DEVELOPMENT OF SELF INSTRUCTIONAL MATERIALS IN MATHEMATICS 101

(College Algebra)

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APPROVAL SHEET

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DEDICATION

To my loving mother, **Isabel Vda. de Montives;**

To my loving husband, **Pompio O. Jaromay;**

To my son and daughter, **Johnny and Ann-ann;**

To my grand daughters, **Reynalyn, Renafe, Reynamay;**

To my sisters and brothers;

To my colleagues in Science and Mathematics;

To them this humble work is dedicated.

A.M.J.

ABSTRACT

This study attempted to determine the steps or process in the development of modules and its validation as to its effectiveness and appropriateness in relation to the students' mathematics skills and performance. The module was designed to meet the students' deficiencies particularly in their class in College Algebra. The target group was the BSIT students of Samar State Polytechnic College, Catbalogan, Samar. This study utilized the experimental method of research using the pre-test/post-test control group design. Fifty BSIT students were chosen as samples/respondents of the study. The students were grouped into two. One group was used as the experimental group which were subjected to the modular approach in teaching. The study revealed that learning took place with the control group as their post-test result was higher than the pre-test. The pre-test mean scores 14.32 and the post-test mean scores was 20.76. Using the t-test for independent samples, the computed t was very much higher than the tabular t. This means that there is a significant difference in their results. Thus the data imply that learning took place in this group. Learning was also facilitated with the experimental group shown in the post-test result. The students learn at their own pace. They do not need to cope with the pace of the class in learning the lessons. Their progress depends on their own and they learn at their own time. Mathematics teachers are encouraged to use modular instruction in the classroom.

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Chapter 1

THE PROBLEM: ITS BACKGROUND

Introduction

The current happening in education in the Philippines today can be aptly called a paradox. The Philippines can be classified as semi-developed and even bordering on being advanced country owing to the considerable number of students in school. Nebres et. al. as cited by Marco (1983) claims that the schools continuously send out caravans of graduates in their respective chosen fields of endeavor but these graduates end up as unemployed professionals. The paradox continues to include that in spite of the great number of educated Filipinos and the rich resources that we have both natural and manpower, the Philippines faces grave problems such as poverty, malnutrition, and unemployment due to economic instability. There is a significant relation between education and progress. But in our case, our country is far behind its neighbors in terms of progress.

Mathematics of which algebra is a branch or field plays a fundamental role in the development of our country because it is the basic component of all scientific and technological researches and technical training. And if our country is to advance economically, our youth should have a strong foundation in Mathematics for they are our future technologists and technocrats. This idea is reinforced then by the former secretary Manuel (1969: 12) pointed out, "The progress of science and technology depends to a great extent upon the development of mathematics. In fact, science and technology can progress only as far as the advance in mathematics is made." Mathematics has become vital to industry and to business and must be part of every one's education. Soriano, D. (1968: 9) commented that the primary cause of the Philippine paradox is the inferior quality of education in our country today. He pointed out that due to the desire of many Filipinos to achieve their economic aspirations through education, our educational system experienced a rapid expansion at the expense of quality. Canares (1967: 345) had the same observation. She affirmed that the graduates are not quality graduates. They are not armed with adequate skills and abilities to man our industries and forward development; thus economic progress has been moving at a snail's pace.

The deteriorating quality of education is true especially in algebra where the students' knowledge is far from being unsatisfactory. It is a common knowledge that mathematics is not everybody's favorite subject. Experience tells us that more often, it is the cause of many students' academic difficulties.

One cause of inferior quality of students, maybe traced back to two negative attitudes of the students. Attitudes affect the learning process of the learner. (Krashen: 1985, 243-247) First, many of them take their subjects lightly without mastering or knowing at least seventy-five percent of the content of the whole course. They pass and finish the course with very little knowledge; consequently they become mediocre graduates. Second, to the great majority of the students mathematics is too abstract to understand and that no matter how much they try, they just cannot understand the concepts. Marco attests this fact when she said: "At present time, rarely could we find a student who is not at once baffled by the interminable streams of numbers and symbols and therefore sulks at the prospect of patient and critical thinking" (Marco: 1983). Hence, the great challenge is for the mathematics teachers to teach the subject within the grasp of all the students. Unless the standard of teaching to the students would be elevated will their negative attitude towards mathematics be changed. Not unless their manipulative skills and analytical thinking would be developed will the country be provided with the pools of craftsmen, technicians and specialist to enhance its technological researches and technical training.

