group and the control group. The findings revealed that there was no significant difference between the pretest mean scores of both experimental and control groups. Also a significant difference existed between the pretest and posttest of the experimental group and the control group. A significant difference in the posttest scores was found between the control and experimental groups. The experimental group did well in the posttest.

The present study is similar to the study of Espano in the following aspects: (1) both studies developed a workbook in mathematics; (2) used the same type of research design; (3) tested the effectiveness of instructional materials; and (4) experimented on teaching method where the workbook was used against the traditional lecture discussion. The difference lies in the target group with Espano's workbook intended for Bachelor of Science in Industrial Technology (BSIT) students while this study's workbook intended for high school students.

Hermosura's (1990) "The Use of Mathematics I Workbook: A Comparative Study" had the following findings:

1. In general, students performed equally in the two

approaches.

2. High ability students performed better when with workbooks. However, the middle ability and the low ability students performed equally with or without workbooks.

3. On sex differences, male and female students performed equally when taught by either of the two approaches.

(4) Age does not affect the performance of students taught by either of the two approaches.

The study came up with the following recommendations:

1. To facilitate the work of the teacher, a workbook should be used.

2. A similar study should be conducted for a longer duration and with a wider scope of subject matter.

3. To find out if findings will be the same, a similar study should be conducted in other mathematics levels and in other subject areas, too.

The study of Hermosura is similar to the present study in the since that both studies experimented on a teaching strategy, developed a workbook in mathematics and compared its effect against the lecture-discussion method. The two studies differ because the present study did not evaluate the effectiveness of the developed workbook considering the different ability groups as was done by Hermosura.

Dacula (1995) in her study "Development and Validation of Module on Percent and Ratio for Mathematics 1" found out that the experimental group showed a significant amount of learning after the respondents were exposed to modularized instruction. She also found out that the developed module was fairly easy and appropriate for first year high school students and that the module was interestingly based on the results of the test.

She concluded that modular approach to teaching is more effective than the traditional lecture-discussion method as far as the topic on Percent and Ratio is concerned. Because the students can go through the modules and learn its contents if needed, they can discover processes and techniques in learning the lesson until the feeling of self-satisfaction is attained.

She recommended the use of modular instruction since it helps the students to learn to be independent, responsible, self-reliant and hardworking.

The study of Dacula is similar to the present study in research design, target respondent group, method used in developing the instrument, used to gather the needed data. The difference is on the selected topics for investigation. The present study is on "Algebraic Expression and Operation" while that of Dacula was on "Percent and Ratio".

Alandino (1996) conducted a study on "Development and Validation of Modules on Exponents and Radicals in Mathematics III" based on difficulties encountered by high school students in Mathematics III of La Milagrosa Academy, Calbayog City, during the school year 1995-1996.

His findings were:

Based on the diagnostic test conducted, the topic on Exponent and Radicals in Mathematics III was found to be difficult by the students. He found out that the experimental group with modules performed better in the posttest than the control group. He also found out that the experimental group taught with the modules had a better retention power than the control group.

He recommended the following:

1. Slow learners and learners with learning difficulties in mathematics should be given learning materials like the modules to give them time to catch-up lessons not learned.

2. Developed instructional materials like modules should be used in the teaching-learning process.

3. Teachers should prepare instructional materials in their field of specialization.

4. School administrators should give their teachers financial support for developing instructional materials.

5. Modules should be used to supplement teaching of the teacher-centered method especially with difficult concepts.

The study of Alandino is similar to the present study, since both studies determined the effect of material-centered instruction compared to the lecture-discussion method. The difference is on the level of the target group. Alandino used third year high school students as target group while the present study made use of first year high school students.

Uy (1992) in her study "Development and Validation of Modules on Circular Trigonometric Functions and Fundamental Identities", found out that there was a significant amount of learning after the respondents were exposed to modularized instruction based on the results of the posttest.

She concluded that modular approach or material - centered instruction was more effective than the traditional lecture-discussion method.

She recommends the use of modules for it serves as an effective resource material for students.

The study of Uy is similar to the present study since both studies tried to use material-centered instruction strategy and compared its effect to lecture discussion.

Gordove (1993) in his study "Effectiveness of Self-Learning Kits in Grade V Mathematics", revealed that the mean score of 26.2 of the experimental group compared with that of the control group mean score of 20.65 implies that the experimental group performed better than the control group. Hence, individualized approach to teaching Geometry to Grade V pupils through self-learning kits is better than the use of any traditional lecture method.

He therefore concluded that based on the findings of his study, teaching with the use of self-learning kits is more effective than the lecture-discussion method.

He recommended the use of self-learning kits since it develops certain mathematical abilities and skills of pupils.

The study of Gordove is similar to the present study since both studies dealt with comparing teaching strategies using the instruction-material-centered approach. The two studies differ in the target group. Gordove's study was directed to catering elementary pupils while the output of this study is for the use of secondary students.

The above studies presented point out the gains of students in terms of learning using the material-centered instruction like the use of workbooks, modules and self-learning kits. These studies have served as foundation of the present study.

Chapter 3

METHODS AND PROCEDURES

This chapter presents the research design, instrumentation, validation of instruments, sampling procedure, data gathering procedure and statistical treatment used in this study.

Research Design

This study made use of the experimental method of research. Two intact classes of high school students from a population of 76 students were used in the experimentation. From these two first year high school classes, only 40 students were chosen to form the two groups. One was the control group taught by the lecture-discussion method while the experimental group from another class was given the developed workbook. To answer the problem posed in the study, the pretest-posttest control group design (Herrin, 1987: 39) was used as shown below:

$$\begin{array}{llll} R & O_{11} & x & O_{12} \\ R & O_{21} & & O_{22} \end{array}$$

Figure 2: Experimental Design of the Study

Where:

O_{11} = refers to the pretest administered to the experimental group;

O_{12} = refers to the pretest administered to the control group;

O_{21} = refers to the posttest administered to the experimental group;

O_{22} = refers to the posttest administered to the control group;

X = refers to the treatment, in this case the use of the developed workbook in Mathematics I; and

R = refers to the randomization procedure used in selecting the members of the intact class to constitute the experimental group or the control group.

In order to control factors which might influence the treatment and to have the same entry behavior for both groups the respondents were matched in terms of age, sex, and achievement in mathematics based on their average Mathematics I rating for two successive grading periods.

The independent variables in this study were the two approaches of teaching mathematics, namely: the traditional approach using the lecture-discussion method and the individualized instruction approach using the developed workbook in Mathematics I. The dependent variable is the student's performance in Mathematics I which is supposed to be affected by the two approaches.

Also, factors like teachers, time and content were controlled in order not to affect the results so that the main focus of the study was on the dependent and independent variables only. The content topics were taught by the researcher to both the control and experimental group using the lecture-discussion method. In testing the attainment of the concepts taught, the researcher gave the developed workbook to the experimental group and the researcher prepared worksheets, activity sheets, and exercises for the control group. The workbook used by the experimental group was prepared by the researcher so as to control the content and teacher factor. The schedule for the two classes were reversed for the last two weeks of experimentation, for both the control and experimental groups to control the time factor. The experimentation lasted for only 20 school days because that was the time schedule per budget of lessons based on the Mathematics I Curriculum and the Desired Minimum Learning Competencies for Mathematics I.

Instrumentation

The needed data were collected using the teacher- made tests, pretest/posttest, documentary analysis and observation sheets as instruments.

Pretest. This is a 30-item multiple choice test given to the respondents before the start of the experimentation.

The pretest was given to both the control and experimental groups to determine the entry knowledge of the respondents on the content topics and to eliminate bias at the start.

Posttest. This is the same as the pretest but the items were rearranged. This was given to both the control and experimental groups after the conduct of the experiment. This was used to determine the amount of knowledge gained after the respondents were exposed to the two approaches.

Teacher-made Test. The set of exercises in the developed workbook which is the same set of exercises given to the control group. The test items in the exercises were intended to gauge the attainment of the concepts taught and to reinforce learning of the content topics. The exercises were prepared by the researcher herself based on the objectives of the lesson.

Documentary Analysis. The researcher used the students' first year report cards and the grading sheets in Mathematics I as basis for the selection of the respondents. In the construction of the test items for the pretest/posttest and the teacher-made test, and in the development of the workbook, the documents used were the Teacher Training Manual for Mathematics I SEDP Series, Mathematics I Textbook SEDP Series, Budget of Lessons, Minimum Learning Competency and Course Objectives.

Observation Sheets. During the experimentation phase the researcher made use of an observation sheet to record the individual responses of the students using the workbook.

Validation of the Instruments

The 50 - item test was constructed based on the prepared Table of Specifications (Appendix H) taking into consideration the suggested time budget in the Mathematics I curriculum and the course objectives. The draft of the test was submitted to the Mathematics I teachers for comments and suggestions. With the suggestions incorporated, the corrected draft was given to the adviser for further improvement. The final draft comprising of 50 items was tried out to 50 sophomore students of Calapi National High School who had taken Mathematics I in the previous year. The students involved in the try-out were given enough time to answer the test.

After the try-out, the test papers were corrected. The following steps as suggested by Ebel (1965: 346) were undertaken by the researcher in conducting the item analysis:

The 50 test papers were scored and arranged from the highest score down to the lowest score. The test papers were separated into three groups - the upper group, middle group and lower group. Twenty-seven percent of the fifty test papers or 14 test papers were counted starting from the top of the pile and another 14 test papers counted starting from the bottom of the pile were each separated to constitute the upper and lower groups respectively. These two groups of test papers were used by the researcher in conducting the item analysis.

The number of correct responses made on every item of the test in the upper and lower groups were determined and tabulated separately.

To compute for the item index of difficulty, the number of correct responses made on every item of the test were determined for both groups. These numbers were added and expressed as a ratio to the sum total of the number of cases for both groups. The quotient obtained was the index of difficulty. The formula used was:

$$p = \frac{U + L}{2(N)}$$

Where:

p = difficulty index

U = upper group number of correct responses on the given item

L = lower group number of correct responses on the given item

N = number of cases in each group

To obtain the discrimination index of the item, the number of correct responses in the lower group was subtracted from the number of correct responses of the upper group and was expressed as a ratio to the number of cases in each group. The quotient obtained was the discrimination index.

$$D = \frac{U - L}{N}$$

Where:

D = discrimination index

U = upper group number of correct responses on the given item

L = lower group number of correct responses on the given item

N = number of cases in each group

The accepted indices of discrimination ranged from 0.30 and up. This acceptance was based on the item selection of Ebel (1965: 374) as shown below:

Index of Discrimination		Item Evaluation
0.40 and up	-	Very good items
0.30 to 0.39	-	Reasonably good but possibly subject to improvement
0.20 to 0.29	-	Marginal items, usually needing improvement

0.19 and below	-	Poor items, to be rejected or improved by revision

As to the index of difficulty, Ebel's interpretations (1965: 376) as shown below was used:

Index of Difficulty		Item Evaluation
86% - 100%	-	Very easy items
71% - 85%	-	Easy items
40% - 70%	-	Moderately difficult items
15% - 39%	-	Difficult items
1% - 14%	-	Very difficult items

In item analysis, Sevilla (1990: 55) recommended to retain items with difficulty indices within .20 and .80 and discrimination indices within .30 to .80 .

The reliability of the test was computed using the Kuder Richardson Formula and interpreted based on the interpretation given by Ebel (1965) as shown below:

Interpretation of the Coefficient of Reliability

Reliability		Degree of Reliability
0.95 - 0.99	-	Very high, rarely found among teacher's made tests.
0.90 - 0.94	-	Highly equaled by few tests.
0.80 - 0.89	-	Fairly high, adequate for individual measurement.

0.70 -	0.79	-	Rather low, adequate for group measurement but not very satisfactory for individual measurements.
Below	0.70	-	Low, entirely inadequate for individual measurement although useful for group average and school survey.

The test items were revised based on the analysis made of the try-out results. The original pool of 50 items were reduced to 30 items in the final form because some items were either rejected or retained and improved.

The final form of the pretest was the result of the revision made based on the analysis conducted (Appendix J).

Sampling Procedure

The researcher utilized purposive sampling technique using the computed average grades in Mathematics I for two successive grading periods - first and second grading periods as basis for the grouping. To avoid bias in the grouping, the said ratings were matched correspondingly.

A total of seventy six first year students were officially enrolled in Calapi National High School during the school year 1997-1998. Section I was composed of 36 students and 20 students from this section were identified to form the control group.

Twenty students formed the experimental group selected readability level of the developed workbook and to make certain that the constructed workbook is appropriate for the target group.

Mean. This statistical measure was used to determine the mean scores of the experimental group both in the pretest and the posttest. The formula recommended by Walpole (1982: 24) was utilized, viz:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Where:

$\bar{x}$ = Mean

x = Pretest/posttest scores of the experimental and control groups

$\sum x$ = Sum of the pretest and posttest scores

n = Number of respondents/samples in each group.

Standard Deviation (s). The variation that existed in each group was computed as follows (Walpole, 1982: 36):

$$s = \sqrt{\frac{n \sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2}{n(n-1)}}$$

Where:

x = pretest/posttest scores of the experimental and control groups

$\sum x$ = sum of the x column

$\sum x^2$ = sum of the x^2 column

n = number of respondents/samples in each group.

t-test for Independent Samples. This statistical tool was applied to test hypotheses 1 and 4 of the study. This particular tool was used to find the significant difference of the experimental and control groups both in the pretest and posttest. The formula suggested by Freund and Simon (1997: 363) was applied.

$$t = \frac{\bar{x}_1 . \bar{x}_2 \quad \delta}{\sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}\left[\frac{1}{n_1}+\frac{1}{n_2}\right]}}$$

Where:

t = computed t value

$\bar{x}_1$ = mean of the pretest or posttest of the experimental group

$\bar{x}_2$ = mean of the pretest or posttest of the control group

n_1 = number of students in the experimental group

n_2 = number of students in the control group

s_1 = standard deviation of the pretest or posttest of the experimental group

s_2 = standard deviation of the pretest or posttest of the control group

t-test for Dependent Samples. This statistical tool was utilized for testing the significant differences between the pretest and posttest of the experimental and control groups, hypotheses two and three. The formula recommended by Bartz (1981: 262) was used:

$$t = \frac{\bar{D}}{S_{x_D}}$$

Where:

t = computed t value for dependent samples

$\bar{D}$ = average difference between the posttest and pretest of the experimental/control group D = D/N

$\sum D$ = summation of the D column

$\sum D^2$ = summation of the square of the D column

N = number of respondents/samples in each group.

The computed t-value using both formulas was compared to the critical or tabular t-value at .05 level of significance. The corresponding hypothesis was rejected because the t-value obtained was greater than the tabular value. Had the obtained value been less than the tabular value, the hypothesis could be accepted.

__Flesch Formula__. This formula was used to determine the readability level of the constructed workbook. The researcher computed the reading ease score (RES) and the human interest score (HIS) of the constructed workbook to make certain that the workbook was appropriate for the target group.

Twenty-six pages were randomly selected from the 130 pages and subjected to the steps in measuring the reading ease score and the human interest score.

1. Selecting the Sample Pages. The sample pages representing twenty percent of the total number of pages were selected at random from the instructional material. Exercises and the cover pages of Lesson 1 up to Lesson 8 were not included.

2. Counting the Number of Words. One hundred words were taken from each page. Counting the one hundred words starts on the first paragraph of each page up to the 100th words. In samples where there were no paragraphs, the first word of the sentence was considered. Figure captions, heading of the lessons, numbers and titles were not included in the counting.

3. Counting the Number of Syllables. The syllables in the 100th words in each sample were counted. The syllables were then counted the way the word was pronounced. The average sentence length was counted. This result was used in computing the reading ease score of the workbook.

4. Counting the Number of Sentences. The total number of sentences in the 100th words in each sample was counted. If the 100th words fell after more than one half of the words of the sentence, it was counted as one, otherwise it was not counted.

5. Finding the Average Word Length. To get the average word length, the number of syllables in all the sample pages were divided by the total number of sample pages.

6. Finding the Average Sentence Length. To get the average sentence length, the number of words in all the sample pages were divided by the total number of sentences.

7. Solving for the Reading Ease Score (RES). The formula of the RES is

$$\text{RES} = 206.835 - (1.015 \times \text{Average Sentence Length} + 0.846 \times \text{Average Word Length}).$$

Where:

$$\text{Ave. Sentence Length} = \frac{\text{Total No. of Words in the Samples}}{\text{Total No. of Sentences}}$$

$$\text{Ave. Word Length} = \frac{\text{No. of Syllables in All Samples}}{\text{Total No. of Sample Pages}}$$

8. Solving for the Human Interest Score (HIS). The formula for computing the Human Interest Score is

HIS = (% Personal Words/100 Words X 3.635) + (% Personal Sentences X 0.314).

Where:

$$\text{\% Personal Words} = \frac{\text{Total No. of Personal Words in All Samples}}{\text{Total No. of Words in All Samples Pages}}$$

$$\text{\% Personal Sentences} = \frac{\text{Total No. of Personal Sentences}}{\text{Total Sentences in All Samples Pages}}$$

Chapter 4

PRESENTATION, ANALYSIS, AND INTERPRETATION OF DATA

This chapter discusses the results of the analysis of the collected data. This includes the profile of the first year high school students, the results of the pretest and posttest administered to them as well as the decision made in relation to the four hypotheses that were tested.

Profile of the Respondents

Table 1 shows the composition of the control group and the experimental group.

It can be gleaned from the table that among the members of the control group, the age ranges from 13 to 21, the oldest member is 21 years old followed by 16 and 15 years of age. Thus the corresponding average age of this group was pegged at 14.70 years old with a standard deviation of 1.87 years. On the other hand, among the members of the experimental group, the oldest was likewise 21 years of age and the youngest members were 13 years of age. Correspondingly, the average age turned out to be 14.65 years old with a standard deviation of 1.81 years.

In terms of sex, the two groups have equal cardinal number of female and male members with a frequency

Table 1

Age, Sex and Grades Profile

Samples Number	AGE EG	AGE CG	SEX EG	SEX CG	Mathematics I Ave. Grades EG	Mathematics I Ave. Grades CG
1	21	16	M	M	75	75
2	15	16	M	M	75	75
3	14	21	M	M	75	75
4	15	13	F	F	75.5	75.5
5	13	13	M	M	75.5	75.5
6	14	14	F	F	77	77
7	13	15	F	F	78	78
8	16	15	F	F	80	80
9	14	13	F	F	82	82
10	16	14	F	F	82.5	82.5
11	14	13	F	F	83	83
12	13	14	F	F	84	84
13	13	15	F	F	84	84
14	16	15	F	F	84.5	84.5
15	13	16	F	M	85	85
16	14	13	F	F	85.5	85.5
17	14	13	F	F	86	86
18	15	14	M	F	86	86
19	15	16	F	M	87	87
20	15	15	M	F	87.5	87.5
TOTAL	293	294	F = 14	F = 4	1627	1627
MEAN	14.65	14.70	M = 6	M = 6	81.35	81.35
SD	1.81	1.87			5.55	5.55

Legend: M - Male
F - Female

of 14 and six, respectively. For their grades in Mathematics I, using the average grade of the first and second grading periods the members of the two groups were exactly matched with one another with a highest grade of 87.5 and lowest grade of 75. Thus, the average grade in Mathematics I of these two groups resulted to 81.35 with a standard deviation of 5.55. It is worthwhile to note at this point that the experimental group and the control group were matched in terms of number, age, sex and grade, hence, their entry behaviors were controlled and were deemed the same.

Pretest Results of the Experimental Group and the Control Group

Table 2 presents the distribution of scores of the two groups in the pretest administered. In the case of the control group, the highest score obtained was 21 followed by a score of 20. The lowest score was nine. The total score of the control group resulted to 290 with a mean of 14.50 and a standard deviation of 3.62. On the other hand, the highest score obtained by the experimental group was 20 and the lowest score was seven. The total number of scores reached a value of 295 with a mean of 14.75 and a standard deviation of 3.96.

In comparing the mean scores obtained by these two groups of respondents, the experimental group turned out to be higher than the control group by 0.25. To find out whether this difference is significant, t-test for

Table 2

Pretest Scores of the Experimental
and Control Group

Respondent Number	PRETEST SCORES	
	Experimental Group	Control Group
1	12	11
2	7	9
3	18	15
4	11	15
5	8	12
6	19	17
7	13	19
8	14	13
9	16	16
10	18	14
11	20	20
12	16	18
13	10	12
14	17	11
15	15	10
16	19	18
17	18	17
18	10	10
19	19	21
20	15	12
TOTAL	295	290
MEAN	14.75	14.50
SD	3.96	3.62
Computed t = 0.208		Tabular t = 1.645
Level of Significance = .05		df = 38

independent samples was applied. The computed t-value of 0.208 turned out to be lesser than the tabular t-value of

1.645 at a = .05 and degrees of freedom = 38. Thus, the first hypothesis which states that there is no significant difference between the pretest results of the experimental group and the control group was accepted. The observed difference between their pretest mean scores was not significant. The findings imply that the entry behaviors of the two groups were the same prior to experimentation.

Also, the researcher noted that some of the respondents with low Mathematics I rating (75) obtained high scores in the pretest and conversely those with high Mathematics I ratings (85) obtained low scores in the pretest. Since the researcher was their teacher in Mathematics I she also examined her class records. She noticed that some of these students were given low ratings in the first/second grading period in Mathematics I because of missed quizzes, exercises, assignments, projects, and recitations due to absences which explains the said contradictory results. The data implies that using the average Mathematics I ratings for two successive grading periods as basis for obtaining the samples in this experiment is not very reliable. But since these observations are not limited to one only group but true to both groups, the researcher concluded that the two groups involved in the experimentation are deemed equal in terms of entry behavior. The pretest administered to the two groups would have differed significantly had the groupings been bias at the start.

Pretest and Posttest Results of the Control Group

Reflected in Table 3 are the pretest and posttest results of the control group. As discussed earlier, the highest score in the pretest of the said group was 21 and the lowest was 9. The resulting mean was 14.50. The highest score in the posttest was 26 and the lowest score was 15. The posttest mean scores was 21.05. The data gave the difference of 6.55 for the posttest and the pretest. Initially, it could be observed that there was an improvement in the performance of the control group after the experimentation.

Subjecting these scores to the t-test for dependent samples the analysis revealed that the computed t-value of 13.56 was greater than the tabular t-value of 2.093 at .05 significant level and degrees of freedom = 19.

Thus, the hypothesis which states that there is no significant difference between the pretest and posttest results of the control group is rejected. This result led to the implication that the control group showed a marked improvement in the posttest with the use of the lecture-discussion method.