Realizing, therefore the great importance of mathematics in the national development of our country and in our personal lives mathematics teachers should make mathematics interesting and attractive to the students. Hence, it is necessary to introduce innovative techniques in our classroom. One innovation in the teaching strategy is the modular approach. It is an individualized instruction aimed at developing the students' skills to the optimum since it is criterion-referenced rather than norm-referenced. To help understand what a module is Greager and Murray (1978: 243) explained that modules are self contained and independent units of instruction with primary focus on well defined objectives. The use of modules in college teaching offers the following advantages:

1. It provides opportunity for organizing a number of sequenced experiences to reflect special interest of the instructor on the students.

2. It allows the instructor to focus on the deficiencies of students in the subject.

3. A module serves to eliminate the necessity of covering subject matter already known to the students.

4. This instructional material assesses the progress of the students' in learning.

5. It reduces routine aspect of instruction giving the teacher a chance to enjoy her personal contact with the students.

In the findings of Perez (1984), she said that through the use of modules students' learning capabilities in the basic skills in mathematics will be very much improved. She further averred that it makes the work of the teacher lighter thus enabling him to give particular attention to the slow learners in his class.

It is along this line of thinking that the researcher planned to develop instructional materials that would help the students understand mathematical concepts better. In this module abstract ideas were concretized and made easy for the students to grasp. Moreover, the developed module will be an addition to few instructional materials available in algebra.

Statement of the Problem:

This study attempted to determine the steps or process in the development of modules and its validation as to its effectiveness and appropriateness in relation to the students mathematics skills and performance. The module was designed to meet the students' deficiencies particularly in their class in algebra. The target group was the first year BSIT students of Samar State Polytechnic College, Catbalogan, Samar. Specifically, it sought to answer the following questions:

1. What is the profile of the samples/respondents as to:

   a. age;

   b. grade in Mathematics IV?

2. On the basis of the pretest and posttest results; how effective are the developed modules?

2.1. What are the pretest and posttest mean scores on mathematical skills performance of control group and the experimental group?

2.2. Is there a significant difference between the mean scores of the control group and experimental group per pretest and posttest?

3. Is there a significant difference between the pretest and posttest mean scores of the control group and the experimental group?

4. Is the developed module appropriate for the BSIT students?

Null Hypotheses

The following null hypotheses were formulated and tested based on the aforementioned specific questions:

1. There is no significant difference between the mean scores of the control group and the experimental group per pretest and posttest.

2. There is no significant difference between the pretest and posttest mean scores of the control group.

3. There is no significant difference between the pretest and posttest mean scores of the experimental group.

Theoretical Framework

In the realm of science, technology and inventions, mathematics is the vehicle which transports one to the newer and broader fields of knowledge. Skills that are supposed to be acquired in algebra need constant drills and experiences for them to be well established. This concept is best expressed in the behavioral concept of learning using the SR theory (stimulus response) as expressed by Thorndike and Skinner as cited by Laird (1986: 101-105). The two psychologist believe that learning as a bond should be strengthened to make it effective.

Furthermore, this study is anchored on Hughes' (1962: 300) theory stating "The learner is called upon to respond frequently in the interaction with an instructional program and the rate of which instruction proceeds is governed individually by each learners responses." An educational technique is then created in which differences among students in background and attitude are taken directly into account in the management of the learning process, in a way that it is hardly possible in the fast paced instruction typical of the classroom. Hughes further enumerated the importance and relevance of mathematics in today's curriculum. To wit:

1. Mathematics should serve as the functional tool involving our individual everyday problems. It is obvious

that effective use of mathematical skills and concepts are basic element of efficient citizenship. In fact it is through life situation that learning of mathematics is first motivated.

The foregoing point of view suggest that society should place more emphasis on mathematical thinking. In other words, every individual should possess some basic mathematical concepts and skills if only to become an enlightened citizen of his community.

2. Mathematics serves as the handmaiden for the explanation of the quantitative situations in other subjects like Biology or even in Arts. Progress brought about by science due to mathematical skills is noticeable and cannot be ignored.

In many ways, science in which mathematics is a part has contributed to a happier and more satisfying life. Indeed mathematics is essential to the scientist, to the engineer, to the surveyor, to the pharmacist, to the navigator and to the astronomer.

Conceptual Model

This study which aimed to develop and evaluate modules for BSIT students based on the students' deficiencies was broken down into three major phases, namely: Phase I - Identification of Students' Deficiencies, Phase II - Development of Modules in Mathematics 101, and Phase III - Validation of the Modules.

The end result of the study is for improved learning and instruction in the classroom. The research environment includes the two groups of BSIT students composing the experimental and control group. It also includes the different tests conducted to the two groups. The modules compose the independent variables while the differences in the performances of the experimental and control groups before and after the period of experimentation are the dependent variables.

Two groups of BSIT students were pretested to assess their knowledge about the identified deficiencies in College Algebra. The experimental group was taught using the modules. The control group was taught using the lecture and discussion methods all belonging to the traditional approach.

A posttest was administered to both groups after instruction using the same instrument to determine whether there was an improvement in the performance of the two groups. The tests scores were analyzed using the appropriate tools to find out any significant difference in the performance of the two groups vis-a-vis in the performance of both groups and the performances of the two groups would establish the effectiveness of the modules and would pave the way to instructional redirection which would ultimately lead to improved teaching and learning in College Algebra. The appropriateness of the developed modules were also tested for readability using Flesch formula.

The conceptual model which guided the researcher is given in the overleaf paradigm:

The three phases of the study progressed as reflected in the paradigm:

Phase I - Identification of Students' Deficiencies is shown in Figure 1 consisting of three frames. Frame I is the input which is composed of course outline, guides, references, textbook and mathematics instructors. Frame II gives the throughput which is the diagnostic test designed to identify the students' deficiencies. Frame III gives the output indicating the identified deficiencies of BSIT students.

Phase II - Development of Modules in Mathematics 101 consisted of three frames. Frame I is the input which includes the course guide in Mathematics 101, textbook, reference materials and the identified deficiencies of the BSIT students. Frame II gives the throughput which is the preparation of course syllabus, and modules on selected topics in Mathematics 101 (College Algebra) taking into consideration the identified deficiencies of students. Frame III the output consists of the developed modules in Mathematics 101.