Table 3

Pretest and Posttest Scores of the Control Group

Respondent Number	Pretest Scores P_{12}	Pretest Scores P_{12}^2	Posttest Scores P_{22}	Posttest Scores P_{22}^2	Difference $(P_{22} - P_{12})$ D	Difference D^2
1	11	121	21	441	10	100
2	9	81	15	225	6	36
3	15	225	21	441	6	36
4	15	225	20	400	5	25
5	12	144	20	400	8	64
6	17	289	23	529	6	36
7	19	361	26	676	7	49
8	13	169	17	289	4	16
9	16	256	23	529	7	49
10	14	196	19	361	5	25
11	20	400	26	676	6	36
12	18	324	22	484	4	16
13	12	144	19	361	7	49
14	11	121	19	361	8	64
15	10	100	20	400	10	100
16	18	324	20	400	2	4
17	17	289	26	676	9	81
18	10	100	18	324	8	64
19	21	441	25	625	4	16
20	12	144	21	441	9	81
TOTAL	290	4454	421	9039	131	947
MEAN	14.50		21.05		SD = 2.16	
Computed t = 13.56					Tabular t = 2.093	
Level of Significance = .05					df = 19	

Pretest and Posttest Results of the Experimental Group

The results of the pretest and posttest of the experimental group are given in Table 4. In the pretest, the highest score obtained by this group was 20 and the

Table 4

Pretest and Posttest Scores of the Experimental Group

Respondent Number	Pretest Scores P_{11}	Pretest Scores P_{11}^2	Posttest Scores P_{21}	Posttest Scores P_{21}^2	Difference $(P_{21} - P_{11})$ D	Difference D^2
1	12	144	21	441	9	81
2	7	49	19	361	12	144
3	18	324	24	576	6	36
4	11	121	21	441	10	100
5	8	64	17	289	9	81
6	19	361	28	784	9	81
7	13	169	26	676	13	169
8	14	196	21	441	7	49
9	16	256	20	400	4	16
10	18	324	24	576	6	36
11	20	400	27	729	7	49
12	16	256	23	529	7	49
13	10	100	23	529	13	169
14	17	289	24	576	7	49
15	15	225	22	484	7	49
16	19	361	26	676	7	49
17	18	324	21	441	3	9
18	10	100	20	400	10	100
19	19	361	25	625	6	36
20	15	225	25	625	10	100
TOTAL	295	4649	314	4454	162	1452
MEAN	14.75		22.85		SD = 2.71	
Computed t = 13.36					Tabular t = 2.093	
Level of Significance = .05					df = 19	

lowest was seven. This resulted to a total of 295 and a mean of 14.75. In the posttest, the highest score obtained by the group was pegged at a value of 28 while the lowest score was 17. Consequently, the total of the scores of the control group in the posttest was 314 with a mean of 22.85. The resulting mean difference between the pretest and posttest scores of the experimental group was 8.1.

To test whether the numerical difference is significant, t-test for dependent samples was utilized. The third hypothesis "There is no significant difference between the pretest and posttest mean scores of the experimental group" was rejected as evidenced by the fact that the computed t-value of 13.36 was greater than the tabular t-value of 2.093 at .05 level of significance and 19 degrees of freedom. Like in the case of the control group, the experimental group showed marked improvement after they were taught the lessons in Mathematics I with the use of the developed workbook.

Posttest Result of the Control and Experimental Group

Reflected in Table 5 are the posttest scores of the experimental and the control group. The highest score obtained by the experimental group was 28 followed by 27, 26, and 25 while the highest score obtained by the control

Table 5

Posttest Scores of the Experimental
and Control Group

Respondent Number	POSTTEST SCORES	
	Experimental Group	Control Group
1	21	21
2	19	15
3	24	21
4	21	20
5	17	20
6	28	23
7	26	26
8	21	17
9	20	23
10	24	19
11	27	26
12	23	22
13	23	19
14	24	19
15	22	20
16	26	20
17	21	26
18	20	18
19	25	25
20	25	21
TOTAL	457	421
MEAN	22.85	21.05
SD	2.87	3.05
Computed t = 1.922		Tabular t = 1.645
Level of Significance = .05		df = 38

group was 26 followed by a score of 25, 23, and 22.

Moreover, the lowest scores for the experimental group and the control group were 17 and 15 respectively.

Futhermore, the data showed that the experimental group had a total of 457 and a mean score of 22.85. While the control group posttest had a total of 421 and a mean score of 21.05. Evidently it can be observed that the mean score of the experimental group in the posttest is higher than that of the control group by 1.8.

To test the significance of the mentioned difference, t-test for independent samples was applied. Inasmuch as the computed t-value of 1.922 proved to be greater than the tabular t-value of 1.645 at .05 level of significance and 38 degrees of freedom the hypothesis "There is no significant difference between the posttest mean scores of the experimental group and the control group" was rejected. This implies that the use of the developed workbook in teaching Mathematics I is better than the lecture-discussion approach. Hence, the use of the developed workbook is an effective approach to teaching Mathematics.

Implication

The findings of the study showed that the moderator variables such as age, sex, and grades were common to the two groups. Since the students were equally paired, one can safely say that taking all things equal the performance of students in Mathematics depended largely on their entry behavior to specifically refer to their previous knowledge in mathematics and their actual absorption of lessons during the experimentation.

The findings revealed that there was a significant difference between the performance of the experimental and control groups. This implies that the use of the developed workbook has a better impact on the students as compared to the traditional method using exercises, worksheets, and activity sheets. The posttest results yielded a higher mean score of 22.85 for the experimental group over the control group having the mean score of 21.05. The difference between the two posttest mean scores is 1.8. The computed t and tabular t yielded a difference of 0.277.

Since the use of the developed workbook has a better impact on the students than the traditional approach, it can be adopted as an instructional material in teaching Mathematics I. Also, the teacher can vary her usual teaching approach by the use of the workbook. Individualized instruction with the use of a workbook has many advantages over the traditional approach hence its use as a strategy is strongly recommended by authorities.

Moreover, aside from the empirical evidence that the use of the developed workbook has resulted to a marked improvement on the part of the experimental group, an special aspect of learning provided by books and other instructional materials is that the developed workbook has succeeded in helping the students to become independent learners.

Analysis of the Readability Level of the Developed Workbook

The readability level of the developed workbook is measured in terms of its appropriateness and how interesting it is to the users. The researcher used Flesch Formula to determine the readability level of the developed workbook.

Table 6

Results of the Reading Ease Score (RES) and Human Interest Score (HIS)

Workbook	RES	Interpretation	HIS	Interpretation
1	71.784	Fairly Easy	12.14	Mildly Interesting

Table 6 reflects the result of the Reading Ease Score (RES) and the Human Interest Score (HIS) obtain from the pages chosen as samples.

As can be gleaned from Table 6, the developed workbook is fairly easy suited and appropriate to the first year and 2nd year high school students. It is also mildly interesting enhancing in the students the desire to go through the material.

Chapter 5

SUMMARY OF FINDINGS, CONCLUSIONS, AND RECOMMENDATIONS

This chapter presents the summary of findings, the corresponding conclusions drawn from the study as well as the recomendations formulated based on the conclusions drawn.

Summary of Findings

After the data were scored, analyzed and interpreted, the following are the salient findings:

1. The control group and the experimental group belonged to more or less the same age level. The average age of the control group was 14.70 and the average age of the experimental group was 14.65.

2. The control group and the experimental group had more or less the same entry behavior in terms of average grade of the two rating periods in Mathematics I. The experimental group had an average of 81.35 and the control group had an average of 81.35.

3. The pretest results of the experimental group and the control group had an average of 14.75 and 14.50 respectively. The computed t for the pretest result of the experimental and control groups was 0.208. This led to the

acceptance of the hypothesis that there is no significant difference between the pretest results of the experimental and control groups.

4. The control group had an average pretest mean score of 14.50. The posttest mean score was 21.05 giving a difference of 6.55. The computed t value was 13.56 which turned out greater than the tabular t-value. Therefore, the hypothesis of no significant difference between the pretest and posttest mean scores of the control group was rejected.

5. The results of the pretest and posttest scores of the experimental group were as follows: the pretest mean score was 14.74 and the posttest mean score was 22.85, resulting in the difference of the means of 8.1. The computed t value was 13.36. Hence, the hypothesis of no significant difference between the pretest and posttest mean scores of the experimental group was rejected.

6. The experimental group had an average of 22.85 in the posttest while the control group had an average posttest score of 21.05. The corresponding computed t-value was 1.922 which was greater than the tabular/critical value. This led to the rejection of the fourth hypothesis which states that "There is no significant difference between the posttest mean scores of the experimental and control groups."

Conclusions

On the basis of the findings just presented, the following conclusions could be drawn from the study:

1. The experimental group and the control group have the same entry behavior on the basis of their age, sex, and average grades in Mathematics I profile, as well as, on the basis of comparison of the result of the pretest mean scores. This implies that the experiment is free from bias in terms of the results arrived at.

2. The use of a developed workbook is an effective method of teaching Mathematics I. This is because the experimental group showed a marked improvement in the posttest.

3. The lecture-discussion method can also be considered as an effective method of teaching Mathematics.

4. The use of workbook (individualized instruction approach) is an effective method of teaching Mathematics compared to the traditional lecture discussion method supplemented by worksheets, activity sheets, and exercises as evidenced by the fact that the fourth hypothesis was rejected.

Recommendations

From the findings arrived at in this study, the following recommendations were made:

1. Teachers should use workbooks in teaching Mathematics so as to make the lessons not boring because of method variation.

2. Students should be exposed to material-centered instruction like the use of workbook to develop their potential for independent learning.

3. Teachers should attend seminar-workshops on making workbooks so that they can prepare their own workbooks and have also time to attend to other functions/activities like individual follow-up and remediation.

4. A similar experiment may be conducted using workbooks on other topics/areas in Mathematics.

5. The use of other teaching approaches or methodologies may be tried out with the developed workbook to prove its effectiveness.

B I B L I O G R A P H Y

A. BOOKS

Ceneso, Samuel A. High School Planning Manual. Quezon City: Rex Printing Company, Inc., 1976.

Constantino, Renato. The Making of a Filipino. Quezon City: Malaya Publishing Company, 1991.

Drucker, Peter F. The Practice of Management. New york: mcMillan, Co., 1993.

Eckles, Robert W. et al. Supervisory Management. New York: McMillan, Co., 1993.

Good, Carter V. Dictionary of Education. New York: McGraw Hill Book Co., 1979.

Gregorio, Herman C. School Administration and Supervision of Quezon City: R.P. Garcia Publishing Co., 1984.

Guiang, Pedro C. et al. Administration and Supervision of New Secondary School. Cebu: Abellan High School Press, 1974.

Jacobson, Paul B. et al. The Effective School Principal. Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1983.

Philips, Herbert H. Education and Development, Economics and Social Aspects of Education Planning. New York: Reinhold Publishing Corp., 1978.

Sison, Perfecto S. Personnel and Human Resources Management. Quezon City: Rex Printing Co., Inc., 1981.

Walpole, Ronald E. Introduction to Statistics. New York: McMillan Publishing Co., Inc., 1982.

B. PAMPHLETS AND PERIODICALS

Code of Ethics for Public School Teachers and Officials, B.F.S. Circular No. 7, s. 1950.

Constitution of the Philippines, 1987.

Constantino, Renato. Miseducation of the Filipino. Quezon City: Malaya Publishing Co., 1969.

C. UNPUBLISHED MATERIALS

Bacho, Cordula P. "Factors Affecting the NCEE Performance of Students of Five Selected Coastal High Schools in Samar", (Unpublished Master's Thesis, Samar State Polytechnic College, Catbalogan, Samar, 1991).

Bulut, Rosario, "Management of Fishery schools in Eastern Visayas", (Unpublished Doctoral Dissertation, Samar State Polytechnic College, Catbalogan, Samar, 1991).

Cairo, Hermogenes N. “Perceptions of the PASKO: Their Relation to the Performance of Key Officials in the Division of Samar”, (Unpublished Master’s Thesis, Samar State Polytechnic College, Catbalogan, Samar, 1986).

Calumpiano, Gail G. “Teaching effectiveness at the Samar Regional School of Fisheries: Perceptions of Teachers and Students”. (Unpublished Master’s Thesis, Samar State Polytechnic College, Catbalogan, Samar, 1986).

Caveiro, Antonio F. “The Administrative and Supervisory Styles of Secondary School Administrators: Bases for Policy Redirection”, (Unpublished Master’s Thesis, Samar College, Catbalogan, Samar, 1997).

Guerra, Letecia R. “Management Style on the Quality of School Governance in Selected Private High Schools and Colleges in Eastern Visayas High Schools and Colleges in Eastern Visayas, Philippines”, (Unpublished Doctoral Dissertation, Samar State Polytechnic College, Catbalogan, Samar, 1990).

Jabiñar, Remedios A. “Motivational Practices of Elementary School Principals in Relation to the Four Major Functions of Management”, (Unpublished Master’s Thesis, Samar College, Catbalogan, Samar, 1989).

Llauder, Gemma D. "Influence of the PAST on the Achievement of Teachers in Wright District, Division of Samar", (Unpublished Master's Thesis, Samar College, Catbalogan, Samar, 1987).

Mabini, Crescencia "The Performance of College Teachers of the Samar State Polytechnic College School Year 1984-85: An Evaluation", (Unpublished Master's Thesis, Samar College, Catbalogan, Samar, 1987).

Namia, Renato C. ``Administration and Supervisory Competencies of Public and Private Secondary school Principals", (Unpublished Master's Thesis, Leyte Institute of Technology, Tacloban City, 1979).

Quitalig, Thelma C. "Management System of Newly Nationalized High School in Samar: Bases for the Development Plan", (Unpublished Doctoral Dissertation, Samar State Polytechnic College, Catbalogan, Samar, 1993).

D. OTHER SOURCES

Philippine Daily Inquirer, Volume IX, Number 163: May 21, 1994.

Philippine Daily Inquirer, Volume IX, Number 167: May 25, 1994.

Bandera, Volume 8, Number 65: December 12, 1997.

Philippine Daily Inquirer, Volume 13, Number 22: December 30, 1997.

APPENDICES

APPENDIX A

Republic of the Philippines
Department of Education, Culture and Sports
Region VIII
Division of Samar
CALAPI NATIONAL HIGH SCHOOL
Calapi, Motiong, Samar

November 7, 1996

The DEAN
Graduate and Post Graduate Department
Samar State Polytechnic College
Catbalogan, Samar

Madam:

In my desire to start writing my thesis proposal, I have the honor to submit for your approval one of the following problems, preferably problem number 1.

1. TEACHING STRATEGY USING KEY WORDS IN MATHEMATICS CONCEPT EMPHASIS: THEIR EFFECTS ON THE STUDENTS' PERFORMANCE.

2. SELECTED CORRELATES IN TEACHING COMPETENCIES OF FIRST YEAR MATHEMATICS TEACHERS IN THE DIVISION OF SAMAR: THEIR IMPLICATION TO MANAGEMENT.

3. THE SEDP MATHEMATICS CURRICULUM PROGRAM; AN EVALUATION.

Hoping for your early and favorable action on this request.

Very truly yours,

(SGD.) JOSEPHINE E. BACSAL
Researcher

APPROVED:

(SGD.) RIZALINA M. URBIZTONDO, Ed.D.
Dean, Graduate & Post Graduate Studies

APPENDIX B

Republic of the Philippines
SAMAR STATE POLYTECHNIC COLLEGE
Catbalogan, Samar

SCHOOL OF GRADUATE STUDIES

APPLICATION FOR ASSIGNMENT OF ADVISER

NAME: BACSAL, (Surname) JOSEPHINE (First Name) EBIAS (Middle Name)

CANDIDATE FOR DEGREE: Master of Arts in Teaching

AREA OF SPECIALIZATION: Mathematics

TITLE/ PROPOSED THESIS/DISSERTATION: TEACHING STRATEGY IN MATHEMATICS, CONCEPT EMPHASIS USING KEY WORDS: THEIR EFFECTS ON STUDENTS' PERFORMANCE

(SGD.) JOSEPHINE E. BACSAL
Applicant

FLORIDA B. MARCO
Name of Designated Adviser

APPROVED:

(SGD.) RIZALINA M. URBIZTONDO, Ed.D.
Dean, Graduate & Post Graduate Studies

Conforme:

(SGD.) FLORIDA B. MARCO
Adviser

In 3 copies: 1st copy - for the Dean
2nd copy - for the Adviser
3rd copy - for the Applicant

APPENDIX C

Profile of Age, Sex and Ave. Grade in Math I of the Control Class (First Year - I)

	No.	Names	Ave. Grade Math I	Sex	Age
*	1.	Ardel J. Bersola	75 Dropped	M	16
	2.	Joel M. Dacles	87	M	13
	3.	Catalino T. Dacutanan	89.5	M	15
*	4.	Jimmy B. Francisco	85	M	16
	5.	Nelcar J. Gabiana	90.5	M	14
	6.	Alberto M. Gabuya	92	M	14
*	7.	Wilson M. Gabuya	75	M	16
	8.	Rolly L. Jabinar	92	M	14
*	9.	Michael G. Jabolin	87	M	16
*	10.	Emelito G. Laboc	75.5	M	13
*	11.	Noli M. Labong	75	M	16
*	12.	Pepito L. Oblino	75	M	21
*	13.	Ritchelle L. Abelleta	82.5	F	14
*	14.	Realiza M. Abarracoso	75.5	F	13
*	15.	Lorena G. Abawag	84	F	15
*	16.	Ann B. Anievas	86	F	14
*	17.	Jimmely D. Babacyao	82	F	13
*	18.	Aileen G. Cabasaris	78 Dropped	F	15
	19.	Gina B. Dacatimban	85	F	15
*	20.	Michelle L. Eclipse	83	F	13
*	21.	Lita M. Fabillar	84	F	14
*	22.	Eleanor C. Gabani	78	F	15
	23.	Elena D. Gabiana	89.5	F	13
	24.	Gina J. Gabiana	84	F	12
*	25.	Mally M. Gabin	77	F	14
	26.	Renalie A. Golondrina	88	F	15
*	27.	Aida G. General	80	F	15
*	28.	Lilia G. Jaba-an	85.5	F	13
	29.	Divina O. Labong	87	F	14
	30.	Arlene G. Mabanan	90.5	F	13
	31.	Wilma G. Mabanan	91	F	13
*	32.	Susana G. Mabubay	84.5	F	13
*	33.	Roselle G. Obinguar	87.5	F	15
*	34.	Nancy G. Pacios	82 Dropped	F	14
*	35.	Emerita B. Paclita	86	F	13
	36.	Victoria G. Paclita	89	F	16

* Respondent

APPENDIX D

Profile of Age, Sex and Ave. Grade in Math I of the Experimental Class (First Year - II)

No.	Names	Ave. Grade Math I	Sex	Age
1.	Ariel L. Abawag	77	M	13
2.	Jonathan L. Bacnutan	76	M	13
3.	Marlon M. Crodua	80.5	M	13
* 4.	Lee F. Cuna	75.5	M	13
5.	Emelito G. Dacles	81	M	14
6.	Rolando G. Fabriag	77.5	M	14
* 7.	Elmer M. Gagbo	75	M	21
8.	Warlito M. Gagbo	76	M	23
* 9.	Jaime M. Lazarra	75	M	14
10.	Melbor L. Mabanan	84	M	15
* 11.	Louie C. Mabilangan	75	M	15
12.	Patrick D. Miraveles	76.5	M	16
13.	Apolonio P. Plana Jr.	78	M	13
* 14.	Alberto G. Pacios	87.5	M	15
15.	Lolito L. Pacle	77	M	21
* 16.	Ariel L. Sumbise	85.5	M	14
* 17.	Maricel O. Bersola	83	F	14
* 18.	Fulgencia A. Cabangunay	75.5	F	15
* 19.	Aileen J. Cabarles	83 Dropped	F	15
* 20.	Marife F. Cabatuan	84.5	F	16
* 21.	Adelina J. Dacles	84	F	13
22.	Vangeline P. Dacutanan	77.5	F	15
* 23.	Aileen P. Enolfe	88	F	15
24.	Fenida L. Gabane	82	F	17
* 25.	Thelma P. Gabane	78	F	13
* 26.	Marina B. Gabiana	77	F	14
* 27.	Marilyn C. Gabin	86	F	14
28.	Janet B. Jaba-an	81.5	F	13
29.	Maricar B. Jaba-an	81	F	16
* 30.	Prudencia G. Jaba-an	80	F	16
* 31.	Analyn D. Labido	82	F	14
* 32.	Amor G. Laboc	85	F	13
* 33.	Rona A. Llanita	80 Dropped	F	18
* 34.	Maricris C. Mabilangan	87	F	15
35.	Michelle L. Mabilangan	78.5	F	13
36.	Wilma C. Mabilangan	78	F	14
* 37.	Hazel O. Obinguar	84	F	13

APPENDIX D (Cont'd.)

No.	Names	Ave. Grade Math I	Sex	Age
* 38.	Mryna J. Obinguar	82.5	F	16
* 39.	Alberta M. Padernos	86	F	15
40.	Junalyn O. Rapiza	82	F	13

* Respondent

APPENDIX E

Age, Sex and Ave. Grade in Math I of the Control Group

Respondent No.	Names	Age	Sex	Ave. Grade Math I
1.	Wilson M. Gabuya	16	M	75
2.	Noli N. Labong	16	M	75
3.	Pepito L. Oblino	21	M	75
4.	Realiza M. Abarracoso	13	F	75.5
5.	Emelito G. Laboc	13	M	75.5
6.	Mally M. Gabin	14	F	77
7.	Eleanor P. Gabane	15	F	78
8.	Aida G. General	15	F	80
9.	Jimmely D. Babacyao	13	F	82
10.	Ritchelle L. Abelleta	14	F	82.5
11.	Michelle L. Eclipse	13	F	83
12.	Lita O. Fabillar	14	F	84
13.	Lorena G. Abawag	15	F	84
14.	Susana F. Mabubay	15	F	84.5
15.	Jimmy B. Francisco	16	M	85
16.	Lilia G. Jaba-an	13	F	85.5
17.	Emerita B. Paclita	13	F	86
18.	Ann B. Anievas	14	F	86
19.	Michael G. Jabolin	16	M	87
20.	Roselle G. Obinguar	15	F	87.5
Total		294	F = 14 M = 6	1627
Mean		14.70		81.35
SD		1.87		5.55

APPENDIX F

Age, Sex and Ave. Grade in Math I of the Experimental Group

Respondent No.	Names	Age	Sex	Ave. Grade Math I
1.	Elmer M. Gagbo	21	M	75
2.	Louie C. Mabilangan	15	M	75
3.	Jaime M. Lazarra	14	M	75
4.	Fulgencia Cabanguanay	15	F	75.5
5.	Lee F. Cuna	13	M	75.5
6.	Marina B. Gabiana	14	F	77
7.	Thelma P. Gabane	13	F	78
8.	Prudencia G. Jaba-an	16	F	80
9.	Analyn D. Labido	14	F	82
10.	Myrna J. Obinguar	16	F	82.5
11.	Maricel O. Bersola	14	F	83
12.	Hazel O. Obinguar	13	F	84
13.	Adelina J. Dacles	13	F	84
14.	Marife F. Cabatuan	16	F	84.5
15.	Amor G. Laboc	13	F	85
16.	Ariel L. Sumbise	14	M	85.5
17.	Marilyn C. Gabin	14	F	86
18.	Alberta M. Padernos	15	F	86
19.	Maricris C. Mabilangan	15	F	87
20.	Alberto G. Pacios	15	M	87.5
Total		293	F = 14 M = 6	1627
Mean		14.65		81.35
SD		1.81		5.55

APPENDIX G

Republic of the Philippines
SAMAR STATE POLYTECHNIC COLLEGE
Catbalogan, Samar

APPLICATION FOR PRE-ORAL DEFENSE

October 15, 1997

The DEAN
Graduate School
Samar State Polytechnic College
Catbalogán, Samar

Madam:

I have the honor to apply for Pre-Oral Defense of my thesis entitled TEACHING STRATEGY USING KEY WORDS IN MATHEMATICS CONCEPT EMPHASIS: THEIR EFFECTS ON THE STUDENTS' PERFORMANCE on the date convenient for your office.

Very truly yours,

(SGD.) JOSEPHINE E. BACSAL
Graduate Student

Recommending Approval:

(SGD.) Prof . FLORIDA B. MARCO
Adviser

A P P R O V E D:

(SGD.) RIZALINA M. URBIZTONDO, Ed.D.
Dean, Graduate & Post Graduate Studies

Date: ____________________
Time: ____________________

APPENDIX H

Table of Specification for Pretest/Posttest Subjected to Validation

TOPICS	Item No.	Cognitive Skills K	C	A	HA	Total
Constants and Variables	1-4	2	0	0	2	4
Mathematical Phrases and Sentences	5-9	0	3	1	1	5
Coefficient, Exponents Base & Power	10-16	1	5	0	1	7
Evaluation of Numerical and Algebraic Expressions	17-23	0	1	4	2	7
Algebraic Expressions & Polynomials	24-29 40-43	1	5	2	2	10
Addition & Subtraction of Monomials & Polynomials	30-33	0	1	3	0	4
Multiplication of Polynomials	34-39	1	2	2	1	6
Division of Polynomials	44-50	0	2	3	2	7
Total		5	19	15	11	50

APPENDIX I

PRETEST/POSTTEST FOR VALIDATION

MULTIPLE CHOICE: Choose the letter of the correct answer and write it on your answer sheet.

1. What are the constants in the algebraic expression $4x^2 -3y$?
   a. 4, –3
   b. x
   c. 2
   *d. 4, –3 and 2

2. What is 6 in the given algebraic expression $2x^2 -3y +6$?
   a. variable
   b. exponent
   c. base
   *d. constant

3. If $n^2 < 12$, what are the values of positive integer n?
   a. n = 1, 2, 3, . . .
   b. n = 1, 2, 3
   c. n = – 3, –2, –1, 0, 1, 2, 3
   *d n = 0, 1, 2, 3

4. A certain number x is less than –3. Which of the following numbers can replace x?
   a. 1
   b. –1
   c. –2
   *d. –4

5. What is the mathematical representation of "five fourths of a number x"?
   a. 4x = 5
   b. x + 5/4
   *c. 5/4 x
   d. 5/4 + x

6. The expression 2 – x means ______ .
   a. 2 is less than the number x.
   *b. 2 minus the number x.
   c. 2 is less than x.
   d. the number x diminished by 2.

7. The area of a square with side s is 25. How is this statement represented mathematically?
   a. s = 25
   b. 2s = 25
   *c. $s^2 = 25$
   d. 4s = 25

8. Which expression represents the cost of 7 pieces of atis at x pesos per piece?
   *a. 7x
   b. x + 7
   c. 7 – x
   d. x / 7

9. Which of these numbers is greater than (–4) but less than 1?
   a. (–5)
   b. (–4)
   *c. 0
   d. 1

10. Which of these algebraic expressions have a numerical coefficient of 2?
   a. x^2
   *b. 2xy
   c. m^2n^2
   d. x/2

11. What is the exponent in the expression, $3x^4m$?
   a. 3
   *b. 4
   c. x
   d. m

12. Which is the same as $(a/b)^3$?
   *a. a^3/b^3
   b. a^3/b
   c. 3a /b
   d. 3a /3b

13. The shorter expression for 2 • 2 • 2 • x • x • x • y • y • y is ______.
   a. $2xy^3$
   b. 3(2xy)
   *c. $8x^3y^3$
   d. 6(3x3y)

14. Which of the following statements is true?
   i. x^4 is the same as 4x.
   ii. y + y can be written as y^2
   iii. $(2x)^3$ is equal to $8x^3$

   a. i only
   *b. iii only
   c. i and ii only
   d. ii and iii

15. What is the coefficient of x^2 in $-1/2\ x^2$?
   a. ½
   *b. –½
   c. 1
   d. 2

16. What is the numerical coefficient of 3y/4?
   a. 3
   b. 4
   *c. 3/4
   d. y

17. What is the value of the expression x + 3 when x is10?
   a. 3
   b. 7
   c. 10
   *d. 13

18. When $x = 2$, which of these expressions gives 12?
   a. $10 - x$
   b. $14 - (-3)$
   *c. $3x^2$
   d. $12x$

19. What is the value of $[(x - 2y)]$ when $x = 8$, $y = 2$, and $z = (-6)$?
   a. 16
   b. 6
   *c. -2
   d. -10

20. When $m = 3$ and $n = (-3)$, what is the value of the expression $9m + 2n$?
   a. 27
   b. (-128)
   * c. (-101)
   d. 155

21. If operation [x] is defined by the equation $a[x]b = a + b^2$ then $2[x]3 =$ ____?
   a. 7
   *b. 11
   c. 12
   d. 25

22. Which expression is **NOT** equal to $4(y) - 1$?
   a. $(4) - 1$
   b. $(4y) - 1$
   c. $4(y - 1)$
   *d. $4y - 1$

23. Which of these statements is true if x^2 and $2x$ are to be replaced by integers?

   i. x^2 is greater than $2x$ when x is greater than 2.
   ii. the two expressions are equal when $x = 2$.

   a. i only
   b. ii only
   *c. both i and ii
   d. neither i nor ii

24. The perimeter of a quadrilateral whose sides are 5n, 3/2n, 2n, and 5/4n is _______?
   a. $15/6n^4$ +
   b. 7n + 8/4n
   *c. 39n/4
   d. $7n^2 + 8n^2/6$

25. The perimeter (P) of a triangle is P = a + b +c where a, b, and c are its sides. If P = 45, which set of values cannot be length of the sides of triangle ABC?
   a. {15, 15, 15}
   b. {12, 10, 23}
   *c. {5, 12, 19}
   d. {10, 10, 25}

26. If A = 3x + 2 and B = x -2, what is A+B?
   a. 4x + 4
   b. 4x – 4
   *c. 4x
   d. $3x^2$

27. The sum of $5x^2y^4$, $10x^2y^4$, and $(-12x^2y^4)$ is _______.
   a. $2x^2y^4$
   *b. $3x^2y^4$
   c. $15x^2y^4$
   d. $27x^2y^4$

28. What is the sum of 5x and –4y?
   *a. 5x – 4y
   b. – x y
   c. 5x + 4y
   d. none of these

29. Given a + b = 5. If b is increased by 3. What will happen to a, if the sum is to remain the same?
   a. became zero
   b. will increase
   *c. will decrease
   d. remain the same

30. What number is added to 3½ which will give 15 4/5?
*a. 12 3/10
b. 12 7/10
c. 11 1/10
d. 11 7/10

31. What must be added to $30mn^2$ to get 25 mn^2?
a. 5 mn^2
b. 20 mn^2
*c. (-5 mn^2)
d. (-10 mn^2)

32. What is the difference if the sum of $3y^2$ and $8y^2$ is subtracted from $15y^2$?
*a. $4y^2$
b. $11y^2$
c. $18y^2$
d. $23y^2$

33. What is the simplified form of $3x - (2y - x)$?
a. $3x - y$
b. $4x - y$
*c. $4x - 2y$
d. $2x - 2y$

34. What is the value of $-x$ if $x = 5$?
a. -10
b. 10
*c. -5
d. 5

35. What are the factors of 5xyz?
a. x, y, z
b. 5
c. 5, z
*d. 5, x, y, z

36. Which of the following statements is true?
a. 12 is a multiple of 5.
*b. 4 is a factor of 20.
c. 15 is divisible by 2.
d. 5 is divisible by 18.

37. What is the square of xy^2?
   a. $2x(4x)$
   b. xy^4
   *c. $x^2 y^4$
   d. $x^4 y^8$

38. What is the product of $3x^2y^4$ and $7xy^2$?
   a. $10x^3y^6$
   b. $21x^3y^8$
   c. $10x^2y^6$
   *d. $21x^3y^6$

39. If $x + 2x + 3x + 4x = 1+2+3+4$, then $x =$ ____?
   a. 0
   *b. 1
   c. 2
   d. 3

40. Which is an example of a monomial?
   *a. $2x^3y^5$
   b. $x + 1$
   c. $(a - b) + 2$
   d. $x + y + z$

41. Which is an example of <u>a term</u> in an algebraic expression?
   a. $3x^2y - 5xy + 7$
   b. $3x^2 + 9x(2y + z)$
   c. $2x - 3y$
   *d. $(-2xy + y - 3)$

42. How many terms has the expression, $2x^2 + \frac{4x + 5y}{3} + 7$ have?
   a. three
   *b. four
   c. one
   d. two

43. What is the degree of the first term of the given polynomial $x^2 - 4xy^2 + 4$?
   *a. two
   b. three
   c. one
   d. none of these

44. What value/values of x (if any) is the expression $\frac{x-1}{x+3}$ undefined?

a. x = 3
*b. x = -3
c. x > 3
d. x <3

45. What value of x will make the denominator equal to zero?

$$\frac{x^2 - 2xy + y^2}{x - 2}$$

a. x = 0
*b. x = 2
c. x = –2
b. none of these

46. What expression must be multiplied by 4x to get $20x^2y^3$?

a. 4xy
*b. $5xy^2$
c. 10xy
d. $20xy^2$

47. If $(-28x^5y^8)$ is divided by $(-4x^2y^3)$, the quotient is ______.

a. $7x^2y^3$
b. $7x^2y^5$
c. $(-7x^2y^3)$
*d. $(7x^3y^5)$

48. What is the quotient when $24x^2y^2$ is divided by –3xy?

a. 8xy
b. $8x^2$
*c. –8xy
c. $8y^2$

49. What expression is equal to 5x/9?

a. 9 + 5x
b. 5 x 9
c. 9/5 x
*d. 5x (1/9)

50. The simplest form of $\frac{8x^3 + 9x^3 - 2x^3}{3x^3}$ is ________.

a. 5x
b. x
*c. 5
c. $5x^3$

APPENDIX J

PRETEST/POSTTEST

MULTIPLE CHOICE: Choose the letter of the correct answer and write it on your answer sheet.

1. What is 6 in the given algebraic expression $2x^2 -3y +6$?
   a. variable
   b. exponent
   c. base
   *d. constant

2. A certain number x is less than –3. Which of the following numbers can be the value of x?
   a. 1
   b. –1
   c. –2
   *d. –4

3. What is the mathematical symbol of "five fourths of a number x"?
   a. $4x = 5$
   b. $x + 5/4$
   *c. $5/4\ x$
   a. $5/4 + x$

4. What is the meaning of the expression $2 - x$?
   a. 2 is less than the number x.
   *b. 2 minus the number x.
   c. 2 is less than x.
   d. the number x diminished by 2.

5. The area of a square with side s is 25. How is this statement represented mathematically?
   a. $s = 25$
   b. $2s = 25$
   *c. $s^2 = 25$
   d. $4s = 25$

6. Which expression represents the cost of 7 pieces of atis at x pesos per piece?
   *a. $7x$
   b. $x + 7$
   c. $7 - x$
   d. $x / 7$

7. Which of these numbers is greater than (–4) but less than 1?
   a. (–5)
   b. (–4)
   *c. 0
   d. 1

8. Which of these algebraic expressions have a numerical coefficient of 2?
   a. x^2
   *b. $2xy$
   c. m^2n^2
   d. $x/2$

9. What is the exponent in the expression, $3x^4m$?
   a. 3
   *b. 4
   c. x
   d. m

10. Which is the same as $(a/b)^3$?
   *a. a^3/b^3
   b. a^3/b
   c. $3a/b$
   d. $3a/3b$

11. What is the shorter expression for $2 \bullet 2 \bullet 2 \bullet x \bullet x \bullet x \bullet y \bullet y \bullet y$?
   a. $2xy^3$
   b. $3(2xy)$
   *c. $8x^3y^3$
   d. $6(3x3y)$

12. What is the numerical coefficient of $3y/4$?
   a. 3
   b. 4
   *c. 3/4
   d. y

13. What is the value of the expression x + 3 when x is10?
    a. 3
    b. 7
    c. 10
    *d. 13

14. When x = 2, which of these expressions gives 12?
    a. 10 – x
    b. 14 – (-3)
    *c. $3x^2$
    d. 12x

15. Which expression is **NOT** equal to 4(y) – 1?
    a. (4) –1
    b. (4y) –1
    c. 4(y –1)
    *d. 4y –1

16. Which of these statements is true if x^2 and 2x are to be replaced by integers?

    i. x^2 is greater than 2x when x is greater than 2.
    ii. the two expressions are equal when x = 2.

    a. i only
    b. ii only
    *c. both i and ii
    d. neither i nor ii

17. The perimeter of a quadrilateral whose sides are 5n, 3/2n, 2n, and 5/4n is _______?
    a. $15/6n^4$ +
    b. 7n + 8/4n
    *c. 39n/4
    d. $7n^2 + 8n^2 /6$

18. What is the sum of $5x^2y^4$, $10x^2 y^4$, and $(-12x^2 y^4)$?
    a. $2x^2 y^4$
    *b. $3x^2 y^4$
    c. $15x^2 y^4$
    d. $27x^2 y^4$

19. Given a + b = 5. If b is increased by 3, what will happen to a, if the sum is to remain the same?
    a. became zero
    a. will increase
    *c. will decrease
    d. remain the same

20. What must be added to $30mn^2$ to get 25 mn^2?
    a. 5 mn^2
    b. 20 mn^2
    *c. (–5 mn^2)
    d. (–10 mn^2)

21. What is the simplified form of 3x – (2y –x)?
    a. 3x - y
    b. 4x – y
    *c. 4x – 2y
    d. 2x – 2y

22. if x = 5, what is –x?
    a. –10
    b. 10
    *c. – 5
    d. 5

23. Which statement is true?
    a. 12 is a multiple of 5.
    *b. 4 is a factor of 20.
    b. 15 is divisible by 2.
    c. 5 is divisible by 18.

24. What is the square of xy^2?
    a. 2x(4x)
    b. xy^4
    *c. $x^2 y^4$
    d. $x^4 y^8$

25. Which of the following choices is an example of a monomial?
    *a. $2x^3y^5$
    b. x + 1
    c. (a – b) + 2
    d. x + y + z

26. How many terms are there in the given expression?

$$2x^2 + \frac{4x + 5y}{3} + 7$$

a. three
*b. four
c. one
d. two

27. What is the degree of the first term of the given polynomial $x^2 - 4xy^2 + 4$?
*a. two
b. three
c. one
d. none of these

28. What expression must be multiplied by 4xy to get $20x^2y^3$?
a. 4xy
*b. $5xy^2$
c. 10xy
d. $20xy^2$

29. What is the quotient when $24x^2y^2$ is divided by –3xy?
a. 8xy
b. $8x^2$
*c. –8xy
d. $8y^2$

30. What expression is equal to 5x/9?
a. 9 + 5x
b. 5 x 9
c. 9/5 x
*d. 5x (1/9)

Appendix K

Item Analysis of Test Instrument (Pretest/Posttest)

Item No.	Key	Correct Responses UG	: LG	Dif.Ind. p = U+L / 2N	Interpretation	Dis.Ind. D = U-L / N	Interpretation	Decision
1.	d	13	12	.89	VE	.07	P	Reject
2	d	13	8	.75	E	.36	RG	Accept
3	d	12	12	.85	E	0	P	Reject
4	d	8	4	.43	MD	.28	NI	Modify
5	c	11	7	.64	MD	.28	NI	Modify
6	b	10	6	.57	MD	.28	NI	Modify
7	c	8	3	.39	D	.36	RG	Accept
8	a	11	5	.50	MD	.43	VG	Accept
9	c	13	4	.61	MD	.64	VG	Accept
10	b	9	2	.39	D	.50	VG	Accept
11	b	8	3	.39	D	.36	RG	Accept
12	a	7	2	.32	D	.36	RG	Accept
13	c	9	5	.50	MD	.28	NI	Modify
14	b	13	12	.89	VE	.07	P	Reject
15	b	10	13	.82	E	-.21	P	Reject
16	c	10	5	.53	MD	.36	RG	Accept
17	d	12	7	.68	MD	.36	RG	Accept
18	c	13	6	.68	MD	.50	RG	Accept
19	c	3	1	.14	VD	.14	P	Reject
20	c	12	11	.82	E	.07	P	Reject
21	b	13	11	.85	E	.14	P	Reject
22	c	9	4	.46	MD	.36	RG	Accept
23	c	8	3	.39	D	.36	RG	Accept
24	c	11	3	.50	MD	.57	RG	Accept
25	c	1	3	.14	VD	-.14	P	Reject
26	c	10	14	.85	E	-.28	P	Reject
27	d	12	8	.71	E	.28	NI	Modify
28	a	6	4	.35	D	.14	P	Reject
29	c	11	6	.61	MD	.36	RG	Accept
30	c	1	0	.04	VD	.07	P	Reject
31	c	9	4	.46	MD	.36	RG	Accept
32	a	2	1	.11	VD	.07	P	Reject
33	c	8	3	.39	D	.36	RG	Accept

Appendix K (Cont'd.)

Item No.	Key	Correct Responses UG	: LG	Dif.Ind. p = (U+L)/2N	Interpretation	Dis.Ind. D = (U-L)/N	Interpretation	Decision
34	c	10	6	.57	MD	.28	NI	Modify
35	d	4	3	.25	D	.07	P	Reject
36	b	9	5	.50	MD	.28	NI	Modify
37	c	13	7	.71	E	.43	VG	Accept
38	d	3	4	.25	D	-.07	P	Reject
39	b	14	14	1	VD	0	P	Reject
40	a	12	8	.71	E	.28	NI	Modify
41	d	4	4	.28	D	0	P	Reject
42	b	9	5	.50	MD	.28	NI	Modify
43	a	11	6	.61	MD	.36	RG	Accept
44	b	2	4	.21	D	-.14	P	Reject
45	b	5	3	.28	D	.14	P	Reject
46	b	11	4	.53	MD	.50	VG	Accept
47	d	14	11	.11	VD	.21	NI	Reject
48	c	10	4	.50	MD	.43	VG	Accept
49	d	12	6	.64	MD	.43	VG	Accept
50	c	14	12	.07	VD	.14	P	Reject

Legend:
VD - Very Difficult
MD - Moderately Difficult
D - Difficult
E - Easy
VE - Very Easy

P - Poor
NI - Needs Improvement
RG - Reasonably Good
VG - Very Good

APPENDIX L

Computation of the t – value of the Difference Between Means of the Pretest Scores of the Experimental and Control Groups

Experimental Group

$$\bar{x}_{11} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{295}{20} = 14.75$$

$$s_{11}^2 = \frac{n\sum x_{11}^2 - (\sum x_{11})^2}{n\,(n-1)}$$

$$= \frac{20(4649) - (295)^2}{20(20-1)}$$

$$= \frac{92980 - 87025}{20(19)}$$

$$= \frac{5955}{380}$$

$$= 15.67$$

Control Group

$$\bar{x}_{12} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{290}{20} = 14.50$$

$$s_{12}^2 = \frac{n\sum x_{12}^2 - (\sum x_{12})^2}{n\,(n-1)}$$

$$= \frac{20(4454) - (290)^2}{20(20-1)}$$

$$= \frac{89080 - 84100}{20(19)}$$

$$= \frac{4980}{380}$$

$$= 13.11$$

$$t = \frac{\bar{x}_{11} - \bar{x}_{12} - \delta}{\sqrt{\frac{(n_{11}-1)s_{11}^2 + (n_{12}-1)s_{12}^2}{n_{11}+n_{12}-2}\left[\frac{1}{n_{11}} + \frac{1}{n_{12}}\right]}}$$

$$t = \frac{14.75 - 14.50}{\sqrt{\frac{(20-1)15.67 + (20-1)13.1}{20+20-2}\left[\frac{1}{20} + \frac{1}{20}\right]}}$$

$$t = \frac{-0.25}{\sqrt{\frac{297.73 + 249.09}{38}(0.1)}}$$

$t = 0.208$

t_{05}, $df = 38$, is 1.645 not significant, $p < .05$

Appendix M

Computation of the t – value of the Difference Between Means of the Posttest Scores of the Experimental and Control Groups

Experimental Group

$$\bar{x}_{21} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{457}{20} = 22.85$$

$$s_{21}^2 = \frac{n\sum x_{21}^2 - (\sum x_{21})^2}{n(n-1)}$$

$$= \frac{20(10599) - (457)^2}{20(20-1)}$$

$$= \frac{211980 - 208849}{20(19)}$$

$$= \frac{3131}{380}$$

$$= 8.24$$

Control Group

$$\bar{x}_{22} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{421}{20} = 21.05$$

$$s_{22}^2 = \frac{n\sum x_{22}^2 - (\sum x_{22})^2}{n(n-1)}$$

$$= \frac{20(9039) - (421)^2}{20(20-1)}$$

$$= \frac{180780 - 177241}{20(19)}$$

$$= \frac{3539}{380}$$

$$= 9.31$$

$$t = \frac{\bar{x}_{21} - \bar{x}_{22} - \delta}{\sqrt{\frac{(n_{21}-1)s_{21}^2 + (n_{22}-1)s_{22}^2}{n_{21}+n_{22}-2}\left[\frac{1}{n_{21}} + \frac{1}{n_{22}}\right]}}$$

$$t = \frac{22.85 - 21.05}{\sqrt{\frac{(20-1)8.24 + (20-1)9.31}{20+20-2}\left[\frac{1}{20} + \frac{1}{20}\right]}}$$

$$t = \frac{1.80}{\sqrt{\frac{156.56 + 176.89}{38}(0.1)}}$$

$t = 1.922$ $\qquad$ t_{05}, $df = 38$, is 1.645 significant, $p < .05$

APPENDIX N

Computation of the t – value of the Difference Between Means of the Posttest and Pretest Scores of the Control Group

$\Sigma D = -131$

$$\bar{D} = \frac{\Sigma D}{N} = \frac{-131}{20} = -6.55$$

$\Sigma D^2 = 947$

$N = 20$

$$S_D = \sqrt{\frac{\Sigma D^2}{N} - \bar{D}^2} = \sqrt{\frac{947}{20} - (-6.55)^2} = \sqrt{47.35 - 42.90}$$

$$= \sqrt{4.45} = 2.1095$$

$$S_{\bar{X}_D} = \frac{S_D}{\sqrt{N-1}} = \frac{2.1095}{\sqrt{20-1}} = \frac{2.1095}{\sqrt{19}} = \frac{2.1095}{4.36} = 0.4838$$

$$t = \frac{\bar{D}}{S_{\bar{X}_D}} = \frac{-6.55}{0.4838} = -13.54 \quad \text{disregard sign}$$

$t_{.05}$, $df = 19$, is 2.093 significant, $p < .05$

APPENDIX N

Computation of the t – value of the Difference Between Means of the Posttest and Pretest Scores of the Experimental Group

$\sum D = -162$

$$\bar{D} = \frac{\sum D}{N} = \frac{-162}{20} = -8.10$$

$\sum D^2 = 1452$

$N = 20$

$$S_D = \sqrt{\frac{\sum D^2}{N} - \bar{D}^2} = \sqrt{\frac{1452}{20} - (-8.10)^2} = \sqrt{72.60 - 65.61}$$

$$= \sqrt{6.99} = 2.64$$

$$S_{\bar{x}_D} = \frac{S_D}{\sqrt{N-1}} = \frac{2.64}{\sqrt{20-1}} = \frac{2.64}{\sqrt{19}} = \frac{2.64}{4.36} = 0.6055$$

$$t = \frac{\bar{D}}{S_{x_D}} = \frac{-8.10}{0.6055} = -13.38 \text{ disregard sign}$$

t_{05}, $df = 19$, is 2.093 significant, $p < .05$

APPENDIX NG

Computation of the Reliability Coefficient (r)

$$r = \frac{k}{k-1}\left(1 - \frac{6\sum pq}{(kd)^2}\right)$$

$$r = \frac{30}{30-1}\left(1 - \frac{6(5.2479)}{(30 \times 0.3988)^2}\right)$$

$$r = \frac{30}{29}\left(1 - \frac{31.4874}{143.13729}\right)$$

$$r = 1.03\ (1 - 0.210)$$

$$r = 0.8410$$

Interpretation: Fairly high, adequate for individual measurement.