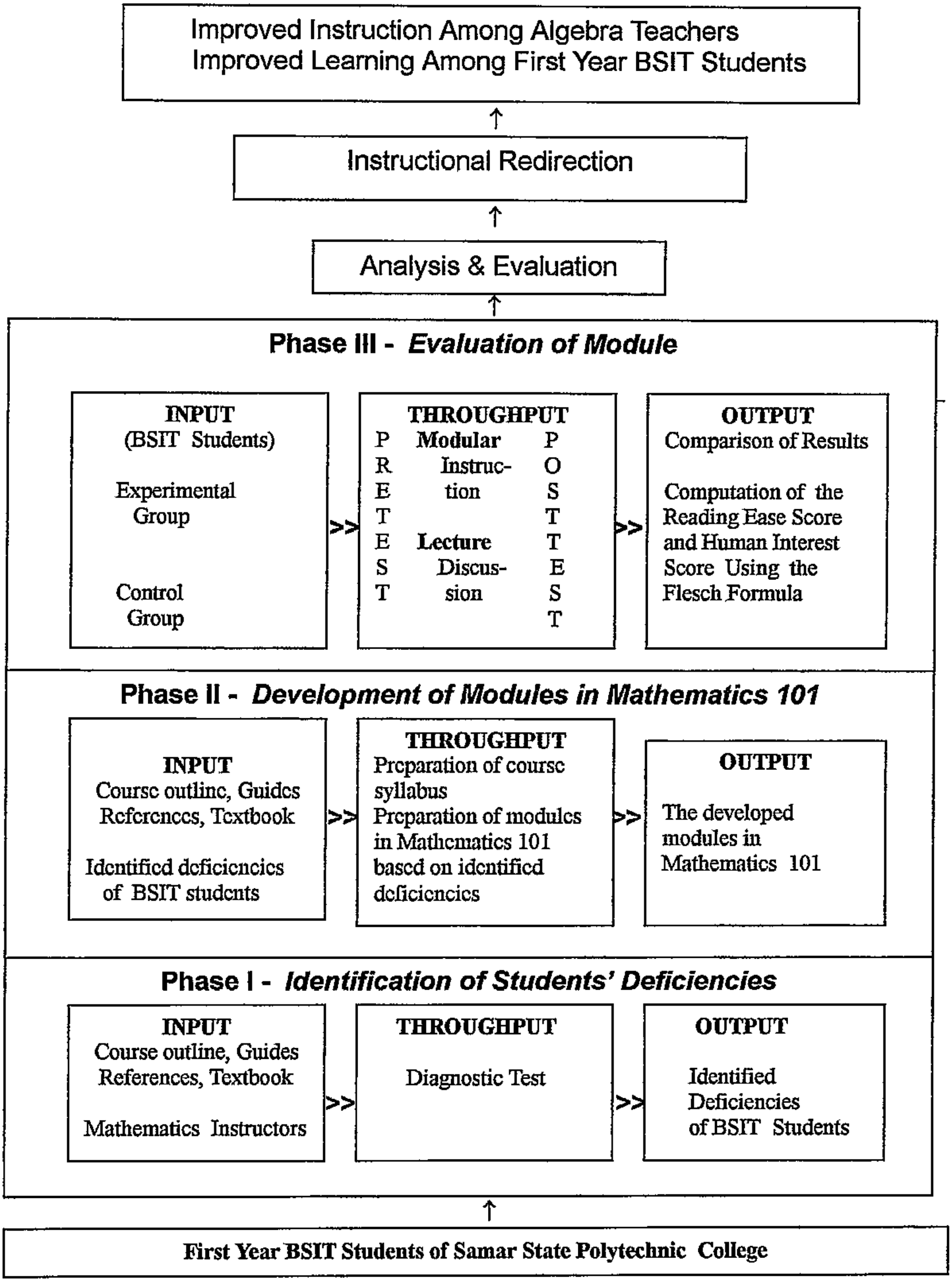


Figure 1. Conceptual Model of the Conduct of the Study

Phase III - Validation of the Developed Modules in Mathematics 101 (College Algebra) consisted of three frames where the input consists of the two groups of students; the throughput is the experimentation phase. The experimental group was given the modules while the control group was taught using the conventional lecture - discussion method. The output consisted of the comparison of the performance of the two groups in their pretest and posttest results. The reading ease score was computed using the Flesch formula to test for the appropriateness of the developed modules.

Significance of the Study

This study is significant for the following reasons: It can serve as a basis for evaluating the mathematics skills performance of SSPC First Year BSIT students and of other students for that matter. It can also serve as a guide for course syllabus improvement in Samar State Polytechnic College to cater to the deficiencies which the students have in College Algebra.

Students will be benefited since the modules will be of great help to them to become independent learners. This will also improve their capabilities, through self-discovery and develop their optimum mathematical potentials at their own pace.

Textbooks and instructional materials are too expensive that many students cannot afford to buy them. With these modules students can avail of less expensive learning materials, that would enrich and improve their learning experiences.

Furthermore, it can develop and enhance the student interest in mathematics thus facilitates better remediation of the difficulties and deficiencies of the students on mathematical performance.

The prepared modules in this study could enhance the understanding of algebraic concepts by the students because it is within their level of understanding.

The teachers will be benefited since he can use the developed modules in teaching his students about algebraic expressions. This would mean lighter work for the teacher hence, giving him more time to give particular attention to the slow learners. It is also anticipated that with the use of modules for enrichment lesson more time can be allotted to actual teaching on more concepts. The teacher will give time and attention to slow learners.

The findings of this study could provide the teacher information on modular approach in the development of mathematical ability among the students of Samar State Polytechnic College. The baseline data are necessary for effective implementation of teaching techniques that can help the students in their studies.

The administrators will also benefit from the results of the study. The results would give the administrator an insight as to the modern trends in teaching algebra. The findings would mean the provision of modules as substitutes for textbooks, thereby reducing expenditures in the school budget.

Scope and Delimitation of the Study

This study was confined to the Development and Validation of Modules in Mathematics 101 (College Algebra) based on the deficiencies of students. The modules were intended for the first year students of Samar State Polytechnic College under the Four-Year Technical Education Curriculum leading to the degree of Bachelor of Science in Industrial Technology (BSIT). The respondents of the study consisted of twenty five (25) BSIT students for the control and another twenty five (25) BSIT students to compose the experimental group which were selected by purposive sampling technique. It made use of the pretest which was given to both the control and experimental groups before use of each modules and the posttest was given after the modular instruction was given to the experimental group and the traditional lecture-discussion method was employed with the control group. The lessons covered were: concepts of polynomials, addition, subtraction, multiplication and division of polynomials and grouping symbols. The study covered the school year 1994-1995.

Definition of Terms

The key variables of this study and their conceptual and operational definitions are as follows:

Control group. This refers to the group in the experiment which is not exposed to the approach/project/technique in question (Herrin, 1987: 39). In this study, this refers to a group composed of 25 BSIT students taught by the lecture-discussion method.

Deficiency. This term refers to a state of being deficient, inadequacy, or shortage of one or more of the thing in question (Webster, 1976: 592). In this study, this refers to anything which presents or constitutes an obstacle to achievement as mastery.

Diagnostic test. A test adapted for furthering the diagnosis or to distinguish, identify or determine the difficulty of the students in a given subject (Webster, 1976: 622). In this study this refers to the 70 items teacher- made test used to establish the difficulty in the students in College algebra. The test was administered to six groups of students which took up College Algebra in their first year of study.

Effectiveness. This refers to the quality or state of being effective or what is brought about especially through successful use of factors contributory to results (Webster, 1976: 725).

Experimental group. Generally, this term refers to the group in the experiment which is exposed to the approach/project/technique in question (Herrin, 1987: 39). In this study, this refers to a group of twenty five BSIT students in Samar State Polytechnic College who were taught using the modules.

HIS. Acronym for Human Interest Score.

Learning. This refers to the psychological activity in development, such as the process of acquisition and extinction of symbolic modifications in existing knowledge, skills, habits and motor skill (Webster Dictionary, 1976: 1286).

Mathematical performance. This term refers to the capacity to achieve a desired outcome/result in Mathematics (Webster, 1986: 1679). In this study, this refers to the scores obtained by the first year BSIT students of Samar State Polytechnic College, Catbalogan, Samar involved as experimental subjects in the pretest and posttest.

Mathematics. The science that deals with the relationship of symbolism of numbers and magnitude and it includes quantitative operations and the solutions of quantitative problem (Webster Dictionary, 1976: 1393).

Mathematics 101 (College Algebra). This is a subject in the BSIT curriculum. The course deals with the study of mathematical concepts, properties and principles involved in College Algebra. It includes algebraic expressions, special products and factoring, rational expressions, linear equations in 1, 2, and 3 unknowns, exponents and radicals, and quadratic equations.

Modular Approach. This refers to the method of teaching wherein modules are used as the medium of instructions (Webster, 1976: 1452).

Module. This refers to a self contained and independent unit of instruction with primary focus on well defined objectives.

Posttest. This term refers to a test given after a period of time (Webster, 1986: 1801). In this study this refers to the test administered to both the experimental and control groups after the modules have been taken up. The posttest contains the same items found in the pretest but with the items rearranged which was also aimed to discover the students achievements through the total applications of skills and knowledge that have been sequenced within the module.

Pretest. This refers to a preliminary test which serves to explore rather than evaluate (Webster, 1986: 1797). In this study this refers to a 30-item test in Mathematics 101 administered to both the experimental and control group prior to the experimentatal activity to determine the extent of knowledge they have on the topics covered in the module.

Rating. A marked indicator of one's study in relation to a perceived criteria for the evaluation of achievement (Webster Dictionary, 1976: 1185).

Readability. The degree of being readable using the Flesch formula as determined by the RES and HIS score.

RES. Acronym for Reading Ease Score.

Skill. Generally, this term means knowledge of the means or methods of accomplishing a task (Webster, 1986: 2133). As used in this study, the term skill refers to the ability of the students to perform the required mathematics operations in a given problem to arrive at the required result.

Target group. This refers to the specific group regarded as the object to be hit or aimed at (Webster, 1976: 2341). In this study this refers to the group of students for whom the modules were prepared and developed. The target group is the first year BSIT students of Samar State Polytechnic College, Catbalogan, Samar.