APPENDIX O

Experimentation Activity Schedule
SY 1997 – 1998

Date		Day	ACTIVITIES
JAN.	19	Mon.	Administration and Validation of Instrument to Second Year Students of Calapi National High School
	21	Wed.	Pretesting
	22	Thurs.	Grouping of the First Year Students and Distribution of Workbook to First Year II
	23	Fri.	Taught Lesson 1.1 to Both Groups
	26	Mon.	Taught Lesson 1.2 to Both Groups
	27	Tues.	Taught Lesson 2.1 to Both Groups
	28	Wed.	Taught Lesson 2.2 to Both Groups
	29	Thurs.	Taught Lesson 3.1 to Both Groups
	30	Fri.	Taught Lesson 3.2 to Both Groups
FEB.	2	Mon.	Taught Lesson 4.1 to Both Groups
	3	Tues.	Taught Lesson 4.2 to Both Groups
	4	Wed.	Taught Lesson 4.3 to Both Groups
	5	Thurs.	Taught Lesson 4.4 to Both Groups
	6	Fri.	Administrators Conference
	9	Mon.	Taught Lesson 5.1 to Both Groups
	10	Tues.	Taught Lesson 5.2 to Both Groups
	11	Wed.	Taught Lesson 6.1 to Both Groups
	12	Thurs.	Taught Lesson 6.2 to Both Groups
	13	Fri.	Taught Lesson 7.1 to Both Groups
	16	Mon.	Taught Lesson 7.2 to Both Groups
	17	Tues.	Taught Lesson 8.1 to Both Groups
	18	Wed.	Taught Lesson 8.2 to Both Groups
	19	Thurs.	Wrap-Up
	20	Fri.	Posttesting

APPENDIX P

Republic of the Philippines
SAMAR STATE POLYTECHNIC COLLEGE
Catbalogan, Samar

APPLICATION FOR FINAL-ORAL DEFENSE

March 4, 1998

The DEAN
Graduate School
Samar State Polytechnic College
Catbalogan, Samar

Madam:

I have the honor to apply for Final Defense of my thesis entitled ______ **EFFECTIVENESS OF WORKBOOK IN TEACHING MATHEMATICS I** ______________________________ on the date convenient for your office.

Very truly yours,

(SGD.) JOSEPHINE E. BACSAL
Graduate Student

Recommending Approval:

(SGD.) Prof. FLORIDA B. MARCO
Adviser

A P P R O V E D:

(SGD.) RIZALINA M. URBIZTONDO, Ed.D.
Dean, Graduate & Post Graduate Studies

Date: ____________________
Time: ____________________

APPENDIX Q

Logbook for Mathematics I
SY 1997 – 1998

Date		Day	TOPICS	Remarks
JAN.	23	Fri.	Constants	Carried
	26	Mon.	Variables	Carried
	27	Tues.	Mathematical Phrases	Carried
	28	Wed.	Mathematical Sentences	Carried
	29	Thurs.	Coefficient and Exponent	Carried
	30	Fri.	Power and Base	Carried
FEB.	2	Mon.	Numerical Expressions	Carried
	3	Tues.	Algebraic Expressions	Carried
	4	Wed.	Evaluation of Numerical Expressions	Carried
	5	Thurs.	Evaluation of Algebraic Expressions	Carried
	9	Mon.	Classification of Mathematical	
	10	Tues.	Expressions	Carried
	11	Wed.	Addition of Monomials and Polynomials	Carried
	12	Thurs.	Subtraction of Monomials	Carried
	13	Fri.	Multiplication of Monomials	Carried
	16	Mon.	Multiplication of Polynomials by Monomials and Another Polynomials	Carried
	17	Tues.	Division of Monomials	Carried
	18	Wed.	Division of Polynomials	Carried

APPENDIX R

INTERPRETATION OF THE READING EASE SCORE AND HUMAN INTEREST SCORE OF THE FLESCH FORMULA

Reading Ease Scale

RES	Description Style	Corrected Grade Level
90 – 100	Very Easy	5th grade
89 – 90	Easy	6th grade
70 – 80	Fairly Easy	1st - 2nd year High School
60 – 70	Standard	3rd – 4th year High School
50 – 60	Fairly Difficult	1st - 2nd year College
30 – 50	Difficult	3rd - 4th year College
0 – 30	Very Difficult	College graduate

Human Interest Scale

HIS	Description of Style
60 – 100	Dramatic
40 – 60	Highly Interesting
20 – 30	Interesting
10 – 20	Mildly Interesting
0 – 10	Dull

APPENDIX S

COMPUTATION OF THE READING EASE SCORE

Page No.	No. of Words	No. of Sentences	No. of Syllables
3	100	11	142
4	100	9	150
5	100	11	159
6	100	14	132
7	100	10	152
8	100	15	155
9	100	9	146
10	100	13	144
11	100	11	137
19	100	13	139
31	100	14	147
32	100	9	164
33	100	10	171
34	100	10	158
35	100	7	120
36	100	11	125
37	100	9	151
38	100	12	120
46	100	13	181
48	100	10	159
70	100	18	158
72	100	17	140
74	100	17	173
75	100	14	143
103	100	11	165
110	100	12	158
Total	2600	310	3889

Appendix S (Cont'd.)

Ave. Sentence Length = 2600/310 = 8.39

Ave. Word Length = 3889/26 = 149.57

RES = 206.835 - (1.015 x 8.39 + 0.846 x 149.570) =
= 206.835 - (8.515 + 126.536)
= 206.835 - 135.051
= 71.784

Interpretation: **Fairly Easy** suited to 1^{st} - 2^{nd} year High School

APPENDIX T

COMPUTATION OF THE HUMAN INTEREST SCORE OF THE FLESCH FORMULA

Page No.	No. of Personal Words	No. of Personal Sentences
3	3	2
4	3	2
5	1	1
6	1	1
7	2	1
8	3	2
9	2	2
10	5	3
11	8	4
19	1	1
31	2	1
32	1	1
33	2	1
34	2	1
35	3	3
36	1	1
37	2	1
38	1	1
46	3	2
48	3	3
70	2	1
72	1	1
74	1	1
75	2	2
103	2	2
110	2	2
Total	58	41

Appendix T (Cont'd.)

% Personal Words = 58/2600 = .022 x 100 = 2.2 %

% Personal Sentences = 41/310 = .132 x 100 = 13.2 %

HIS = (2.2 x 3.635) + (13.2 x 0.314)

= 7.997 + 4.1448

= 12.14

Interpretation: Mildly interesting

WORKBOOK in Mathematics I

Algebraic Expressions & Operations

Overview

Algebra is actually an extension of arithmetic. It uses the same operations and symbols. Algebra differs from arithmetic in its use of letters to stand for numbers.

This workbook discusses in details major topics in Algebra such as translating ideas in English into mathematical symbols, evaluating algebraic expressions, performing arithmetical operations with integers, simplifying expression with particular emphasis on the function of exponents, performing the following operations: addition, subtraction, multiplication, and division of monomials and polynomials.

This workbook begins with an introduction of constants and variables followed by how variables are used and how operations are performed with algebraic expressions.

General Objectives

1. Demonstrate understanding of algebraic expressions.
2. Acquire skills in translating expressions.
3. Perform operations on monomials and polynomials.
4. Show desirable attitudes like accuracy and patience in performing operations.

TABLE OF CONTENTS

TABLE OF CONTENTS
(Cont'd.)

Lesson 1

Constants and Variables

Lesson 1. CONSTANTS AND VARIABLES

Objectives:

Content:
1. To identify constants and variables.
2. To describe a constant and a variable.
3. To find the value of the variable of an expression.

Process:
1. To recognize difference between constants and variables.
2. To give examples of constants and variables.

Affective:
1. To show appreciation in the value of constants and variables in daily life situation.

1.1 CONSTANTS

Constants are numbers or symbols that have a fixed value. They can be real numbers, rational numbers, irrational numbers, integers, whole numbers, natural or counting numbers and others.

The Real Number System
(A System of Constants)

Real Numbers

The set of **real numbers** contains the set of *rational numbers* and the *irrational numbers*. This set is an **infinite set**. An infinite set is a set whose elements cannot be counted. This means that you cannot reach up to the last element when you start counting the elements of the set. The three successive dots indicate that there are *some elements which are not written.*

{. . ., –2, –1, –½ , –2/5, –1/3, 0, 1/3, ½ , 4/5, √2, e, ∏, . . .}

Irrational Numbers

The set of **irrational numbers** contains numbers which cannot be expressed as a *terminating* and a *non-terminating repeating* decimal. This set of numbers is also an infinite set. The word infinite means that we have so many numbers which cannot be expressed as a terminating and a non-terminating repeating decimal.

{. . ., 3∏, ∏, e, √3, √2, . . .}

The √2 and ∏ are irrational numbers. Take note that the value of √2 = 1.414213562 and ∏ = 3.141592654 is a sequence of digits that do not repeat.

Rational Numbers

The set of **rational numbers** contains numbers which can be expressed as a *terminating* and a *non terminating repeating decimal.* In other words, the set of rational numbers are the fractions. Included in the set of rational numbers are the numbers which can be expressed as a ratio or a quotient of two numbers. This set of numbers is also an infinite set. So, do not try to list or enumerate all the fractions because you will only be wasting your time and effort. The word "infinite" means that it is not countable. We have so many numbers which can be expressed as a terminating and a non-terminating repeating decimal.

{ ..., –2/1, –1/1, –½ , –2/5, –1/3, 0, 1/3, ½ , 4/5, 1/1, 2/1, ... }

Examples of ratios expressed as terminating decimals are the following: { ½, ¼, ¾ , 1/5, 4/5, 3/2, 3/1 }

The values of the ratios/fractions in decimals are as follows:

½ = 1 ÷ 2 = 0.5

¼ = 1 ÷ 4 = 0.25

¾ = 3 ÷ 4 = 0.75

4/5 = 4 ÷ 5 = 0.8

3/2 = 3 ÷ 2 = 1.5

3/1 = 3 ÷ 1 = 3.0

The set of numbers { ½, ¼, ¾ , 1/5, 4/5 } are also called **proper fractions**. These are fractions in which *the value of the numerator is less than the denominator.* The fractions 3/2, and 3/1 are called **improper fractions.** Improper fractions are fractions in which *the value of the denominator is less than the numerator or the value of the numerator of the fraction is greater than the denominator.*

Examples of ratios expressed as repeating non-terminating decimals are the following: { 1/3, 2/3, 6/9, 1/6, 1/7}

The value of the ratios/fractions in decimals are as follows:

1/3 = 1 ÷ 3 = 0.3333 The digit 3 repeats.

2/3 = 2 ÷ 3 = 0.6666 The digit 6 repeats.

1/6 = 1 ÷ 6 = 0.1666 The digit 6 repeats.

Integers

The set of **integers** is also called the set of *signed numbers.* The numbers that belong to this set are the positive numbers, the negative numbers, and zero. The set of integers is also an infinite set.

{ ..., –3, –2, –1, 0, +1, +2, +3, ... }

The set of integers can be partitioned into two sets. The set of **even integers** and the set of **odd integers**. The sets of odd and even integers are infinite sets. The set of even integers can be denoted by thc following symbol, { ..., –4, –2, 0, +2, +4, ... } and the set of odd integers by the symbol, {..., –5, –3, –1, +1, +3, +5, ...}.

Whole Numbers

The set of **whole numbers** includes the set of counting numbers and zero.

{ 0, 1, 2, 3, 4, 5, ... } .

Counting Numbers

The set of **counting numbers** contains the smallest element, the number l, as its first element or member followed by the number 2, and so on. This set is an infinite set.

{ 1, 2, 3, 4, 5, ... }

Other Symbols for Constants

Some symbols that represent real numbers such as Π and e are also constants. These symbols have a **fixed value.** **Fixed value** means a value that does not change. A number is a constant. It is equal to itself. The value of the number 2 is not equal to –2. It is equal to 2.

Types of Constants

There are two types of constants: ***numerical constants*** and ***arbitrary constants***.

Numerical constants are constants which are numbers. The real numbers, rational numbers, irrational numbers, integers, whole numbers, and counting numbers are used as numerical constants in Algebra.

Examples:

2x +3 ---- 2 and 3 are the numerical constants.
5x +a = 12 ---- 5 and 12 are the numerical constants.

Arbitrary constants are constants which are usually represented by any letter of the English alphabet except the last three letters, x, y, and z. The small letter "a' may represent a real number in an algebraic expression involving the letters x, y, and z.

Examples:

$5x + a = 12$ ----- 5 and 12 are numerical constants.
a is an arbitrary constant.

$at^2 + bt + c = 0$ ----- 0 is the numerical constant.
a, b, and c are arbitrary constants.

$y = mx + b$ ----- m and b are the arbitrary constants.
y is a variable.

$C_1x + C_2y = 4$ ----- C_1 and C_2 are arbitrary constants.
4 is a numerical constant.

The letters a, b, c, m, C_1, C_2, are examples of arbitrary constants, which mean a *replacement letter* for a constant in a given term. These letters have fixed values in a given particular algebraic expression. The letter "a" is a replacement letter in our given algebraic expression involving the letter "x" and "t" in our examples. It can also be used as a replacement letter for another algebraic expression. In other words, arbitrary constants are used to replace assigned numerical constants to variables.

1.2 VARIABLES

Variables are letters and symbols that *represent numbers*. A variable may represent a set of numbers or a set of possible values. A set or group of numbers which can replace the variable is called the **value** of the variable. This set whose elements are the possible values of the variable is called a **replacement set**. The replacement set can be a subset of the set of real numbers or the real number itself.

The mathematical sentence, $2 + x = 4$ is true when x takes the value of the real number 2. The set of real numbers can be chosen as our replacement set. The set containing the real number 2 as its only element is called the **solution set.** The solution set for $2 + x = 4$ is $\{2\}$. The number 2 is the value of the variable x which will make the mathematical sentence true. Number 2 is the **solution.** The solution is the value of the variable (the constant) which will make the mathematical sentence true when this value is substituted to the variable in the given equation or inequality.

The box □, the question mark ?, and the blank space ____, are also symbols which refer to unknown values, hence, represent variables.

Replacement Set

Consider the set of all possible replacements for the blank in the following sentence.

____ is the author of Tom Sawyer.

Obviously we are to replace the blank with the name of the person. Why?

In mathematics, however, the set from which we may choose a replacement for a variable is not obvious. Before we can proceed to make replacement for the variable, we must know first what set of numbers we are allowed to use.

For example, let us use the open sentence $2n + 3 = 1$. You may be told that you are allowed to use only natural numbers or only integers, or only fractional numbers as replacement for n.

If you are allowed to use only the set of natural numbers, you might replace n with 5. Then the mathematical sentence becomes 2(5) +3 = 1, which is false.

If you use only the natural numbers, can you find a replacement for n which makes the sentence true?

If you are allowed to use only the set of integers, you might replace n with -3. Then the mathematical sentence becomes 2(–3) +3 = 1, which is false.

If you use only the integers, can you find a replacement for n which makes the sentence true?

A set of numbers whose elements are to be used as replacement for a variable is called a **replacement set.**

Practice Task:

I. Tell whether or not each replacement set below contains a replacement for k which will make the sentence $7+k = 11$ true.

1. Set of Natural Numbers
2. Set of Integers
3. Set of Negative Integers
4. Set of Odd Numbers
5. Set of Prime Numbers
6. Set of Even Numbers
7. Set of Multiples of Two
8. Set of Composite Numbers
9. Set of Rational Numbers
10. Set of Real Numbers

II. A **replacement set** is specified for each open sentence below. Make one replacement of the variable resulting in a true sentence and one replacement resulting in a false sentence.

1. $3 - r = 6$ (integers)
2. $3n + 2 = 14$ (natural numbers)
3. $k - 8 = 5$ (positive integers)
4. $6y < 0$ (integers)
5. $4n - 6 \neq -2$ (odd numbers)
6. $3(k + 2) = 16$ (prime numbers)
7. $5y + 6 < -4$ (integers)
8. $30 - x = 10$ (negative integers)
9. $2 = 3m - 3$ (rational numbers)
10. $z - 5 = 11$ (counting numbers)

Name: ____________________ Date: ________ Score: _________

Exercise 1.1

I. Determine the constants (numerical and arbitrary) and the variable(s) of the following expressions.

Algebraic Expressions/ Equations	Constants Numerical	Constants Arbitrary	Variable(s)
1. $4x - m = -8$	________	________	________
2. $ay - z - 5$	________	________	________
3. $9 = 4ax$	________	________	________
4. $n < 8$	________	________	________
5. $A = 2y + 2$	________	________	________
6. $37 - z$	________	________	________
7. $4cx - y$	________	________	________
8. $x - 4y > 25$	________	________	________
9. $3 \bullet 5 = 15$	________	________	________
10. $14 - x = 2$	________	________	________
11. $y > -25$	________	________	________
12. $2a + xyz$	________	________	________

13. $2x - 4y$ ________ ________ ________

14. $15 = 7axy$ ________ ________ ________

15. $-5 < 2x < 6$ ________ ________ ________

16. $xy > -25$ ________ ________ ________

17. $ab + 3yz$ ________ ________ ________

18. $2z - 4a$ ________ ________ ________

19. $A = lw$ ________ ________ ________

20. $P = 2l + 2w$ ________ ________ ________

21. $D = 2r$ ________ ________ ________

22. $4s = A$ ________ ________ ________

23. $xy - 4c$ ________ ________ ________

24. $2 = 4axy$ ________ ________ ________

25. $-502x < 6$ ________ ________ ________

Name: ______________________ Date: ________ Score: __________

Exercise 1.2

I. Give the variables of the following expressions.

1. □ + 25 ______________
2. ___ + 16 ______________
3. 36 – ? ______________
4. 12 + x ______________
5. y – 9 ______________
6. 34 / m ______________
7. 12xy ______________
8. 2(lw) ______________
9. 4s ______________
10. □ –45 ______________
11. 8 – 6y ______________
12. +2x ______________
13. m – 3n ______________
14. 6x – 4y ______________
15. A – 3C ______________

II. Give the possible value or values of the given variable (s) that will make the given mathematical sentence true.

1. □ + 25 = 25 ____________

2. ___ + 67 = 60 ____________

3. 36 – ? = 12 ____________

4. 12 + x = 80 ____________

5. y – 9 = – 1 ____________

6. 30/m = 2 ____________

7. 12xy = 12 ____________

8. 25(lw) = 25 ____________

9. 4(st) = 16 ____________

10. rst = 25 ____________

11. 25w – 5 = 20 ____________

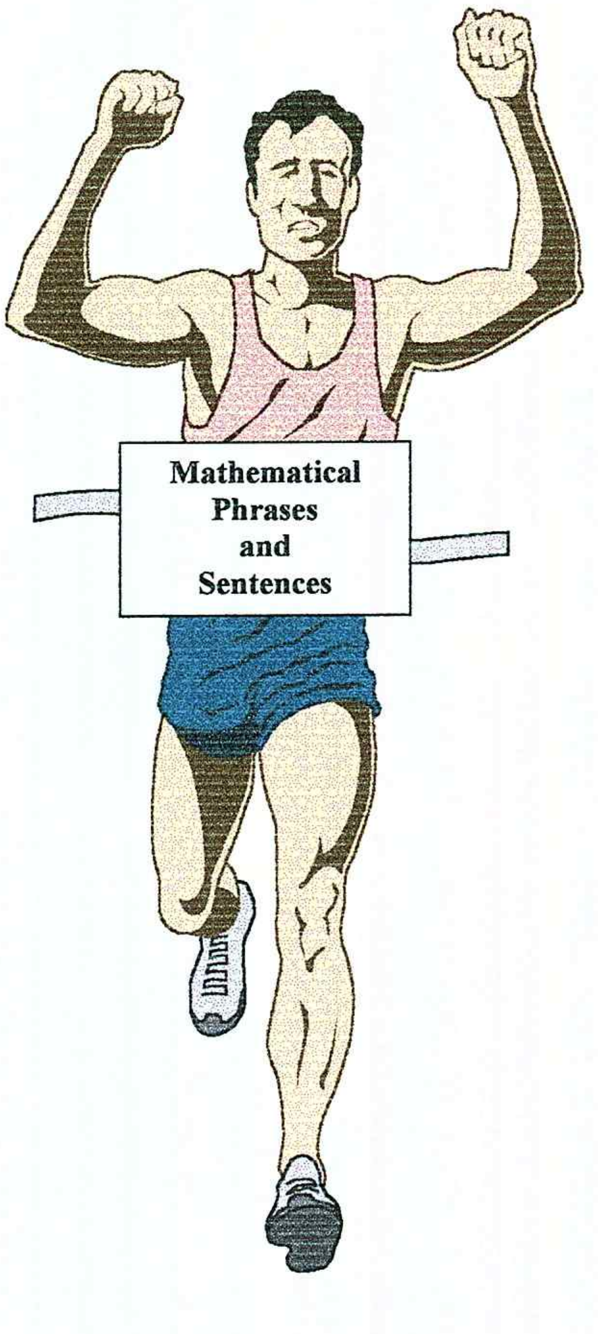
12. 125/x = 5 ____________

13. 5y – 5z = 10 ____________

14. □ + 2 = 100 ____________

15. □ + 25y – 50 = 0 ____________

Lesson 2



Lesson 2. MATHEMATICAL PHRASES AND SENTENCES

Objectives:

Content:

1. To identify mathematical phrases and sentences.

Process:

1. To determine mathematical phrases and sentences.
2. To give examples of mathematical phrases and sentences.

Affective:

1. Resourcefulness helps in the study of mathematical phrases and sentences.

2.1 MATHEMATICAL PHRASES AND SENTENCES

Mathematics has a language of its own which can be used to show the connection between related quantities. The connection between related quantities is best expressed in terms of mathematical phrases and sentences. The use of symbols in stating mathematical phrases and sentences is very important because aside from providing a more compact way of writing very long statements it has a beauty of its own. If one is to study and learn Algebra one must be adept in using symbols for Algebra uses symbols to a greater extent.

Sentence

A *sentence* is a word or group of words that expresses a complete thought. Sentences can be constructed about the world, universe, people, things, objects, events, relationships, numbers, etc.

Examples: Sentences about numbers.

1. The difference of twelve and five is seven.
2. The sum of four and five is nine.
3. Three times the number x is equal to twelve.
4. The square of four is sixteen.
5. The negative of a is symbolized by –a.

Phrase

A group of words that does not have a complete thought is a *phrase*. Phrases are not sentences since they do not make a complete statement in the sense that there is no verb and consequently they do not have a subject and a predicate.

Examples:

1. between twelve and five
2. greater than five and nine

3. three times the number x
4. square of four
5. almost one thousand

MATHEMATICAL SENTENCES

Sentences about numbers and number relationships are mathematical sentences. Mathematical sentences are usually written in mathematical symbols.

Examples:

1. $12 - 5 = 7$.
2. $4 + 5 = 9$.
2. $3x = 12$.
3. $4\sqrt{9} = 2 \times 3$.
5. $(-1)\, a = -a$.

MATHEMATICAL PHRASES

Mathematical phrases do not express a complete thought. Operation symbols and numbers, number relations, and etc. which are either written in words or symbols are examples of mathematical phrases.

Examples:

1. < 12
2. $4 + 5$
3. $\equiv 23x$
4. $4\sqrt{9}$
5. $(-1)\ a$
6. $\neq 19$

A mathematical phrase like "twice a number less three" can be translated into symbols, that is $2x - 3$ and vice versa.

Some examples of mathematical phrases and their corresponding symbols are:

Mathematical Phrases	**Symbols**
Eight added to the product of a number and 3	$8 + 3x$

A number divided by 8	X/8
4 times a number divided by 5	4x/5
A number diminished by 2	X - 2
Six times a number added to 7	6y + 7

Symbols	**Mathematical Phrases**
x - 6	a number reduced by 6
15 – x	a number subtracted from 15
x + 2	a number added to 2
3x - 20	Three times a number less 20
x/6	a number divided by 6

Mathematical Symbols

There are three groups of symbols used to represent mathematical ideas. They are the numbers, operation symbols, and symbols of relationship. The subsequent table gives the verbal interpretations of mathematical symbols of operations and relationships.