Teaching. In a classroom situation, it deals with the process of stimulating, directing, guiding, and encouraging learning activities (Gregorio, 1983: 9).

Technique. This means one's ability to use several methods and procedures in performing a particular task

(Webster, 1986: 2349). In this study this refers to the mathematics teacher's way of doing his task at hand according to the plans, specifications, or objectives associated with it.

Traditional approach. Generally, this term means the use of an inherited or established way of doing (Webster, 1986: 2422). As used in this study this refers to the combined lecture-discussion method which is used by the teacher in teaching the content of Mathematics 101 (College Algebra).

Validation of the module. This refers to the process of determining the degree of the validity of the module (Webster, 1976: 2530). In this study this refers to the process of testing the effectiveness of the module.

Validity. This refers to the degree to which a test or measuring instrument measures what it is intended to measure (Calmorin, 1994: 63). As used in this study, validity refers to the ability of the pretest and the posttest to evaluate the performance of the student in Mathematics 101.

Chapter 2

REVIEW OF RELATED LITERATURE AND STUDIES

This chapter presents the researcher's readings regarding her study. She consulted several books, journals and periodicals. She came up with the compilation of information found to be relevant to this study. The readings that she had guided her in the development of modules in Mathematics 101.

Related Literature

Harbison and Myers (1973: 3-5) opined that programmed teaching is teaching in which the instruction follows a prearranged plan or program in detail. The programme prescribes not only what is to be taught but also specific procedures for teaching which take a number of forms, one of which is the modular teaching. Programme teaching specifically in modular instructions, forms one such technology. It is self contained and independent unit of instruction with a primary focus on a few well defined objectives. The boundaries of a module are definable only in terms of the stated objectives.

Torralba (1983: 121 - 128) observed that learning module is the answer to the needs of a developing country like the Philippines with inadequate logistics for a rapidly growing school population. Through it, instruction can be individualized and from it pupils and students learn even out of school.

A programme instruction defined by Strenchan (1969: 155-158) is a device which leads the students through a set of specified behavior designed and sequenced to make it probable that students will learn what the programmed study is designed to teach.

According to Howes (1971: 55-60) individualized instruction is an adjusted instruction where elements in the classroom are altered in an attempt to reach the learners more successfully by rearranging or reducing the more extreme differences within the group.

Bautista (1978: 17) defined module as the teaching system that is self-contained, self-pacing and self-participation and allows them to repeat segment of the content until a maximum level of performance is achieved.

Norman (1974: 3) defines individualization as tailoring, the curriculum and the days activities to the needs and interest of the learner. It puts emphasis on how the student can become and how he interact with the people and object around him.

Charles (1980: 117) defines instruction as a way of organizing materials and activities so that students

1. Know what specific objectives they are suppose to reach.

2. Have optional activities from which they can choose, enabling them to reach the objectives.

3. Can direct themselves through learning activities with minimum assistance from the teacher.

4. Can pace themselves, that is, work at a rate of speed that suits each person.

Socrates (1981: 3-4) supports that there maybe manuals and other supplementary materials, but modules alone, if prepared well, can suffice as models or patterns for programmed method concept.

The literature reviewed lent support to the researcher about the importance of modules as an aid to learning and instruction.

Related Studies

The survey and review of the related studies provided materials that serve as foundation for the study.

Calud (1982) in her study on "The Modular Approach in Teaching Introductory Concepts on Algebraic Expressions" attempted to produce effective modules which could possibly take the place of the teacher and classroom so that agricultural students can learn their lesson even outside the classroom to meet the demands of the works in the field.

Her study sought answers to the following questions:

1. What is the significant difference between the mean of the pretest and the mean of the posttest of the control

and experimental group?

2. How do the experimental group compare in their performance before the treatment and as a consequences of the application of the modular approach in teaching the introductory concepts in operations of algebraic expressions?

The subjects involved in the study were selected based on their IQ and achievements in Science, Mathematics and English.

A pretest was administered to the students or respondents before they were exposed to modular instruction. Immediately after finishing the module, the students were given the posttest. The result in the pretest provided the data needed in her study.. Her study showed that both groups of respondents performed significantly better in the posttest than in the pretest but the experimental group performed a lot better than the control group, after the treatment. This led to the conclusion that students can learn better through the module and that the module is effective as an instructional material. She recommended that all subjects in the academic department should be modularized and that a further study on the effectiveness of the modular approach in teaching should be done.