A. Operation Symbols

Symbols	Interpretations
+	Plus, the sum of, increased by, added to, more than, total of.
-	Minus, less than, subtracted from, difference of, decreased by, taken away from, deducted from, diminished by.
•, x, ( )	Multiplied by, the product of, times, twice, thrice, etc.
÷,) , x/y	Divided by, the quotient of x and y, the ratio of x and y.
$X \bullet x = (x)^2 = x^2$	The square of the number x, x to the second power.
$X \bullet x \bullet x = (x)^3 = x^3$	The cube of the number x, x to the third power.
√	The square root.

B. Symbols of Relationship

Symbols	Interpretations
$>$	Greater than
$<$	Less than
$=$	Equals, is equal to
$\neq$	Is not equal to
$\geq$	Greater than or equal to, at least
$\leq$	Less than or equal to, at most
$\cong$	Is congruent to, measures the same as
$\equiv$	Is equivalent to, similar to

Name: ____________________Date: ________ Score: _________

Exercise 2.1

I. Determine whether the following are sentences or phrases.

1. The quotient of twenty-four by four.
2. Five less than eight.
3. Six is more than four.
4. Seven decreased by five equals two.
5. Nine added to two
6. The difference between twelve and eight.
7. Fifteen divided by five is not equal to four.
8. The sum of four and twelve is equal to four squared.
9. Six subtracted from ten is less than four times three.
10. The product of five and minus two.
11. A number increased by nine.
12. Ten subtracted from a number.
13. The sum of m + n is equal to n + m.
14. Five times a number added to seven.
15. The product of x and y divided by z.

16. The sum of ab and cd divided by bd.

17. The quotient when x is divided by y.

18. Twice the length (l) plus twice the width (w).

19. The side (s) multiplied by four.

20. A number which is two greater than twelve.

21. A number less ten is equal to twice the same number.

22. Eight less a number added to six.

23. A number less five is eight.

24. Six is four times more than a number.

25. Seventy – five decreased by five equals two.

Name: ____________________ **Date:** ________ **Score:** _________

Exercise 2.2

I. Write the mathematical symbols of the phrases/sentences in Exercise 2.1.

1.

2.

3.

4.

5.

6.

7.

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14.

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28

16.

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24.

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Lesson 3

Coefficients, Exponents, Base and Powers

Lesson 3. Coefficients, Exponents, Base and Powers

Objectives:

Content:

1. To distinguish the exponents, base and coefficients in a monomial.
2. To raise a number to an indicated power.
3. To evaluate monomials with exponents.

Process:

1. To identify the exponent, base, coefficient and power in a given expression.
2. To differentiate terms like exponent, base, coefficient, and power from one another.
3. To simplify monomials with exponents.

Affective:

1. To demonstrate correctness in identifying the base, power, exponent, and coefficient.

3.1 Coefficients

In *arithmetic*, if the product of **a** and **b** is equal to **c** or **a x b = c** then if c is divided by b the quotient is a and if c is divided by a the quotient is b. The division yields a zero remainder. The numbers a and b are called the factors of c.

Example:

2 x 3 = 6, 2 and 3 are called the factors of 6.

In *algebra*, if two or more numbers (symbols) are multiplied we do not call them factors but each number (symbol) is called the **coefficient** of the other. The factor which is a constant is called the *numerical coefficient* and the factors which are letters or variables are called *literal coefficients*.

Examples:

1. 2 • x • y = 2xy — 2 is the coefficient of xy

   x is the coefficient of 2y

   y is the coefficient of 2x

2. 5 • a = 5a — 5 is the coefficient of a

   a is the coefficient of 5

3. x — 1 is the coefficient of x

   x is the coefficient of 1

The number factor in the example above such as 2, 5, and 1 is called the numerical coefficient. The letter factor, such as a, x, and y is called the literal coefficient. When no number is written before a letter such as in example 3, the numerical coefficient is understood to be the number 1.

Examples:

1. –z	–1 is the numerical coefficient of z. z is the literal coefficient of –1.
2. 8xyz	8 is the numerical coefficient of xyz. xyz is the literal coefficient of 8.
3. (2x –3y)	1 is the numerical coefficient of (2x –3y). (2x –3y) is considered as a single term.
4. –(4 +3a –b +c)	–1 is the numerical coefficient and the expression enclosed in parenthesis is considered as one term.

The product of a constant and a variable is indicated by writing the constant first, then the variable. Usually the symbol for multiplication is omitted. Thus the expression 3y is a product of the number 3, a constant and the variable y. Instead of writing 3 x y, 3 • y or (3)(y) we simply write it as 3y.

The product of a constant and another constant is indicated by writing the operation symbol for multiplication (•, x, ( )) between the two numbers to be multiplied. The operation symbol for multiplication is not omitted. In multiplying 2 by 3 we write it as 2 x 3 not as 23 because if the two numbers to be multiplied are beside each other as 23, it is different from 2 x 3. Thus the operation symbol of multiplication is omitted only if the numbers to be multiplied are constants and variables not for both constants.

In the product of a constant and a variable, the constant is the numerical coefficient (or simply the coefficient), and the variable is the literal coefficient. Hence, when we say "coefficient" we are referring to the numerical coefficient.

In the product of a constant and two or more variables, the constant is the numerical coefficient (or simply the coefficient), and the variables are the literal coefficients. 3xyz is a product of 3 and the variables x, y, and z. 3xyz is (3)(x)(y)(z) which is simply written as 3xyz. The multiplication operation symbol is omitted.

In the case of the product of two or more constants and variables, we must simplify it by obtaining the product of the constants which is multiplied to the product of the variables. In this example, (2)(3)(x)(y)(z) the expression is simplified by (1) getting the product of the constant; (2) getting the product of the variables, and (3) getting the product of the constant and the variables.

In the example (2)(3)(x)(y)(z), the numerical coefficient is (2)(3) or 6 and the literal coefficient is xyz. Hence, the product of (2)(3)(x)(y)(z) is equal to 6xyz.

If an expression contains only variables such as xyz this does not mean that there is no constant present. The numerical coefficient is not expressly written but only omitted. In this case the numerical coefficient of the expression is number 1. By omitted we mean we simply do not write it before the variables. In –xyz the numerical coefficient is –1 and the variables are x, y and z. –xyz is (–1)(xyz) or (1)(–xyz) this expression is simply written as –xyz.

The product of constants and certain variables occurring several times, like (2)(x)(x)(y)(y)(y)(z)(z)(z)(z) can be written in more compact form by applying knowledge of powers. Hence the given expression can be written as $2x^2y^3z^4$. In writing the product of constant and variables occurring several times, the constant is indicated first followed by the variables with exponents. The variables maybe written in any order in view of the commutative and associative properties of real numbers. However, for practical purposes we may write algebraic expressions with the variables arranged in alphabetical order as in this example.

$$(5)(w)(x)(x)(y)(m)(y)(z)(w)(n)(m) = 5m^2nw^2x^2y^2z$$

3.2 Exponents and Powers

The process of multiplying the number n by itself two or more times such as n x n x n x n ... is called raising n to a **power.** The notation n^n means n is used as a factor n (many) times. When n is used as a factor two times, such as, n x n, we say that n is raised to the second power. When n is used as a factor three times, such as, n x n x n, we say n is raised to the third power. Hence if we multiply 3 by itself two times, as in 3 x 3, 3 is raised to the second power, in symbol $3^2 = 9$. If 2 is multiplied by itself three times, 2 x 2 x 2 in symbol is $2^3 = 8$. Thus:

$3 \times 3 = 3^2 = 9$ $\quad\quad\quad$ $2 \times 2 \times 2 = 2^3 = 8$

The process of raising a number to an indicated power involves three numbers or quantities – base, exponent and power.

Base – is the number symbol (constant or variable) taken as a factor one or more times by itself. The base is the factor that is being repeated.

Examples:

3^2	means	(3)(3),	the base is 3
$(-4)^2$	means	(–4)(–4),	the base is –4
a^4	means	a x a x a x a,	the base is a
n^3	means	n x n x n,	the base is n

Exponent – is a number written at the upper right hand of a symbol (base). It tells how many times the base will be multiplied by itself or it tells how many times the symbol (base) may be used as a factor.

Examples:

3^2	the exponent is 2,	(3)(3),	repeated two times
$(-4)^2$	the exponent is 2,	(–4)(–4),	repeated two times
a^4	the exponent is 4,	axaxaxa,	repeated four times
n^3	the exponent is 3,	n x n x n,	repeated three times

The exponent of a number or symbol is applied only to the base to which it is attached. If we want it to be applied to a group of symbols (more than one symbols) we have to enclose the group of symbols in parenthesis.

In 4^2bc the exponent 2 applies only to 4 but in $(4bc)^2$ the exponent 2 applies to 4, b, and c. $(4bc)^2 = 4^2 b^2 c^2$.

Examples:

1. $3^2 = 3 \times 3 = 9$ — The exponent 2 is applied to 3.
2. $-3^2 = -(3 \times 3) = -9$ — The exponent 2 is applied to 3 not to –3, the symbol immediate ly preceding 2 is 3.
3. $(-3)^2 = (-3)(-3) = 9$ — The exponent 2 is applied to –3.
4. $3x^2 = 3 \bullet x \bullet x$ — The exponent 2 is applied to x, the symbol immediately preceding it.
5. $(3x)^2 = 3x \bullet 3x$ — The exponent 2 is applied to 3x.

Power - is the result (number or symbol) of multiplying the base by itself several times. It is the result of raising the base to the given exponent. Since it is the result operation multiplication, power is therefore a *product*. It is the product of the same number being repeatedly multiplied or power is the *product of equal factors*.

Examples:

1. $3^2 = 3 \times 3 = 9$ 9 is the second power of 3.
2. $(-3)^2 = -3 \times -3 = 9$ 9 is the second power of –3.
3. $2^3 = 2 \times 2 \times 2 = 8$ 8 is the third power of 2.
4. $(-1)^5 = (-1)(-1)(-1)(-1)(-1) = -1$
   –1 is the fifth power of –1.
5. $x^2 = x \bullet x$ x^2 is the second power of x.

Finally, to sum up what we have learned so far, the factor being repeated is the *base*. The symbol written at the upper right hand of another symbol is the *exponent*. The result of the repeated multiplication of the base is the *power*.

Exponent

↓

Base → $3^2 = 9$ ← Power

Name: ____________________ Date: ________ Score: _________

Exercise 3.1

I. Give the coefficient of x in each of the following expressions and write it on the blank space before each number.

__________ 1. x

__________ 2. $3x^2$

__________ 3. $x + 3$

__________ 4. $x(a + b)$

__________ 5. $3x(y - 3)$

__________ 6. $-4(3x + 2y)$

__________ 7. $-2xy$

__________ 8. $\frac{1}{2}xy$

__________ 9. $9xy^2$

__________ 10. $\frac{3}{4}xyz$

__________ 11. $ax^2 - bx$

__________ 12. $2x(3 - 4y - z)$

__________ 13. $1/x + 3x$

__________ 14. $4 - 2x + 3x^2$

__________ 15. $-(x - 3y)$

__________ 16. $-x - 4$

__________ 17. $3xy(-4z)$

__________ 18. $-3(4x - 6)$

__________ 19. $(-a + b + c)x$

__________ 20. $-x + \frac{1}{2}x^2$

II. Give the numerical coefficient of x in test I.

__________ 1.

__________ 2.

__________ 3.

__________ 4.

__________ 5.

__________ 6.

__________ 7.

__________ 8.

__________ 9.

__________ 10.

__________ 11.

__________ 12.

__________ 13.

__________ 14.

__________ 15.

__________ 16.

__________ 17.

__________ 18.

__________ 19.

__________ 20.

II. Give the numerical and literal coefficients of the following expressions.

Numerical Coefficients	Literal Coefficients	Algebraic Expressions
__________	__________	1. $10abc$
__________	__________	2. $4xy$
__________	__________	3. $-a^2 bc^2$
__________	__________	4. $21mn$
__________	__________	5. $-opr$
__________	__________	6. $(abc)/5$
__________	__________	7. $-1/2abc$
__________	__________	8. $210a^{23}b^{12}c$
__________	__________	9. $(abc)(3x)$
__________	__________	10. $(abc)^2$
__________	__________	11. $(10 - abc)$
__________	__________	12. $-(10x - 20y)$
__________	__________	13. 10^2abc
__________	__________	14. $-132(abc)^2$
__________	__________	15. $-100ab$
__________	__________	16. $39abc$
__________	__________	17. $+(x - 2y)$
__________	__________	18. $9a^2bc$
__________	__________	19. $-32abc^2$
__________	__________	20. $-3/4\ cab$

Name: ____________________ **Date:** ________ **Score:** _________

Exercise 3.2

III. Rewrite the following expressions using exponents. Write each on the blank space provided before each number.

____ 1. 4• 4 • 4 • 4 • 4 • 4•4

____ 2. x • x • x • x • y • y • y

____ 3. 4a • 4a • 4a • 4a

____ 4. S • S • S

____ 5. x•x • x• x • (4y) • (4y)

____ 6. 3 • x • y • y • z • z • z

____ 7. xy•xy•xy•xy•yx•yx•yx

____ 8. rst • rst • rst

____ 9. tu • tu • tu • tu • tu

____ 10. –ab• –ab• –ab• –ab

II. Simplify the expressions with powers and write them on the blank spaces provided for.

__________ 1. 2^5

__________ 2. $(2a)^5$

__________ 3. $(-2x^3\ )$

__________ 4. $2(4)^2$

__________ 5. $3^2 + 2^3$

__________ 6. $(-2)^2$

__________ 7. $-(3)^2 + (-2)^3$

__________ 8. $-(-1)^8 + 2^3$

__________ 9. $-3(8 - 2)^2$

__________ 10. $-4^2 + 3^3$

III. Identify the base and the exponent.

Base	Exponent	Algebraic Expressions
________	________	1. 5^2
________	________	2. $(-5)^2$
________	________	3. $-(1/2)^2$
________	________	4. $-(6y)^3$
________	________	5. $(Y/3)^2$
________	________	6. $-(xy)^2$
________	________	7. $-(12)^2$
________	________	8. 210^0
________	________	9. $(a - b)^2$
________	________	10. a^{2-x}
________	________	11. b^{-y}
________	________	12. $-(x - y)^2$
________	________	13. $(10abc)^{22}$
________	________	14. $(abc)^{2-3y}$
________	________	15. $(-100)^{abc}$

Lesson 4

Evaluation of Numerical and Algebraic Expressions

Lesson 4. Evaluation of Numerical and Algebraic Expressions

Objectives:

Content:

1. To define numerical and algebraic expressions.
2. To translate English phrases into algebraic or numerical expressions and vice versa.
3. To differentiate numerical and algebraic expressions.
4. To evaluate numerical and algebraic expression.

Process:

1. To give examples of situations that are within the experience of students.
2. To use mathematical operations, relation symbols, and grouping symbols.
3. To find the value of numerical and algebraic expressions.
4. To verify results.

Affective:

1. To show accuracy and orderliness in translating numerical and algebraic expressions into verbal statements.
2. To show accuracy in evaluating numerical and algebraic expressions.

4.1 Difference of Numerical and Algebraic Expressions

Mathematical expressions can be classified into (1) numerical expressions, and (2) algebraic expressions.

Numerical Expressions are expressions which consist of a single number with or without operation symbols, or of two or more numbers with operations and grouping symbols. They can be numerical phrases or sentences.

A numerical expression may consist of a single number with or without operation symbols, or two or more numbers with operations and grouping symbols.

The operation symbols are the following:

1. fundamental operation symbols, +, –, x and ÷ ;
2. the symbol used to represent a number x raised to a certain power n, x^n ; and
3. symbol representing the nth root of a certain number x, $\sqrt[n]{x}$.

The grouping symbols are as follows:

1. parenthesis, ( )
2. brackets, [ ]; and
3. braces, { }.

Examples of numerical expressions are:

(a) Single numbers ....

1, 2, -39, π, 2/6, 4.789, √36, etc.

(b) Two or more numbers with operation symbols ....

$3 - 5, -9 - 89, 3 \times 29 - 1, 45 + \sqrt{2}, 4 \div 2, 3^2 + 2^3$

(b) Two or more numbers with operation and grouping symbols ....

$(13 \times 3), [(12) \div 2] - 7, (-1)(-56) + (23 - 34),$

$(2 - 45)^2, \{[(15 \times 3) - 3] + (-8 + -9)\}, 1^2\, 2^3\, 3^4 - 2^2$ etc.

An algebraic expression may consist of numbers and one or more variables joined by a symbol of operation and sometimes using the grouping symbols.

Algebraic expressions contain letters (or variables). They may be open phrases or open sentences.

An expression consisting of number (constant) and one or more letters (variables) joined by any symbol of operation and sometimes with grouping symbols is an algebraic expression.

Examples :

1. $x^2 + 2x + 4$
2. xyz
3. $2x - 3y + z$,

4. $3(4x - 5y) = 6$
5. $(x - y + z) = 12x - 15y$

In an algebraic expression, it is not necessary to write the symbol of multiplication, x or a dot midway between the numbers, (•). Also, it is not necessary to enclose the letters/variables in parentheses as in numerical expressions. The product of the variables x and y may be written simply as xy. In cases where the expression contains a number and a variable, the number always precedes the variable. The product of 2 and y, therefore is written as 2y. The product of 1 and z is written as z only. Most often when it is a product of 1 and any variable we prefer not to write 1. So it is implied that whenever we see variables only in the expression the number that precedes it is 1..

In writing algebraic expressions you must always follow the order prescribed.

For a group of symbols consisting of numbers, letters, operation signs and grouping symbols to form an algebraic expression, it must represent a real number (specific or unspecific).

$2x^2 + 3x - 1$ is an algebraic expression for if the variable x is substituted by a specific real number the result is a real number in view of the closure property of real numbers.

$3/4x^3 + x/4 + y^3$ is an algebraic expression for if the variables x and y is substituted by a specific real number the result is a real number .

$3x^2 / y + 3x + 6$ is an algebraic expression if $y \neq o$. Since division by zero is not defined.

Some examples of algebraic expressions are $3x-y+3z$, $a-2b+c$, $2x^5 y^2 - 7$, $at^2 + b\,t + c$, $3y^2 - (x+2y)+6$, $(x^3 - 2y-8)$ and etc.

Not all collection of symbols made up of numbers, variables, operation signs and grouping symbols are algebraic expressions.

$2x^4 - 2x + \sqrt{} - 4$ is not an algebraic expression because the symbol $\sqrt{}$ does not represent a real number.

$3x^2 - \sqrt{y} + 2$ is not an algebraic expression if y is negative since the $\sqrt{y}$ is imaginary, hence not real.

$2x/y - 3x^2 - 7$ is not an algebraic expression if y is zero since division by zero is not defined or it is meaningless.

The table gives the difference between numerical expressions and algebraic expressions.

Numerical Expressions	**Algebraic Expressions**
A.) Numerical Phrases	A) Algebraic Phrases
$4 = 8 - 7$	$x + 3$
$9 (5)$	$7y - y$
$8 - 2 \times 9 / 3$	$\sqrt{5a / 2b}$
$(3 + 7) - 4$	$(2x + 1)(3x - 4)$
B.) Numerical Sentences	B) Open Sentences
1. Equations	1. Equations
$3 + 5 = 4 \times 2$	$2x - 8 = 6$
$4 + 8 - 7 = 5$	$n + 3 = 8$
$9 \times 5 = 45$	$7y - y = 6y$
2. Inequalities	2. Inequalities
$5 + 6 \neq 2 \times 6$	$x > 3$
$-8 < 0$	$y - 2 < 3$
$(5)(2) < 100$	$x > 7 - y$

4.2 Evaluation of Numerical Expressions

To evaluate a given numerical expression is to find its value in the simplest form. This is the same as to find the single real number that is equal to the given numerical expression.

In simplifying numerical expressions certain rules are followed regarding the order of operation. Otherwise we might not arrive at the correct answer.

Examples:

1. Evaluate $(3 + 7) - 2$

   Solution: $10 \quad - 2 = 8$

2. Evaluate $[(8)(2)] - 7$

   Solution: $16 \quad - 7 = 9$

3. Evaluate $24 \div [(2)(3)]$

   Solution: $24 \div \quad 6 \quad = 4$

Order of Operations

1. If there are any parentheses in the expression, that part of the expression within a pair of parentheses is evaluated first. Then the entire expression is evaluated.

2. Any evaluation always goes in three steps:

First: Powers and roots are being done in any order.

Second: Multiplication and division are done in order from left to right.

Third: Addition and subtraction are done in order from left to to right.

The *order of operation* provides us with specific order in which an expression with several operations is being evaluated to ensure a correct answer. The commutative and associative properties make it possible to evaluate addition and multiplication in more than one order yet get the same result. It may possible to perform the operations, addition and multiplication, in an expression in more than one order. Yet get the same answer.

For example, it is important to realize that an expression such as "8 –6 –4 +7" is being evaluated by doing the addition and subtraction in order from left to right. Subtraction is not commutative or associative so, it is necessary to follow the order of operation.

The same expression is a sum: (8) + (-6) + (-4) + (7). If the expression is a sum, then we can add these terms in any order (because addition is commutative and associative).

<u>Evaluated only left to right</u>	<u>Added in any order</u>
$8 - 6 - 4 + 7$	$(8) + (-6) + (-4) + (7)$
$= 2 - 4 + 7$	$= (8) + (7) + (-6) + (-4)$
$= -2 + 7$	$= 15 + (-10)$
$= 5$	$= 5$

When only division and multiplication are to be done, as in $75 \div 5 \bullet 3$. It is equally important that the left-to-right order be followed (because division is neither associative nor commutative). Thus we have

$$75 \div 5 \bullet 3$$

$$15 \bullet 3 = 45$$

Example 1. Use order of operation rules to evaluate expressions:

(a) $(7 + 3) \bullet 5$ We do the part in parenthesis first

$= 10 \bullet 5 = 50$

(b) $7 + 3 \bullet 5$ Multiplication is done before addition

$= 7 + 15 = 22$

(c) $4^2 + \sqrt{25} - 6$ Powers and roots are done first

$= 16 + 5 - 6$

$= 21 - 6$

$= 15$

(d) $16 \div 2 \bullet 4$ Here division is done first because in

$= 8 \bullet 4$ reading from left to right the division

$= 32$ comes first.

(e) $\sqrt{16} - 4(2 \bullet 3^2 - 12 \div 2)$ First evaluate the expression inside parenthesis

$= \sqrt{16} - 4(2 \bullet 9 - 12 \div 2)$ Do the powers inside the parenthesis

$= \sqrt{16} - 4(18 - 6)$ Do the x and ÷ inside ( )

$= \sqrt{16} - 4(12)$ Do the subtraction (–) inside( )

$= 4 - 48 = -44$ Next, find the root

(f) $(-8\ ½) \div 2 - (-4\ ½)$ Division is done before

$= (-17/2) \div 2 - (-9/2)$ subtraction

$= (-17/2) \bullet ½ + 9/2$

$= -17/4 + 18/4 = ¼$

(g) $\sqrt[3]{-8}\,(-3)^2 - 2(-6)$ Roots and powers are done first

$= -2\,(9) - 2(-6)$ Multiplication is done before

$= -18 - (-12)$ subtraction

$= -18 + 12$

$= -6$

4.3 Evaluating Algebraic Expressions

An algebraic term is only a part of an algebraic expression. An algebraic term alone can be called an algebraic expression of one term called **monomial.** But when there are two or more algebraic terms connected by their assigned algebraic signs of plus or minus (positive and negative) and the operation involved are those of addition, subtraction, multiplication, raising to a power or exponents and extraction of roots, then this series of algebraic terms forms what is called an **algebraic expression.**

Examples:

$-x$	-----	monomial or algebraic term
y	-----	monomial or algebraic term
$2x + 3y + 4z$	-----	algebraic expression
$5ax^2 + 6by^2 - 2cz$	-----	algebraic expression
$-6xy^2z^3$	-----	algebraic expression (monomial)

Therefore algebraic expressions and terms are merely symbolic representations of known and unknown quantities or numbers which are separated by plus and minus signs.