The study of Calud is similar to the present study since both studies tried to develop modules in algebra and the sample respondents were selected based on grades in mathematics.

Soriano, M. (1984) in his study entitled "Deficiencies in College Algebra of BSIE Students at the Bicol College of Arts and Trades: Module Development of Selected Results" sought to answer the following questions:

1. Is there a significant difference in the mean scores of the posttest of the experimental and control group?

2. What difficulties do students encounter in College Algebra?

To find out the difficulties encountered by the respondents, he constructed an achievement test by following the steps in test construction. He found out that freshmen in the BSIE curriculum had varying degrees of difficulties in College Algebra. They were deficient in the following topics: relations, functions and graphs, exponents and radicals and in operations of algebraic expressions.

He concluded that a significant difference existed between the mean scores of the posttest of the experimental and the control group. It indicated that modular instruction is more effective in attaining the objectives in the learning program on any topic than in the traditional instruction learning process. He recommended that teachers and instructors should be trained in modular instruction so that they can prepare modules to be used in their instruction.

The study of Manuel is similar to the present study since both studies constructed learning modules based on identified difficulties in College Algebra.

Castillano (1980) in her study entitled "Solutions to Systems of Linear Equations by Determinants: A Modular Approach" aimed to determine the effectiveness of the use of modules on determinants and in solving systems of linear equations found out that modular teaching is not only effective to the above average students but also to the average students provided that the module is well prepared and expressed in a very simple language which could be easily understood. Another finding was there was a significant difference between the mean scores of the posttest of the experimental and control group.

She recommended that teachers be trained in module construction so as to encourage them to prepare module to break the monotony of the usual or daily discussions and also for them to have a chance to adopt the more effective method in teaching mathematics. She further recommended that topics which are not compulsory for classroom discussions be presented in modular form to give the fast learners a better grasp and wider scope of coverage on the subject and to give the slow learners a time to learn at their pace without putting them under time pressure.

The study of Castillano is similar to the present study since both studies constructed learning modules in a topic/lesson in algebra.

Also, the studies conducted by Calud, Manuel Soriano and Castillano are related to the present study because they are all concerned with module construction which can be used as effective instructional material in teaching concepts in College Algebra and as a solution to the problem on scarcity of textbooks which is also true not only in Samar State Polytechnic College in particular but also in other schools in general.

Reyes (1984) in her study about the deficiencies in Mathematics I of first year high school students in Division of Cagayan I, attempted to improve three serious identified deficiencies.

To find out the weaknesses on the subject she made use of a test which was developed by the researcher while she was in the Ministry of Education Culture and Sports.

The significant findings on her study are as follows:

1. Most of the students who took the achievement test were most deficient in computing fraction, finding the ratio, proportion, percent and solving area, circumference and time.

2. Skills which obtained the three highest ranks were

selected as the most serious deficiencies. Modules were developed based on the most deficient skills and were used as treatment of the identified weaknesses.

She concluded that the use of modules produced significant differences on the scores of the students. In view of the said significant findings, she recommended that teachers should give greater emphasis on the areas/skills in Mathematics I where the students are weak. More guided activities should be provided to improve students computational skills. Furthermore, she also recommended that teachers should adjust instruction on the needs and characteristics of individual learners. Development and use of instructional materials like modules are strongly recommended to cope with these deficiencies in teaching of selected topics as compared to the ordinary method of teaching.

Nones (1985) in her study on "Development of Modules on Selected Topics in Mathematics 102 (Trigonometry) for DIT Students at NVSPC" said, that the use of modules serves as an effective way of imparting knowledge to the students because modules are self-learning kits and they provide an immediate feedback for them. She also recommended the following:

1. Modularized materials on different areas in mathematics should be developed and validated to alleviate

the lack of textbook and reference materials in the school.

2. The modular approach of instruction should go hand in hand with the traditional method of teaching to make teaching of mathematics more effective and interesting to the respondents.

Labro (1984) in his study on self-instructional materials that meet selected deficiencies in physics found out that there was a significant difference between the mean of the pretest and the posttest for both of the experimental and the control group. The significant increase, in the posttest scores for both groups showed that the developed instructional materials were effective in attaining topic objectives either for self-instructional or remedial purposes. He further recommended that similar studies should be conducted in the other areas of physics such as heat, electricity, etc.