To evaluate an algebraic expression, substitute in place of each literal quantity its numerical value and perform the indicated operation. It is very easy to find the value of any given algebraic expression if it is in its simplest form. This process of simplifying an algebraic expression is called simplification.

Example 1. Find the value of $3x - 5y$ if $x = 10$ and $y = 4$.

Solution: $3x - 5y = 3 \bullet x - 5 \bullet x = 3(10) - 5(4) = 30 - 20$

Notice that we simply replace each variable by its number value, then carry out the arithmetic operations as we have done before.

> When replacing a variable by a number, enclose the number in parenthesis to avoid the following common errors.

Evaluate: $3x$ when $x = 2$

CORRECT	**COMMON ERROR**
$3x = 3(-2) = -6$	$3x = 3 - 2 = 1$

Evaluate $4x^2$ when $x = -3$

CORRECT	**COMMON ERROR**
$4x^2 = 4(-3)^2 = 4(9) = 36$	$4x^2 = 4 - 3^2 = 4 - 9 = -5$ or
	$4x^2 = 4 - 3^2 = 4 + 9 = 13$

Example 2. Find the value of $2a - [b - (3x - 4y)]$ for $a = -3$, $b = 4$, $x = -5$ and $y = 2$.

Solution: $2a - [b - (3x - 4y]$

$= 2(-3) - [4 - \{3(-5) - 4(2)\}]$ Notice {} were used in

$= 2(-3) - [4 - \{-15 - 8\}]$ place of ( ) to clarify the

$= 2(-3) - [4 - \{-23\}]$ grouping

$= 2(-3) - [4 + 23]$

$= -6 - 27$

$= -33$

Example 3. Evaluate $b - \sqrt{b^2 - 4ac}$ when $a = 3$, $b = -7$, and $c = 2$.

Solution: $b \; - \sqrt{b^2 - 4ac}$

$= \; -7 - \sqrt{(-7)^2 - 4(3)(2)}$

$= \; -7 - \sqrt{49 - 24}$

$= \; -7 - \sqrt{25}$

$= \; -7 \; - 5$

$= \; -12$

A common error often made is to mistake $(-3)^2$ for -3^2

$(-3)^2 \; = \; (-3)(-3) \; = \; 9$ The exponent 2 applies to -3.

$-3^2 \; = \; -(3)(3) \; = -9$ The exponent 2 applies to 3.

Let us find the numerical value of algebraic expressions in the following examples:

Let $x = 1$, $y = 2$, and $z = 4$. Find the numerical value of the given algebraic expressions.

1. $2x^2 + 3x \; -1$
2. $3/4x^3 + x/4 + y^3$

3. $3x^2/y \ +2x \ +z$

Solution: To find the numerical value of the given algebraic expressions substitute the specific value for each letter and simplify.

1. $2x^2 + 3x - 1$

   $= 2(1)^2 + 3(1) - 1$

   $= 2 + 3 - 1$

   $= 4$

2. $3/4x^3 + x/4 + y^3$

   $= \frac{3}{4}(1)^3 + \frac{1}{4} + (2)^3$

   $= \frac{3}{4} + \frac{1}{4} + 8$

   $= 1 + 8$

   $= 9$

3. $3x^2/y \ +2x \ +z$

   $= 3(1)^2/(2) \ +2(1) \ +(4)$

   $= 3/2 \ +2 \ +4$

= 3 / 2 +6

= 1 ½ +6

= 7 ½

Let us solve some problems about the evaluation of expressions. But we should take note that in evaluating algebraic expressions the specific real number representing a particular variable should be the same throughout the whole process.

The content of the statement can be explained using the given example.

The numerical value of the expression $2x^2 + 3xy + y^2$ is 15 when x = 2, and y = 1. The expression is a sum of three terms $2x^2$, 3xy, and y^2. There is an x in the first term of the expression so we have to substitute it with the real number 2. An x also appears in the second term so we have to substitute the real number 2 again. It is not correct practice if another real number is substituted for x in the second term since the second term is part of the given algebraic expression. In the same manner, with the variable y, if 1 is substituted for y in the second term, it must be the same number that will be substituted for y in the third term of the expression not any other number. However, for these algebraic expressions such as $x - 2y$, $2xy - 4$, $y^2 - x^2$, and a lot of others we can substitute other real numbers for the variables x and y unless otherwise stated.

Remember:

(1) To evaluate an algebraic expression is to find the numerical value of the given algebraic expression.

(2) Specific value given to a letter varies from one problem to the next, but it remains the same for that letter throughout any one problem.

(3) To find the numerical value of an algebraic expression we need specific real numbers to be substituted for the variables in the expressions. The next steps are substitution and computation.

(4) Calculations are easier and the likelihood of an error to occur is reduced when each letter in the expression is being substituted by the given specific values. A given value for a variable in a particular expression is enclosed

in parenthesis in a distinct step before doing the operations.

(5) After we substitute the specific value for each letter in the expression, the resulting numerical expression can be simplified by carrying out the operations according to the **Order of Operations Rule.**

Name: ____________________Date: ________ Score: _________

Exercise 4.1

I. Use the numbers 1, 2, 3, ... 9, the fundamental operation symbols and the grouping symbols to write ten numerical expressions.

II. Use the numbers 1, 2, 3, ... 9, the fundamental operation symbols and the last three letters of the alphabet to write ten algebraic expressions.

III. Write the given phrases as algebraic expressions using "n" for any variable not specified.

1. a number increased by 2
2. ten subtracted from a number
3. the sum of m and n
4. five times a number added to seven
5. The product of x and y divided by 2.
6. The quotient when x is divided by y.
7. The product of x and y increased by nine.
8. The sum of ab and cd divided by 2.
9. Twice the length (l) plus twice the width (w).
10. The side (s) multiplied by 4.

Name: ____________________Date: ________ Score: _________

Exercise 4.2

I. Simplify the following expressions:

1. (20 – 14) + 8
2. 3 – (68 ÷ 2)
3. 146 + (56 x 12)
4. (23 • 300) + (12 – 7)
5. [9 – (2 x 12)] – (2 x –3)
6. (40 – 13) x (12 + 14)
7. (13 x 12) ÷ (13 – 10) + 2
8. [(12 + 16 – 5) x (–2 + 15)
9. (12 – 4 –4 + 4) + (–2 + 20 –7)
10. [(12 ÷ 2) x (4 ÷ 2)] – (120 –25)
11. 4(2 +4) + 8(2 –3)
12. 36 – (68 ÷ 2)
13. 16 + 2(56 x 12)
14. (2 x 30) + 2(12 –12)
15. [9 –2(2 x 2)] – 2(2 x –3)
16. (40 –2) x (12 + 4)
17. (3 x 12) ÷ (30 –10) + 12
18. [(12 + 16 – 5) x (–1 + 13)]
19. (12 – 4 +2) + (–20 + 20 – 7)
20. [(12 ÷ 6) x (4 ÷ 12)] – (12 ÷ 4) –25

Name: ____________________ **Date:** ________ **Score:** _________

Exercise 4.3

I. Evaluate the given algebraic expressions when $x = 1$, $y = 2$ and $z = 3$.

1. $2x^2y^3 - 3xy^2 + (x - 3y)^2$
2. $(3x + 2y - z)^2 - x^2y^3z^2$
3. $2x^2y^2z^2 - xyz + 7$
4. $(2x - y)^2 + (x - 3y)^3$
5. $x^2 - 4y/3 + 3xy^2$
6. $3x^2y^2 - [xy^2 + (x - 3y)]^2$
7. $(3x + 2y - z) - (x^2y^2z^2)$
8. $2(xyz)^2 - x^2y^2z^2$
9. $[(2x - y)^2 - (x - 3y)^2$
10. $x^2/3 - 4y^2/5 + 3z/2$
11. $(x^2)^3 - (xy)^2 + z^2$
12. $(x + y - z)^2 + (x^2y^2z^2)^2$
13. $x^2y^2 - x^2y^2$
14. $[(x + y)^2]^2 - [(x - y)^2]^2$
15. $(x^2 - 2xy + y^2) \div (x - y)^2$

II. Write the algebraic expressions in symbols and evaluate.

1. Six times a number added to six is twelve
2. If 5 is subtracted from 3 times a number the result is 19. Find the number.
3. Six times a number increased by five equals 23.
4. The product of a number and five decreased by 9 equals 11. What is the number?
5. Think of a certain number, which when divided by four will give eight. What is the number?

Name: ____________________ **Date:** ________ **Score:** _________

Exercise 4.4

I. Give the possible value(s) of the variables in the given algebraic expressions.

1. $xy + y^2 = 4$

2. $3x + 2y = 5$

3. $2x - yz = 7$

4. $2x - y^2 + z = 10$

5. $x^2 - 2y = 12$

6. $3x + y^2 - 2y^2 = 21$

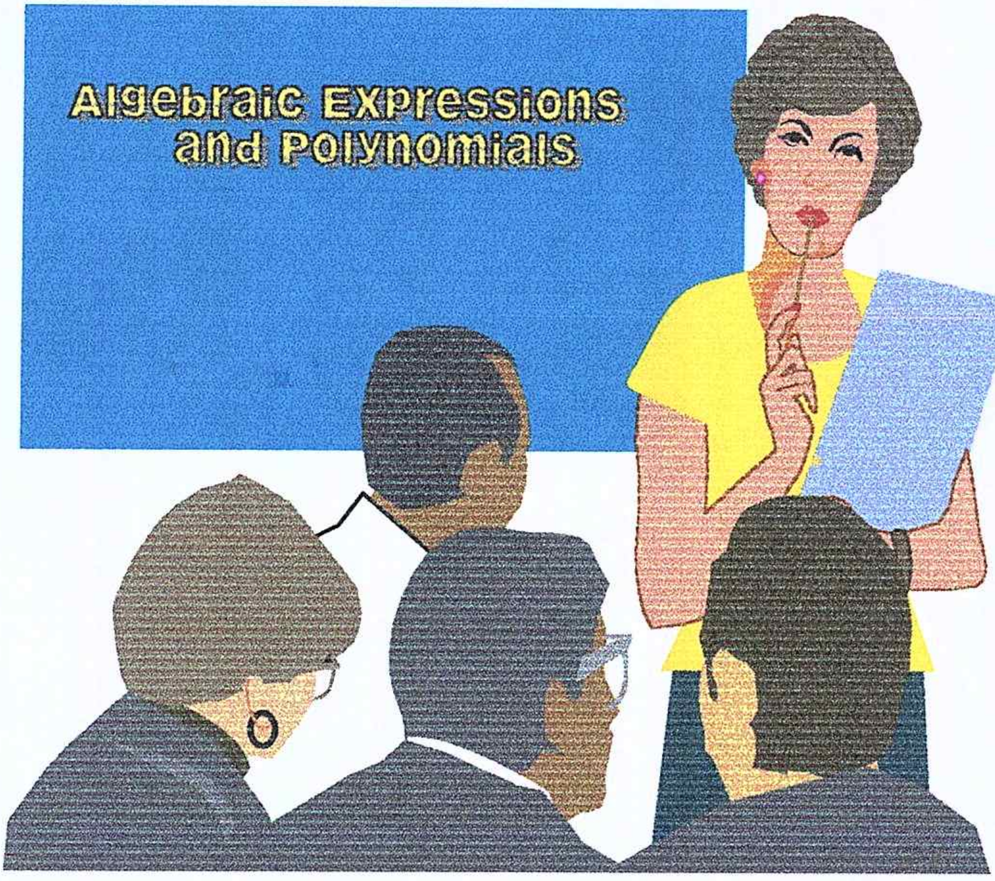
7. $(2x - y)^2 = 1$

8. $x^2/4 - 4 = 0$

9. $(x^2)^3 - x^2 = 0$

10. $(x + y - z)^2 = 16$

Lesson 5



Algebraic Expressions and Polynomials

Lesson 5. ALGEBRAIC EXRESSIONS AND POLYNOMIALS

Objectives:

Content:

1. To classify algebraic expressions.

Process:

1. To recognize differences between monomials, binomials, and polynomials.

Affective:

1. To demonstrate correctness in identifying monomials, binomials, and polynomials.

5.1 Classification of Mathematical Expressions

Mathematical expressions called **polynomials** play an important role in the application of mathematics to science and technology. Polynomials can consist of any number of variables. In this lesson, we will discuss mainly the polynomials in one variable.

A polynomial can be classified according to:

(a) the number of terms it contains.

(b) the degree of the polynomial.

5.2 Classification of Polynomials According to Number of Terms:

1. **Monomial** – an expression with only *one term.* It may be a constant with a variable, or a variable with exponent that is a positive integer, or a constant with a variable with positive exponent. It is also called a special polynomial.

   Examples:

   1. 5 ----- constant
   2. xyz ----- variables
   3. 6x ----- constant with variable
   4. $7x^2$ ----- constant with variable with positive exponent

2. **Binomial** – a polynomial with two terms.

   Examples:

Binomials (Two terms)	First Term	Second Term
1. $3^2 + 5$	3^2 – constant	+ 5 – constant
2. xyz + 7	xyz – variables	+ 7 – constant
3. $x^2 + y^2$	x^2 – variable with positive exponent	+ y^2 – variable with posiitive exponent

4. $8x^2y + 6x^2y^2$ — $8x^2y$ – constant and variables with positive exponent; $+6x^2y^2$ – constant and variables with posiitive exponent

3. **Trinomial** – a polynomial with three terms.

Examples:

1. $3^3 + 2^2 - 1$
2. $x + y + z$
3. $2x - 4y + 5z$
4. $7x^3 + 8y^3 + 9z$

4. **Quadronomial** – a polynomial with four terms.

Examples:

1. $3^3 + 2^2 - 1 + \sqrt{2}$
2. $ax + by + cz - 5$
3. $2x - 4x^2 + 5x^3 - 10x^4$
4. $72xy^4 - 4x^2y^3 + 5x^3y^3 - 10x^4$

5. **Multinomial** – a polynomial with four or more terms.

5.3 Classification of Polynomials According to Its Degree:

1. **Linear Polynomial** – is one in which the highest exponent of the variables involved is one.

   Examples:

   1. $x + 5$ ----- the variable involved is x and the exponent of x is 1.
   2. $x + y - z$ ----- the variables are x, y, and z and the exponent of x is 1, y is 1 and and z is 1.
   3. $6w$ ----- the variable is w and the exponent of w is 1.
   4. $7ab - ab$ ----- the variables are a and b with exponent of 1.

2. **Quadratic polynomial** – a polynomial in which the highest exponent of the variables involved is two.

   Examples:

   1. $3x^2 + 5$ --- quadratic polynomial in x highest exponent of x is 2.
   2. $z + 7y^2$ — quadratic in y highest exponent of y is 2.

      linear in z highest exponent of z is 1.

3. $x^2 + y^2$ --- quadratic in x and y
highest exponent of
x is 2, y is 2.

4. $at^2 + bt + c$ --- quadratic in t
highest exponent of t is 2.

5. $8x^2 + 6y^2$ --- quadratic in x and y
highest exponent of
x is 2, y is 2.

3. **Cubic polynomial** – a polynomial in which the highest exponent of the variables involved is three.

Examples:

1. $3x^3 + 2x^2 - 1$ ----- cubic in x
highest exponent of x is 3.

2. $7x^3 + 8y^3 + 9z$ ---- cubic in x
quadratic in y
linear in z

3. $7x^3 + 8y^3 + 9z^3$ ---- cubic in x, y, and z.

5.4 Leading Monomial and Coefficient

The term or monomial with the highest degree of the variable is called the **leading monomial**. The coefficient of the leading monomial is called the **leading coefficient**. It is customary to write a polynomial in standard form, that is, the leading monomial is written first, followed by the monomial of the next degree down to the lowest degree.

Examples:

Analyze the following polynomials according to the different terms just defined.

$7x^3 + 9x + 3$ It is a trinomial for it contains 3 terms.
Cubic in x since its highest exponent is 3.
$7x^3$ is the leading monomial or term.
The number 1 is the leading coefficient.
It is in standard form.

$At^3 + bt + c$ It is a trinomial for it contains 3 terms.
Quadratic in t since its highest exponent is 2.
At^3 is the leading monomial or term.
A is the leading coefficient.
It is in standard form.

$X^2 + xy + y^2$ It is a trinomial for it contains 3 terms.
Quadratic in x and y since its highest exponent for x and y is 2.
X^3 is the leading monomial or term.
1 is the leading coefficient.
It is in standard form. The exponents of x is decreasing and y is increasing.

5.5 Degree of a Monomial

The **degree** of a monomial is **n.** "n" is the sum of the positive exponents of the variables in the monomial. If the monomial has no variables or if it is a constant only, then the degree of the monomial is **0**. Any real number for that matter, when used as a coefficient or constant in a monomial or polynomial, has a degree of 0.

Examples:

1. $2x$ --- is of degree 1
   The exponent of x is 1.

2. $-6y^4$ --- is of degree 4
   The exponent of y is 4.

3. $24ab^2c^4$ --- is of degree 7
   The exponent of a is 1, b is 2, c is 4.
   The sum of the exponents is 7.
   $1 + 2 + 4 = 7.$

4. 69 ---- is of degree 0.

5. $22\,x^4y^2z^3$ ---- is of degree 9.
   The exponent of x is 4, y is 2, z is 3.
   The sum of the exponents is 9.
   $4 + 2 + 3 = 9.$

5.6 Degree of a Polynomial

The **degree** of a polynomial is **n.** "n" is the degree of a term of the polynomial which is the highest. If the polynomial is a one term polynomial then the degree of the polynomial is the degree of the monomial. If it has several terms then the degree of the polynomial is found by determining the degree of each of the terms of the polynomial and finding the highest degree. The term with the highest degree is the degree of the given polynomial.

Examples:

1. $2x - 10$ --- is of degree 1
   The degree of $2x$ is 1, -10 is 0.
   The term with the highest degree is $2x$.

2. $-6y^4$ --- is of degree 4
   The degree of $-6y^4$ is 4. This is a one-term polynomial.

3. $24 - ab^2c + d^4$ --- is of degree 4
   The degree of 24 is 0, $-ab^2c$ is 4 and d^4 is 4.
   The terms with the highest degree are ab^2c and d^4.
   ab^2c is degree 4 and d^4 is degree 4.

4. $69 - 22x^4y^2z^3$ ---- is of degree 9.
   The degree of 69 is 0, and $-22x^4y^2z^3$ is 9
   The term with the highest degree is $22x^4y^2$

Name: ____________________ **Date:** ________ **Score:** _________

Exercise 5.1

I. Tell whether each of the following mathematical expression is monomial, binomial, trinomial or polynomial with the number of terms given.

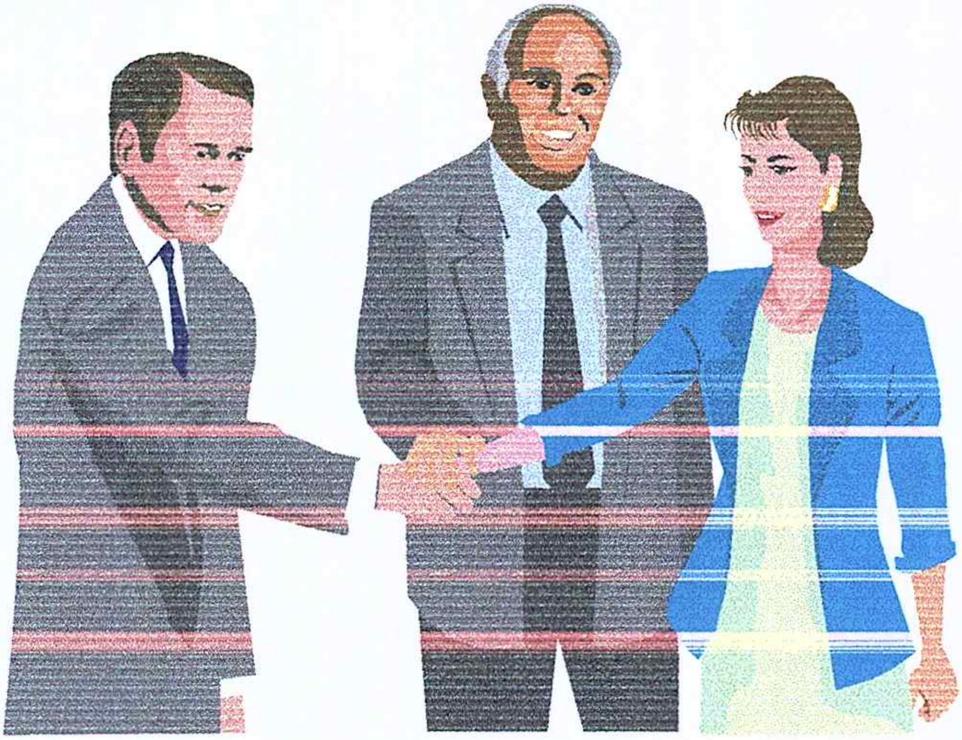
1. $x - 5y$
2. $x^4yz^3w - 209$
3. $ax^4 - by^2 - cz^3$
4. $x - x^2 + x^3$
5. $y^4 - y^2 + y - 10$
6. $a/bx^2 - b/cxy + c/dy^2$
7. $abc + xyz - 123$
8. $2ax - 3bx - 4cx + 5dx$
9. $mx + b$
10. $2l + 2w$

II. State the degree of the monomial or polynomial in the following expressions.

1. -5.0005
2. $-6x^4yz^3 + 8yz - 20$
3. $ax^4 - 2$
4. $11x^4y^2z^7$
5. $ax^3 - by^2x - cz^3$
6. $94x^4y^2z^3 - 123$

7. $x^4/3^2 - 4z^3/5$
8. $at^3 - bt^2 + ct - 5^7$
9. $ax^4 - bx^3y + cx^2y^2 - dxy^3 + ey^4$
10. $abcx^4 + dx^4yz - 123ax^9$
11. 895.05
12. $-46x^6yz^3 + 87yz - 120$
13. $7x^4 - 2y$
14. $211x^4y^3z^4$
15. $ax^4 - by^5x - cz^5$
16. $4x^4y^6z^3 - 12$
17. $x^5/3 - 4z^3/45$
18. $at^4 - bt^2 + ct - 5t^7$
19. $ax^5 - bx^6y + cxy^2 - dxy^3 + ey^4$
20. $abcx + dxy - 123ax^9$

Lesson 6



Addition and Subtraction of Monomials and Polynomials

Lesson 6. Addition and Subtraction of Monomials and Polynomials

Objectives:

Content:

1. To add and subtract monomials and polynomials.

Process:

1. To apply skills in adding and subtracting integers.
2. To make generalizations.

Affective:

1. To show patience in adding and subtracting monomials and polynomials.

5.1 Addition of Monomials and Polynomials

In lesson 5, we stated that a monomial which contains only one term can be called an algebraic term and the polynomial or a series of terms can also be related to algebraic expressions.

Quantities to be added or subtracted must always be of the same kinds. We can add 33 cats and 25 cats and the sum is 58 cats. But we cannot add 33 cats and 25 rats. In algebra quantities represented by the same variables or **similar terms** can be added or combined together. 33cats + 25cats = 58cats. Consider "33cats" as an algebraic term or (monomial). The numerical coefficient is 33 and the variables or the literal coefficients are the letters c, a, t, and s. "25cats" is another algebraic term (monomial) having the same variables but with different numerical coefficient. We can add the numerical coefficients 33 and 25 and the result is 58 and copy the common literal coefficients (variables), "cats".

In adding monomials it is important that the literal coefficients (variables) are the same for the terms to be added or combined. **Similar terms** are terms or monomials which have the same literal coefficient. By the same literal coefficient we mean the letters together with their exponents are the same. x and x^2 are not similar terms because although the letters are both x the exponents of x are different for x it is 1, and for x^2 it is 2.

Going back to the given problem, can we add 33 cats and 25 rats? What is the result? Is it 58 cats? 58 rats? Or 58 animals with bodies half of that of a cat and the other half that of a rat? In combining quantities, we can only combine quantities of the same kind. Addition of terms which are dissimilar cannot be performed but addition can be indicated by placing the "plus sign". So, 33 cats added to 25 rats can be written in symbol as "33cats + 25rats".

What is the general rule or procedure in adding monomials?

Rule: Addition of Monomials

To add monomials, first, write the terms in vertical column then add the numerical coefficients using the rule of addition of integers, then copy the same literal coefficients.

Examples:

1. Add: 7xy and 6xy

Solution: Write in vertical form then add the coefficients and copy the common literal coefficient.

7xy	7	coefficients are
+	+	added
6xy	6	(algebraic addition)
	13 xy	Literal coefficient xy is copied.

2. Add +abc , –4abc, –3xyz, 5xyz

Solution: Write similar terms in the same vertical column then add the numerical coefficients. Copy the literal coefficients. Use the plus sign to show that addition cannot be performed on the partial results (in case of dissimilar terms).

$$\begin{array}{r} +abc \\ + \quad \\ -4abc \\ \hline \end{array} \qquad \begin{array}{r} 1 \\ + \quad \\ -4 \\ \hline -3\ abc \end{array} \qquad \begin{array}{r} -3xyz \\ + \quad \\ 5xyz \\ \hline \end{array} \qquad \begin{array}{r} -3 \\ + \quad \\ 5 \\ \hline +2\ xyz \end{array}$$

(+abc) + (–4abc) +(–3xyz) +(5xyz) = –3abc +2xyz

3. Add $7x^3y^4z$, $10x^3y^4z$, $-12x^3y^4z$, $-9x^3y^4z$

Solution: Write in vertical form then add the coefficients and copy the common literal coefficients.

$$\begin{array}{rr} & 7x^3y^4z \\ & 10x^3y^4z \\ + & -12x^3y^4z \\ & -9x^3y^4z \\ \hline \end{array} \qquad \begin{array}{rr} & 7 \\ & 10 \\ + & -12 \\ & -9 \\ \hline & -4\ x^3y^4z \end{array}$$

4. Add 2a, +4a, +6a, –a, and +8a = 19a

$$\begin{array}{rr} & +2a \\ & +4a \\ + & +6a \\ & +8a \\ \hline & \\ & +20a \\ + & -\quad a \\ \hline & +19a \end{array}$$

When the algebraic expressions to be added consist of more than one terms or polynomials, will the rule for finding the sum of two monomials still apply? How do we add polynomials?

Rule: Addition of Polynomials

To add polynomials, first, write them so that similar terms are in vertical column. Second, add each column separately. In adding the columns add only the numerical coefficients using the rule of addition of integers, then copy the same literal coefficients or apply the rule for addition of monomials. Lastly, arrange the partial sum in ascending or descending power of a letter or variable.

Examples:

1. Add 7x +4y and 6x -8y

7x +4y	Arrange the x terms in one column
+ 6x -8y	and the y terms in another column.
13x -2y	Add separately each column and combine the results.

2. Add +a –4b +c, –4a +5b +4c, –3a –6b +7z

Solution: Write similar terms in the same vertical column then add the coefficients and copy the literal coefficient. Use the plus sign to show that addition cannot be performed on the partial results (in case of dissimilar terms)

+ a	–4b	+ c
–4a	+5b	+4c
–3a	–6b	+7c
–6a	–5b	+12c

3. Add $7x^3y^4z - 30$, $10x^3y^4z + 56$, $-12x^3y^4z - 19$ $-9x^3y^4z$ = ?

Solution: Write separately similar terms in vertical columns then add the columns.

$$
\begin{array}{rrr}
 & 7\,x^3y^4z & -30 \\
 & 10\,x^3y^4z & +56 \\
+ & -12\,x^3y^4z & -19 \\
 & -\ 9\,x^3y^4z & \\
\hline
 & -\ 4\,x^3y^4z & +\ 7
\end{array}
$$

2. Add 2a –b, –4a +c, +6a –d, a +3b, 8a +4c, 9a +7d.

2a	– b		
–4a		+ c	
+6a			– d
a	+3b		
8a		+4c	
9a			+7d
21a	+2b	+5c	+6d

The sum is "21a +2b +5c +6d".

6.2 Subtraction of Monomials and Polynomials

Quantities to be subtracted must be of the same kind. For example, if we have 100 papayas and 8 papayas are spoiled, then we can subtract 8 papayas from 100 papayas and the difference is 92 papayas. The 92 papayas are the good papayas.

Usually, we subtract smaller quantities from greater quantities but because of the invention of the **integers and algebra**, we can now subtract bigger or greater quantities (numbers) from smaller quantities (numbers).

In subtraction, the quantity to which we subtract is called the **minuend** and the quantity which is being subtracted is called the **subtrahend** and the result is the **difference.**

	100	papayas	----- minuend
−	8	papayas	----- subtrahend
	92	papayas	----- difference

This concept of subtraction of the same kind of quantities is used in the subtraction of monomials. Consider “papayas” as the literal coefficients (variables) of the given monomials. We copy "papayas" but we subtract the coefficients of the monomials (100 – 8) to get the difference which is 92. The final answer is "92 papayas". Remember, we only copy the literal coefficient, "papayas".

We cannot subtract quantities made up of different kinds but we can only indicate the subtraction process by affixing the **minus sign** (–) in the result.

Example:

> If you have 30 apples and have eaten 5 oranges, can you subtract the 5 oranges from the 30 apples? Of course, you cannot and you are not allowed to do the subtraction process.

In the subtraction of monomials if the terms to be subtracted are dissimilar or unlike we cannot perform the subtraction. But we can indicate the subtraction by affixing a minus sign in the answer or result.

It is important that we know the subtrahend and the minuend in subtraction. Some phrases used in subtraction are the following:

"subtract ____ from ____ ", "____ minus ____", and "the difference of ___ and ____".

It is very clear which is the minuend in "subtract ____ from ____". The quantity after the word from is the minuend and the subtrahend is the quantity that follows the word "subtract".

In " ____ minus ____" the first quantity is the minuend and the quantity after the word minus is the subtrahend.

In the subtraction phrase "the difference of ___ and ___", the first ____ is taken as the minuend and the second ____ is the subtrahend.

Also, very important in subtraction of monomials is knowledge of subtraction of integers. The process of subtraction of monomials involves the subtraction of the coefficients of the monomials. Subtraction of the coefficient is **algebraic subtraction**. We have to change mentally the sign of the subtrahend and proceed as in algebraic addition.

We have two methods of subtraction of monomials: (1) horizontal method, and (2) the vertical method.

Rule: Subtraction of Monomials

To subtract monomials:

1. Determine the minuend and the subtrahend.
2. Change the sign of the term of the subtrahend mentally and proceed as in algebraic addition.
3. Copy the common literal coefficient (variable).

Examples:

1. Subtract 7x from 10x.

Horizontal method:

$$10x - 7x = (10 - 7)x = 3x$$

Vertical method:

$10x$	10	Coefficients are subtracted.
$-\ 7x$	-7	
	$3x$	Literal coefficient (variable x) is copied.

2. Subtract.

		Coefficients of the monomials	Algebraic subtraction of the coefficients	
1.	$+7xy$	$+7$	$+\ 7$	
	$+3xy$	$+3$	$(-)\ 3$	
			$+\ 4xy$	Literal coefficient copied, xy

2. −21az	−21	− 21	
6az	+ 6	(−) 6	
		− 27az	Literal coefficient copied, az
3. 10xyz	10	− 10	
− 8xyz	−8	(+) 8	
		− 2xyz	Literal coefficient copied, xyz
4. −7(x+y)	− 7	− 7	
−3(x+y)	− 3	(+) 3	
		− 4(x+y)	Literal coefficient copied.

Algebraic expressions with more than one term or polynomials can also be subtracted. In the subtraction of polynomials it is important that we can distinguish the minuend from the subtrahend. Also, equally important is knowledge of how to arrange the terms of the minuend in descending order or ascending order based on a single letter or variable. If we are to arrange the terms of the minuend in **descending order** based on a single letter or variable, this means that if the terms of the minuend contains the letters x, y, z, etc. and we would like to arrange the terms in descending powers of x,. we have to look for the exponent of x in each of the terms of the minuend. The terms which has an x, and the x in that term which has the highest exponent will be written first together with other letters (variables) in the term followed by the x term which has the next highest exponent up to the term containing an x which has the lowest exponent (usually this is the constant, x). You can also arrange the minuend in decreasing power of y or z. To arrange in ascending order is to do the opposite arrangement. You will have to start with the term with the lowest exponent, followed by the next lowest up to the highest.

Rule: Subtraction of Polynomials

To subtract polynomials:

1. Determine the minuend and the subtrahend.
2. Arrange the terms of the minuend in standard form (descending or ascending according to a single letter or variable) then below each term write the subtrahend placing all like terms in similar column.
3. Change all the sign of the terms of the subtrahend and add algebraically.

Examples:

1. Subtract $7x + 3y$ from $10x - 8y$

Horizontal method:

$$
\begin{aligned}
(10x - 8y) - (7x + 3y) &= \\
10x - 8y - 7x - 3y &= \\
(10x - 7x) + (-8y - 3y) &= \\
(10 - 7)x + (-8 - 3)y &= \\
3x \quad -11y
\end{aligned}
$$

Vertical method:

$10x$	$-8y$	
$+(-)\,7x$	$+(-)3y$	sign of the subtrahend is changed mentally.
$3x$	$-11y$	algebraic addition of terms

2. Subtract.

1. $4 \quad +7x \quad + 6x^2$ — Terms of the minuend and subtrahend
   $8 \quad +3x \quad + 3x^2$ — are arranged in increasing order of x

Solution:

4	$+7x$	$+6x^2$	
$+(-)\,8$	$+(-)3x$	$+(-)3x^2$	Change all signs of the terms of the subtrahend mentally.
-4	$+4x$	$+3x^2$	Add algebraically each vertical column.

2. $-21x^4z \quad + 3x^3y^2 \quad - 2xy^3$
   $-21x^4z \quad + 3x^3y^2 \quad - 2xy^3$

   0

3. $-4x^2y^2 \quad -3x^4y^4 \quad +2xy \quad + 2x^3y^3$
   $+ 3x^2y^2 \quad +4xy \quad -3x^4y^4 \quad - 2x^3y^3$

Solution: First: Arrange the terms of the minuend in descending power of x.
Second: Write each term of the subtrahend below each term of the minuend placing all like terms in similar column.
Third: Change the sign of the terms of the subtrahend and proceed as in algebraic addition.

$$\begin{array}{rrrr} -3x^4y^4 & + 2x^3y^3 & - 4x^2y^2 & +2xy \\ -3x^4y^4 & -2x^3y^3 & + 3x^2y^2 & -4xy \\ \hline & 4x^3y^3 & -7x^2y^2 & -2xy \end{array}$$

3. Subtract the sum of $(-14x^2y^2 \quad -3x^2y^2)$ and $(+2xy \quad + 2xy)$ from the sum of $(3x^2y^2 \quad -13x^2y^2)$ and $(40xy \quad -22xy)$

Solution: First: Determine the minuend and the subtrahend.
Second: Combine and simplify the minuend and the subtrahend.
Third: Arrange the terms of the minuend in descending power of x.
Fourth: Write each term of the subtrahend below each term of the minuend placing all like terms in similar column.
Fifth: Change the sign of the terms of the subtrahend and proceed as in algebraic addition.

Subtrahend

$$+\begin{array}{r} -14x^2y^2 \\ -\ 3x^2y^2 \\ \hline -17x^2y^2 \end{array} \quad + \begin{array}{r} +2xy \\ +2xy \\ \hline +4xy \end{array}$$

Minuend

$$+\begin{array}{r} 3x^2y^2 \\ -13x^2y^2 \\ \hline -10x^2y^2 \end{array} \quad + \begin{array}{r} 40xy \\ -22xy \\ \hline +18xy \end{array}$$

Subtract:

$$\begin{array}{rr} -10x^2y^2 & +18xy \\ -17x^2y^2 & +\ 4xy \\ \hline 7x^2y^2 & +14xy \end{array}$$

4. Subtract the sum of $(+14x^2y^2 \quad +3x^2y^2)$ and $(+12xy \quad +42xy)$ from the sum of $(30x^2y^2 \quad -23x^2y^2)$ and $(4xy \quad -2xy)$

Subtrahend

$$+\begin{array}{r} 14x^2y^2 \\ 3x^2y^2 \\ \hline 17x^2y^2 \end{array} \quad + \begin{array}{r} +12xy \\ +42xy \\ \hline +54xy \end{array}$$

Minuend

$$+\begin{array}{r} 30x^2y^2 \\ -23x^2y^2 \\ \hline 7x^2y^2 \end{array} \quad + \begin{array}{r} 4xy \\ -2xy \\ \hline +2xy \end{array}$$

Subtract:

$$\begin{array}{rr} 7x^2y^2 & +\ 2xy \\ 17x^2y^2 & +54xy \\ \hline -10x^2y^2 & -52xy \end{array}$$

Name: ____________________ **Date:** ________ **Score:** _________

Exercise 6.1

I. Add.

1. $2b$
$4b$

2. $-8mn$
$-6mn$

3. xyz
$-2xyz$

4. $-x^4$
$-7x^4$

5. $-3x^4yz^3$
$-x^4yz^3$

6. $15(a+b+c)$
$34(a+b+c)$

7. $-2xy(z+1)$
$7xy(z+1)$

8. $x^4 - 2$
$-x^4 - 4$
$-3x^4 + 10$

9. $-6w - 6x - 12y - 5z$
$-9w - 8x + 11y + 4z$
$+3w + 7x - 21y - 15z$

10. $5x^4y + 2x^3y^2 + 6xy^3 - 2y^4$
$-7x^4y - 4x^3y^2 - 7xy^3 + 8y^4$
$-3x^4y + 10x^3y^2 - 5xy^3 + 8y^4$

11. $2x^4 y^4 z^4 \quad +2x^3 y^3 z^3 \quad +69$
$-4x^4 y^4 z^4 \quad -4x^3 y^3 z^3 \quad -76$
$+3x^4 y^4 z^4 \quad +6x^3 y^3 z^3 \quad -50$

12. $109 \quad + x \quad - x^2 \quad +x^3$
$-10 \quad + y^4 \quad - y^2 \quad +y$

13. $2x^4 \quad +2x^3 \quad +y^3 \quad -z^3 \quad +9$
$-2y^4 \quad 6z^3 \quad 2y^3 \quad x^3 \quad -76$
$+ z^4 \quad +6x^3 \quad z^3 \quad - x \quad +50$

14. $12x^4 \quad +2x^3 \quad + \ 89$
$-4x^4 \quad - 4x^3 \quad + \ 65$
$13x^4 \quad + x^3 \quad - \ 40$

15. $2t^4 \quad +2t^3 \quad +t \quad +6$
$-t^4 \quad - 4t^3 \quad -7t \quad +2$
$+3t^4 \quad +3t^3 \quad +4t \quad -5$

II. Arrange the following polynomials in descending power of the letters, then add.

1. $a + 3b$; $5b + 4c$, $-3c + 4a$
2. $3ab - 5xy + c$, $-2xy + 9ab$; $c + 5$
3. $x + y - 2$; $9 + 3x - 6x$, $-32y + 43x$
4. $2abc - 18$; $5a + 5abc$, $6b - abc$; $9 + c$
5. $-x + 2y$; $-7 + z$, $7x + y$, $z + 4x$; $12z + 25y$
6. $2x - 4 + 6x^2$; $3x - 7x^2 + 8$
7. $13x^3 - 10 - 12x^2 + 3x$; $11 - 5x^2 + 7x^3$
8. $ax^3 + 3a^2b + 2ab^2 + b^3$; $3a^2b - 3a^3 + 3ab^2 + b$

9. $5y^2 + xy - x^2; \quad +3xy - 4x^2 - y^2$

10. $2z + 3w - 6x + 2y; \quad 12z + 30w - 6y + 9x$

11. $-24(a+b) + 46(c-d); \quad +32(a+b) + 42(c-d)$

12. $-12x^4 - 34x^3 - 7; \quad + x + 11x^4; \quad -24x^3 - 2x + 21$

13. $2ab + 3bc - bd + 4gh$ and $8ab - ac + 7bc + 8ef$

14. $7a^2b - 4ab^2 - 3a^3b^2$ and $19a^2b^2 - 6ab + 12ab^2$

15. $-12x^4 + 10 - 4x^3 - 7x - x^2; \quad -4x^3 - 7x - 4x^4 - 41$

Name: ____________________ **Date:** ________ **Score:** _________

Exercise 6.2

III. Perform the following subtraction.

1. $2a$
   $4a$

2. $-5x^4$
   $-3x^4$

3. $1.8xy^3z^2$
   $-2xy^3z^2$

4. $-3(x-1)$
   $-7(x-2)$

5. $-35abx^4$
   $-4abx^4$

6. $15(a+b+c)$
   $34(a+b+c)$

7. $-2xyz$
   $7xyz$

8. $x^4 \quad -2$
   $-x^4 \quad +2$

9. $-12y \quad -5z$
   $+11y \quad +4z$

10. $5x^4y^2 \quad -2y^4$
    $-7x^4y^2 \quad +8y^4$

11. $2x^4y^4z^4 - 3x^4y^4z^4$

12. $(109 + x) - (x^2 + x)$

13. $2x^4y^3z^3 - 2x^4y^3z^3$

14. $(12x^4z^3 + 8) - (4x^4z^3 + 1)$

15. $(2t^3 - 7t) - (+2t^3 + t)$

II. Subtract.

1. 43abc from +13abc
2. 3abc from 2xyz
3. – 3yz from – 2zy
4. –12abc from $45a^3b^3c^3$
5. (2ac + 3ac) from – 5ac
6. (2y + 5y) from (– 3y + 4y)
7. (x +2y) from [(7x +2y) + (x +y)]
8. [(2x – 4) + (6x –9)] from [(3x –7) + (x +8)]
9. [(x – 24) + (3x –7)] from x – 4
10. $(ax^3 + 3x^3 + 2x^3)$ from $(+b^3 + b - 3)$

III. Subtract the first quantity from the second.

1. $5y^2 + xy - x^2$, $+3xy - 4x^2 - y^2$

2. $2z + 3w - 6x + 2y$, $12z + 30w - 6y + 9x$

3. $-24(a+b) + 46(c-d)$, $+32(a+b) + 42(c-d)$

4. $-12x^4 - 34x^3 - 7$, $+x + 11x^4 + 21$

5. $2ab + 3bc - bd + 4gh$, $8ab - ac + 7bc + 8ef$

Lesson 7

Multiplication of Monomials and Polynomials

Lesson 7. Multiplication of Monomials and Polynomials

Objectives:

Content:

1. To find the product of monomials and polynomials.

Process:

1. To search for patterns.
2. To formulate rules in multiplying monomials and polynomials.

Affective:

1. To show perseverance in making use of patterns to arrive at a conclusion.

7.1 Multiplication of Monomials and Polynomials

Multiplication just like addition and subtraction is a binary operation. We need two elements of a set before the operation can be performed. For example, in the set of integers, we need two integers to have a product. And if we are to multiply three integers, we have to get the product of any two of the three integers first and the partial product is to be multiplied to the third integer to get the final answer. Hence, we cannot multiply all the three numbers simultaneously. The two numbers multiplied to obtain a product are called **multiplicand** and **multiplier** or we call them also as **factors** and the result of the operation is called **product.**

2 multiplied to 3 will result in 6. 2 and 3 are the factors of 6. 6 is the product of 2 and 3.

The symbols used to denote multiplication are (1) dot (•), (2) **x**, and (3) parenthesis, **( )**.

Multiplication of monomials and polynomials is very easy if you possess the following mathematical skills: (1) skills in multiplication of integers or signed numbers, (2) skills in applying the laws of exponents (positive, negative, fractional).

Let us review multiplication of integers or signed numbers. The first step in the multiplication of monomials is to obtain the product of the numerical coefficients of the factors. Hence, we have to multiply the numerical coefficients of the factors.

Knowledge of multiplication of integers is important because the product of two monomials is a product of (1) the numerical coefficients of the factors, and (2) the product of the literal coefficients of the factors. The first requires knowledge of multiplication of integers while the second requires knowledge of applying the laws of exponent.

The following rules give the sign of the product in multiplication of integers. Of course it is assumed that you know how to multiply two numbers.

Rule: Multiplication of Integers

1. The product of two numbers having like sign is positive.
2. The product of two numbers having unlike sign is negative.

The product of two numbers having like sign is positive. This means that the sign of the product is positive if the factors are both positive or both negative.

Examples:

1. (+2)(+6) = +12
2. (-2)(-6) = -12

The product of two numbers having unlike sign is negative. The product of a negative and a positive number is negative. Also the product of a positive number and a negative number is negative.

Examples:

1. $(+2)(-3) = -6$
2. $(-2)(+3) = -6$

Exponent plays a very important role in multiplication. In fact because of the exponents we can express products in more compact form.

Examples:

$x \bullet x = x^3$

$x \bullet x \bullet x = x^3$

$x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x = x^8$

Laws of Exponents:

1. $(x^m)(x^m) = x^{m+n}$ **3.** $(x^m)^n = x^{mn}$

2. $(x^m)(y^m) = x^m y^n$ **4.** $(xy^n) = x^n y^n$

Law 1. $(x^m)(x^m) = x^{m+n}$

Examples:

1. $(x^2)(x^3) = x^{2+3} = x^5$
2. $(x^4)(x^7) = x^{4+7} = x^{11}$
3. $(2^m)(2^m) = 2^{m+m} = 2^{2m}$
4. $(5^5)(5^4) = 5^{5+4} = 5^9$
5. $(4^2)(4) = 4^{2+1} = 4^3$

To multiply numbers with the same base copy the base and add the exponents.

Law 2: $(x^m)(y^m) = x^m y^n$

Examples:

1. $(x^2)(y^3) = x^2y^3$
2. $(2^4)(x^7) = 2^4x^7 = 16x^7$
3. $(2^3)(3^4) = 2^33^4 = 8 \times 81 = 648$
4. $(5^2)(4^3) = 5^24^3 = 25 \times 64 = 1600$
5. $(4^2)(2^4) = 4^22^4 = 16 \times 16 = 256$

To multiply numbers with different bases copy the corresponding base and exponents or just copy the factors omit the sign of multiplication and simplify if necessary.

Examples

1. $(x^2)^3 = x^{2x3} = x^6$
2. $(2^4)^7 = 2^{4 \times 7} = 2^{28}$
3. $(3^3)^4 = 3^{3 \times 4} = 3^{12}$
4. $(y^3)^3 = y^{3 \times 3} = y^9$
5. $(4^2)^4 = 4^{2 \times 4} = 2^8 = 32$

To raise a power to a power copy the base, multiply the exponent and simplify if necessary.

Law 4. $(xy^n) = x^n y^n$

Examples:

1. $(xy)^3 = x^3y^3$
2. $(2a)^7 = 2^7a^7$
3. $(3b)^4 = 3^4b^4$
4. $(xyz)^3 = x^3y^3z^3$
5. $(4x^2)^4 = 4^4x^{2 \times 4} = 4^4x^8$

To raise a product to a power raise each factor to the indicated power or exponent then multiply the factors. Simplify if necessary.

Monomials multiplied by itself (the same monomial) can be written in more compact form by using exponents. The process of raising a monomial to a power is the quickest way to arrive at the product or result. This is made possible by applying the second and third law of exponent.

Examples:

1. $(-8a)(-8a) = (-8a)^2$
2. $(3a^2)(3a^2) = (3a^2)^2$
3. $(-6a^2)(-6a^2)(-6a^2) = (-6a^2)^3$
4. $p^2p^2p^2p^2p^2 = (p^2)^5$

Rules: Multiplication of Monomials

1. **Multiply the numerical coefficients.**
2. **Add the exponents of the letters (variable) that are alike and give the sum as an exponent of that letter in the product.**
3. **Include in the product all letters that occur in one factor only.**

Examples:

I. Multiply

1. $(-8a)(2a) = (-8)(2)(a^{1+1}) = 16a^3$

2. $(3a^2)(2a^4) = (3)(2)(a^{2+4}) = 6a^6$

3. $(-6a^2)(-a^4) = (-6)(-1)(a^{2+4}) = 6a^6$

4. $4p^2q^2(-7p^3) = (4)(-7)(p^{2+3})q^2 = -28p^5q^2$

5. $3(a+b)^2 \bullet 7xy(a+b) = (3 \bullet 7)(a+b)^{2+1}xy$
$= 21(a+b)^3xy$

6. $(2a^3y^2)(-12xy^4) = (2)(-12)(x^{3+1})(y^{2+4})$
$= -24(x^4)(y^6)$
$= -24x^4y^6$

7. $(2abc)(-ac) = (2)(-1)(a^{1+1})b(c^{1+1}) = 2a^2bc^2$

8. $9(-xyz^2) = (9)(-1)xyz^2 = -9xyz^2$

9. $(-2xy^2z^3)^2 = (-2)^2x^2(y^{2\times 2})(z^{3\times 2}) = 4x^2y^4z^6$

10. $(4x^3)(-2y)(3x) = (4)(-2)(3)(x^{3+1})y = 36x^4y$

7.2 Multiplication of Polynomial by Monomials and Another Polynomials

Multiplication of a **polynomial by a monomial** uses the same rule as in multiplication of monomial by a monomial. Only this time we makes use of an additional property of the real number, the distributive property of multiplication over addition.

Examples:

1. $a(b+ c) = a(b) + a(c) = ab + ac$
2. $4(a +4b) = 4a + 4(4b) = 4a + 16b$
3. $7x(x^2 + y^2) = 7x(x^2) + 7x(y^2) = 7x^3 + 7xy^2$

In multiplying a **polynomial by another polynomial** we use the same rule as in multiplying a polynomial by a monomial. But since, the multiplier is a polynomial so, each term of the multiplicand is multiplied by each term of the multiplier. What is very important is to arrange first the terms of the multiplicand and multiplier in either descending or ascending power of your choice of letter before you multiply.

Examples:

1. Multiply $5x + 8$ by $3x - 5$

$$\begin{array}{rl} & 5x + 8 \\ \times & 3x - 5 \\ \hline \end{array}$$

$15x^2 + 24x$	Multiply 3x(5x+8), 1st partial product
$-25x - 40$	Multiply -5(5x+8), 2nd partial product
$15x^2 - x - 40$	Add similar terms and complete the product.

2. Multiply $(x + 2)(x - 5)$

$$\begin{array}{rl} & x + 2 \\ \times & x - 5 \\ \hline \end{array}$$

$x^2 + 2x$	Multiply x(x+2), 1st partial product
$-5x - 10$	Multiply -5(x+2), 2nd partial product
$x^2 - 3x - 10$	Add similar terms and complete the product.

3. Multiply $x^2 + x + 2$ by $2x - 3$

$$\begin{array}{rl} & x^2 + x + 2 \\ \times & 2x - 3 \\ \hline \end{array}$$

$2x^3 + 2x^2 + 4x$	Multiply $2x(x^2 + x + 2)$, 1st partial product
$-3x^2 - 3x - 6$	Multiply $-3(x^2 + x + 2)$, 2nd partial product
$15x^2 - x - 40$	Add similar terms and complete the product.

Name: ____________________ **Date:** ________ **Score:** _________

Exercise 7.1

I. Find the product.

1. $(2c)(-2)$
2. $(-6w)(-6x)$
3. $(-x^2)(-x^4)$
4. $(-y^4)(7y^3)$
5. $(-2/3x^4)(1/2\,x^2)$
6. $(-3x^4y^3)(-y^3z)$
7. $(2x^4y^4z^4)(4x^3y^3z^3)$
8. $(-2xyz)(+2x^3y^2z)$
9. $(5xy^4z)(-3wx^4z)$
10. $[(-12x^4y)(-4x^3y^2)](-7xy^3)$
11. $12(-3x^4y)^3(-5xy^3)$
12. $[(4x^4z)[(3xy^4)^4](2x^3y^3)^3$
13. $(2xy)[(2xy)^2(2x^2y^2)^3]$
14. $-4(2x^4yz^3)^3[(9y^4z^3)(2y^3z^3)]^3$
15. $[12x^4(+2x^3)^3][(13x^4y^3)(2x^4yz)^3]$

II. Perform the indicated multiplication.

1. $a + 3b;\ 5b + 4c,\ -3c + 4a$
2. $3ab - 5xy + c,\ -2xy + 9ab;\ c + 5$
3. $x + y - 2;\ 9 + 3x - 6x,\ -32y + 43x$
4. $2abc - 18;\ 5a + 5abc,\ 6b - abc;\ 9 + c$
5. $-x + 2y;\ -7 + z,\ 7x + y,\ z + 4x;\ 12z + 25y$
6. $2x - 4 + 6x^2;\ 3x - 7x^2 + 8$
7. $13x^3 - 10 - 12x^2 + 3x;\ 11 - 5x^2 + 7x^3$
8. $ax^3 + 3a^2b + 2ab^2 + b^3;\ 3a^2b - 3a^3 + 3ab^2 + b$
9. $5y^2 + xy - x^2;\ +3xy - 4x^2 - y^2$
10. $2z + 3w - 6x + 2y;\ 12z + 30w - 6y + 9x$
11. $-24(a+b) + 46(c-d);\ +32(a+b) + 42(c-d)$
12. $-12x^4 - 34x^3 - 7;\ + x + 11x^4;\ -24x^3 - 2x + 21$
13. $2ab + 3bc - bd + 4gh$ and $8ab - ac + 7bc + 8ef$
14. $7a^2b - 4ab^2 - 3a^3b^2$ and $19a^2b^2 - 6ab + 12ab^2$
15. $-12x^4 + 10 - 4x^3 - 7x - x^2;\ -4x^3 - 7x - 4x^4 - 41$

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Exercise 7.2

I. Raise each monomial to the indicated power.

1. $(y)^3$
2. $(5xy)^4$
3. $(-a^2)^3$
4. $(-5x^2 y^3 z^4)^2$
5. $6(x^3)^2$
6. $(8ab^2)^3$
7. $(5^2x^3y^2)^3$
8. $(-4ax^3)^4$
9. $20(x^2y^3)^2$
10. $(7x^4y^5z^2)^3$

II. Multiply.

1. $(x + 4)$ by $(x - 1)$
2. $(a - 3)$ by $(a + 5)$
3. $2a(10x^5 - 3x + 7x^3)$
4. $7x^2 (10x^3 - 3xy + 4y^2)$
5. $m + 6$ by $m - 6$
6. $(x + 6)(x - 4)$
7. $(5r - 3)(2r + 7)$
8. $(x^3 + 5 + 2x^2)(x + 2)$

9. $(x + 7)(4x - 5 + x^2)$

10. $(3x + 7)(5x^2 - 1)$

11. $(x + 6)(2x - 1)$

12. $(3r - 5)(2r - 4)$

13. $(4x - 3)(x+7)$

14. $(x^3 + 8 + 2x^2)(2x+2)$

15. $(2x + 7)(5x + 5 + x^2)$

Lesson 8

Division of Monomials & Polynomials

Lesson 8. Division of Monomials and Polynomials

Objectives:

Content:

1. To find the quotient of two polynomials.
2. To perform the fundamental division of monomials and polynomials.

Process:

1. To apply the laws and of exponent in dividing polynomials.
2. To apply the rules in dividing integers.

Affective:

1. To show speed and accuracy in doing the activity.

8.1 Division of Integers

Like multiplication, division is also a binary operation. It needs a dividend and a divisor. The result of the operation is called the quotient. It is the inverse operation of multiplication. The symbols are ÷, /, —, a bar and $)\overline{\ }$. Unlike multiplication, division is not commutative. Therefore, it should follow the order of operation, that is, it must be performed from left to right, and division by zero cannot be determined.

Division of algebraic expression requires the knowledge of the following:

1) Division of integers

Rule 1. The quotient of two numbers having the same sign is positive.

Examples:

$4 \div 2 = 2$

$+4 \div +2 = +2$

$-4 \div -2 = 2$

$6 \div 2 = 3$

$+6 \div +2 = +3$

$-6 \div -2 = +3$

Rule 2. The quotient of two numbers having unlike signs Is negative.

Examples:

$-4 \div 2 = -2$

$-4 \div +2 = - 2$

$+6 \div -3 = - 2$

$-6 \div +2 = -3$

2) Knowledge on the laws of exponent.

Laws of Exponent:

1. $a^m \div a^n = a^{m-n}, \quad m > n \text{ and } a \neq 0$
2. $a^m \div a^n = a^{-(m-n)} = 1/a^{m-n}, \quad m < n \text{ and } a \neq 0$
3. $a^n \div a^n = a^{n-n}, = a^0 = 1$
4. $(a/b)^n = a^n / b^n$

Examples:

$7^4 \div 7^2 = 7^{4-2} = 7^2 = 49$

$8^4 y^3 z^2 \div 8^3 y^2 z^2 = 8y$

$xy \div xy = (xy)^0 = 1$

$x^4 \div x^7 = x^{(4-7)} = x^{-3} = 1/x^3$

$(x/y)^3 = x^3 / y^3$

8.2 Division of Monomials

Rules:

1. *Divide the coefficient of the dividend by the coefficient of the divisor.*

2. *Subtract the exponent of the literal coefficient in the divisor from the exponent of the same literal coefficient in the dividend (apply the laws of exponent).*

3. *Omit any literal coefficient with the same exponent in the dividend and divisor (apply laws of exponent $x/x = x^0 = 1$).*

Examples:

1. $$\frac{10a^4}{2a^3} = \frac{10}{2} \bullet \frac{a^4}{a^3} = 5a^{4-3} = 5a$$

2. $$\frac{10a^4b^2}{2a^3} = \frac{10}{2} \bullet \frac{a^4}{a^3} \bullet b^4 = 5a^{4-3}b^4 = 5ab^4$$

3. $$\frac{-20x^4y^2}{4xy} = \frac{-20}{4} \bullet \frac{x^4}{x} \bullet \frac{y^2}{y} = -5x^{4-1}y^{2-1} = -5x^3y$$

4. $$\frac{32m^2n^4}{-16mn} = \frac{32}{-16} \bullet \frac{m^2}{m} \bullet \frac{n^4}{n} = -2m^{2-1}n^{4-1} = -2mn^3$$

5. $$\frac{(-2xy^2)^2}{(xyz^3)^2} = \frac{4}{1} \bullet \frac{x^2}{x^2} \bullet \frac{y^4}{y^2} \bullet \frac{1}{z^6} = 4x^{2-2}y^{4-2}z^{0-6}$$

$$= 4x^0y^2z^{-6} = 4y^2z^{-6} = 4y^2/z^6$$

8.3 Division of Polynomials

A. Division of a Polynomial by a Monomial.

To divide a polynomial by a monomial, divide each term of the polynomial successively by the monomial, beginning at the left. Follow the same procedure of division in arithmetic.

Example:

1. Divide $\dfrac{-18a^4 - 6a^5 - 12a^3}{6a^2} = \dfrac{-18a^4}{6a^2} \; \dfrac{-6a^5}{6a^2} \; \dfrac{-12a^3}{6a^2}$

$$= -3a^{4-2} \; -1a^{5-2} \; -2a^{3-2}$$
$$= -3a^2 \; -1a^3 \; -2a$$
$$= -3a^2 \; -a^3 \; -2a$$

B. Division of a Polynomial by a Polynomial

To divide a polynomial by a polynomial, we follow exactly the same procedure of the long division in arithmetic.

RULES:

1. Arrange the terms of the dividend and divisor according to either ascending or descending powers of a common letter.

2. Divide the first term of the dividend by the first term of the divisor to get the first term of the quotient.

3. Multiply the entire divisor by the first term of the quotient and subtract the result from the dividend bringing down the other terms of the dividend.

4. Divide the first term of the remainder by the first term of the divisor.

5. Multiply the entire divisor by the second term of the quotient, subtract and continue until there is no remainder or until the remainder can no longer be divided by the divisor.

6. Check by multiplying the divisor by the quotient.

Example1. Divide $3x^2 + 5x + 2$ by $x + 1$

$$
\begin{array}{r|l}
 & 3x + 2 \\
\hline
x + 1 & 3x^2 + 5x + 2 \\
 & 3x^2 + 3x \\
\cline{2-2}
 & \quad 2x + 2 \\
 & \quad 2x + 2 \\
\cline{2-2}
 & \quad 0
\end{array}
$$

Steps:

1. Divide $3x^2$ by x and record the quotient.

$$\frac{3x^2}{x} = 3x$$

2. Multiply the term of the divisor by 3x.

$$\begin{array}{r} x + 1 \\ 3x \\ \hline 3x^2 + 3x \end{array}$$

3. Subtract the product from the dividend and bring down 2.

4. Divide the first term which is the new dividend by x and record the result as quotient.

$$\frac{2x}{x} = 2$$

5. Multiply the divisor by two.

$$\begin{array}{r} x + 1 \\ 2 \\ \hline 2x + 2 \end{array}$$

6. Record the result then subtract it to the new dividend.

Example 2. Divide $x^2 + 3xy - 5y^2$ by $x + y$

$$
\begin{array}{r l}
 & x \quad + 2y \\
x + y \; / & \overline{x^2 + 3xy \; - 5y^2} \\
 & \underline{x^2 + xy} \\
 & \quad 2xy \; - 5y^2 \\
 & \quad \underline{2xy \; - 2y^2} \\
 & \qquad\quad -3y^2
\end{array}
$$

remainder remainder

The quotient may be written as $x + 2y - \dfrac{3y^2}{x + y}$

Example 3. Divide $(a^3 - 1)$ by $(a - 1)$

Note: The dividend does not include an $\mathbf{a^2}$ term and an **a** term therefore you must leave some space between a^3 and -1.

$$
\begin{array}{r l}
 & a^2 \; + a \quad + 1 \\
a - 1 \; / & \overline{a^3 \qquad\qquad -1} \\
 & \underline{a^3 \; - a^2} \\
 & \quad a^2 \\
 & \quad \underline{a^2 \qquad\quad} \\
 & \qquad a - 1 \\
 & \qquad \underline{a - 1} \\
 & \qquad\quad 0
\end{array}
$$

The quotient is $\mathbf{a^2 + a + 1}$.

Name: ____________________ **Date:** ________ **Score:** _________

Exercise 8.1

I. Find the quotient.

1. $(2c) \div (-2)$
2. $(-6w) \div (-6w)$
3. $(-4x^2) \div (2x^4)$
4. $(-4y^4) \div (12y^3)$
5. $(-2/4x^4) \div (1/2x^2)$
6. $(30x^4y^3) \div (-6xy^3)$
7. $(2x^4y^4z^4) \div (4x^3y^3z^3)$
8. $(-2xyz) \div (+2x^3y^2z)$
9. $(5xy^4z) \div (-3wx^4z)$
10. $[(-12x^4y)(-4x^3y^2)] \div (-7xy^3)$
11. $12(-3x^4y)^3 \div (-5xy^3)$
12. $[(4x^4z)[(3xy^4)^4] \div (2x^3y^3)^3$
13. $(2xy)(2xy)^2 \div (2xy)^3$
14. $-4(2x^4yz^3)^3(9y^4z^3) \div (2y^3z^3)$
15. $[12x^4(+2x^3)^3] \div [(13x^4y^3)(2x^4yz)^3]$

Name: ____________________ **Date:** ________ **Score:** _________

Exercise 8.2

I. Perform the indicated division:

1. $\frac{5x + 10}{5}$

2. $\frac{4x + 2}{2}$

3. $\frac{12x^4 - 8x^5}{2x^2}$

4. $\frac{x - x^2}{x}$

5. $\frac{-5x^3 + 15xy^4}{-5xy^2}$

6. $\frac{36x^3y - 18xy^2}{3xy^2}$

7. $\frac{-15m^4n^2 - 20m^2n^4}{-5mn}$

8. $\frac{12p^2q^2 - 6pq}{6pq}$

9. $\frac{x^4z^4 - y^2z^2}{z^2}$

10. $\dfrac{25a^2c^2 - 20a^4c^2 + 10a^4c^3}{-5ac^2}$

II. Divide the first polynomial by the second and check by multiplication.

1. $(x^2 + 3x - 2)$ by $(x + 2)$
2. $a^4 + 8a^2 + 16$ by $a^2 + 4$
3. $3 + 12a^2 + 10a$ by $2a - 3$
4. $6x^2 + 3x + 40$ by $2x + 5$
5. $m^2 - 5m + 6$ by $m - 3$
6. $m^4 + 3m^3n + n^4$ by $m + 2n$
7. $d^2 + 14d - 49$ by $d - 7$
8. $c^4 - d^4$ by $c - d$
9. $m^3 - n^3$ by $m - n$
10. $r^2 - 7rs + 12s^2$ by $r - 3s$

CURRICULUM VITAE

CURRICULUM VITAE

PERSONAL DATA

NAME	:	JOSEPHINE EBIAS BACSAL
ADDRESS	:	132 Guindapunan District Catbalogan, Samar
DATE OF BIRTH	:	December 23, 1962
PLACE OF BIRTH	:	Fabilla Hospital Sta. Cruz, Manila
CIVIL STATUS	:	Married
HUSBAND	:	Rolando Dacoco Bacsal
CHILDREN	:	Jocelyn E. Bacsal Jennelyn E. Bacsal Jovelyn E. Bacsal Jessalyn E. Bacsal Jay Roland E. Bacsal
PRESENT POSITION	:	Head Teacher I
STATION	:	Calapi National High School Calapi, Motiong, Samar

EDUCATIONAL BACKGROUND

Primary	:	Old Balara Elementary School Balara, Quezon City 1973
Elementary	:	Calapi Elementary School Calapi, Motiong, Samar 1975

Secondary	:	Wright Vocational School Lipata, Paranas, Samar 1979
College	:	Sacred Heart College Catbalogan, Samar 1982 BS Accounting Samar College Catbalogan, Samar 1986 Professional Education Subjects 24 units
Graduate	:	Samar State Polytechnic College Catbalogan, Samar
Degree Pursued	:	Master of Arts in Teaching (MAT)
Major	:	Mathematics

CIVIL SERVICE ELIGIBILITIES

Professional Board Examination for Teachers (PBET)

WORK EXPERIENCE

1. Secondary School Teacher I – Calapi National High School, Calapi, Motiong, Samar, SY 1986 – 1987.

2. Secondary School Teacher In - charge I – Calapi National High School, Calapi, Motiong, Samar, SY 1987 – 1988 to SY 1992 - 1993.

3. Secondary School Teacher II – Calapi National High School, Calapi, Motiong, Samar, SY 1993 – 1994 to SY 1994 - 1995.

4. Secondary School Head Teacher I – Calapi National High School, Calapi, Motiong, Samar, SY 1995 – 1996 up to the present.

AWARDS AND ACCOMPLISHMENT

1. Certificate of Appreciation – for invaluable support given by 34th Infantry Battalion, 8th Infantry Battalion, Philippine Army, Calapi, Motiong, Samar, July 26, 1991,

2. Certificate of Recognition – as Demonstration Teacher in the Division Seminar Workshop on Curriculum Writing in Mathematics, Aug. 26-28, 1992.

3. Certificate of Recognition – as In-charge of Registration in the Seminar Workshop for Secondary Mathematics Teachers, Division Level, June 30, 1993.

4. Certificate of Recognition – for exemplary services rendered during the Content Based Regional In-house Conference of Social Studies Teachers and Secondary School Heads, October 16, 1994.

5. Certificate of Recognition – for meritorious and outstanding services rendered to the 3rd Provincial Jamborette '95 at SNAS, San Jorge, Samar, BSP Samar Council, March 14, 1995.

6. Certificate of Recognition – for spearheading the establishment of the Pupils Government Center, July 20, 1995.

7. Certificate of Recognition – as coach in the Second Division DAMATH Competition on October 1, 1995 at SNS, Catbalogan, Samar.

8. Bronze Medal of Merit - for meritorious and outstanding services rendered to the BSP, Samar Council, October 31, 1995.

9. Certificate of Recognition – as facilitator of the Adult Class, Non-formal Education in Calapi National High School, March 27, 1996.

10. Plaque of Recognition – for moral and physical development of the youth, given by the BSP Samar Council on March 14, 1997.

11. Certificate of Recognition – for meritorious and outstanding services rendered to the BSP 4th Provincial Jamborette and GSP Encampment, Barangay Panayaron, Calbiga, Samar, October 31, 1996.

TRAININGS, SEMINARS AND WORKSHOP ATTENDED

1. SEDP Leadership Training for Brgy. High School Administrators, Teachers Camp, Baguio City, September 26 –29, 1988.
2. SEDP Mass Training for Mathematics I Teachers, SNS, Catbalogan, Samar, April 1-28, 1989.
3. SEDP Mass Training for Science I Teachers, SNS, Catbalogan, Samar, May 1-28, 1989.
4. SEDP Mass Training for Mathematics II Teachers, SNS, Catbalogan, Samar, May 14 – June 2, 1990.
5. Seminar – Workshop on Research in Teaching Learning Math, GSP Catbalogan, Samar, December 4 – 6, 1991.
6. Seminar Workshop on Curriculum Writing in Mathematics I and II, GSP Building, Catbalogan, Samar, August 26 –28, 1992
7. 1991 Plan-budget Workshop, RELC (Regional Educational Learning Center) Government Center, Candahug, Palo, Leyte, July 12 – 23, 1993.
8. Seminar Workshop for Secondary Mathematics Teachers, GSP Building, Catbalogan, Samar, June 28 - 30, 1993.
9. School Based Management Development Program, RELC, Government Center, Candahug, Palo, Leyte, July 12 – 23, 1993.
10. Division In-Service Orientation on Mathematics Teaching, GSP Building, Catbalogan, Samar, October 6 – 7, 1994.
11. Regional Seminar on Process, Monitoring, Care and Maintenance of School Physical Facilities, COA, Government Center, Candahug, Palo, Leyte, November 11, 1994.
12. Conference - Workshop on Reviving and Developing Desirable Values in Scouting, Capitol Hills Scout Camp, Cebu City, February 17 – 19, 1995.
13. Seminar - Workshop on the Preparation of FY 1996 Budget Proposal, GSP Building, Catbalogan, Samar, March 20 – 21, 1995.
14. Seminar - Workshop on TRM Utilization in Secondary Mathematics, BSP Building, Catbalogan, Samar, July 14 -15, 1995.

15. School In-Service Program for Mathematics, Pinabacdao National High School, Pinabacdao, Samar, January 29; 1996.

16. Orientation Conference – Workshop on the Centennials of Philippine Revolution, Maqueda Bay Hotel, Catbalogan, Samar, October 17–19, 1996.

17. Orientation Workshop on the Regular Annual Collection and Processing of Basic Education Data, RELC, Government Center, Candahug, Palo, Leyte, December 4 – 5, 1996.

18. 17th National Congress of Secondary Schools Administrators, Teachers Camp, Baguio City, April 28 to May 1, 1997.

LIST OF TABLES

LIST OF FIGURES