The study of Nones, Reyes, and Labro is related to the present study which is modularization of selected topics in order to maximize learning skills needed by the students. Hence, related to aims and objectives of this study.

Perez's (1985) had a seminar paper entitled "Development and Validation of a Module on Progression, A Topic in Math for Tech 201 for DIT Students at Samar State Polytechnic College." She developed, tried out, and validated a module in Mathematics for Technology 201. She found out that the module is readable, appropriate and useful as source materials for teaching.

The study sought to answer the following questions:

1. What difficulties do the students encounter on Progression?

2. Is there a significant difference between the pretest and the posttest mean scores of the experimental group and the control group in the same learning content?

3. Is there a significant difference between the posttest mean scores of the experimental and control groups?

4. Is the developed module appropriate to the respondents in terms of readability.

The findings implied that the students learned better under modular instruction than the traditional method of instruction. She further recommended that students with identified difficulties should be given the learning materials like the module to give them time to catch up with the lessons not well learned in the classroom.

Avila (1984) recommended that students with identified difficulties should be given resource materials to bring home so that they will have more time to read and study them. These resource materials refer to the instructional materials developed by the individual instructors when books are not found to be more effective than the lectures given by the teachers, then these materials should be made accessible to the students who are with identified mathematical deficiencies.

A dissertation conducted by Filamor (1983) entitled "The Development and Validation of Modules in Technical Writing for College Students in Vocational Schools and Technical Schools" states that the general posttest result in technical writing indicated higher mean score of 70, obtained by the college students who were taught in technical writing by the traditional lecture method acquired a mean score of 149.40. He further added that encouragement be given to teachers in order to allow them to exploit further the advantages offered by the modular instruction as innovation in providing individualized instruction.

A study which was conducted by Lacambra (1985) about the development and validation of module in Applied Science 102 (Electrochemistry) in Samar State Polytechnic College, Catbalogan, Samar have the following conclusions:

1. The college students in SSPC encountered difficulty in the sub topics on electrochemistry specifically oxidation, reduction and chemical effects of an electric current.

2. (a) The control and the experimental groups had the same level of entry behavior.

(b) Students gained knowledge about oxidation reduction and chemical effects of an electric current with the use of modular approach.

(c) Students learned oxidation reduction and chemical effects of an electric current with the use of modular approach.

(d) The modular approach of teaching is more effective than the lecture method in so far as the subtopic oxidation reduction and the chemical effects of an electric current are concerned. The fact is that the students can go through the modules at his own pace, repeat some sections of the work if needed and progress at his own rate until the feeling of self satisfaction is obtained.

3. The modules are appropriate and interesting to the first year students.

She further recommended that teachers and instructors should be encourage to prepare modules on other areas of chemistry.

The study of Perez, Avila, Filamor, and Lacambra is similar to the present study since their studies involve development and validation of modules for college students. They differ only in the choice of topics to be modularized.

A study conducted by Aguilar (1989) on the effects of self instructional materials on the reading levels of grade four pupils in Mercedes Elementary Schools during the school year 1988-1989 made the following conclusions based on the findings:

1. The forty-five (45) self-instructional materials were found to be effective for the used in the development of word recognition, comprehension and vocabulary skills that lead to the improvement of the reading levels of the grade four pupils.

2. The materials were found to be within the reading level of the grade four pupils.

3. All the self instructional materials were acceptable and appropriate in terms of physical aspects; instructions to learners, learning activities and evaluative measures. She further suggested that the grade four pupils be exposed to the use of the forty five instructional materials to develop in them the feeling of independence in learning a lesson without teachers aid, and if possible preparation of self instructional materials in Reading and other subjects be undertaken.

Another study was conducted by Dacuro (1982) on "Self Instructional Materials in Reading of Grade Four Pupils" focused in the construction and validation of a set of multi levels and self instructional materials which were intended to develop skills in word recognition, comprehension and vocabulary. The essential development were patterned after Science Research Associate (SRA) and were tried out within a group of grade four pupils. He found out that larger variation were scattered through the multilevel characteristics of self instructional materials and were good as supplementary and complementary resources in the classroom aside from their being developmental in nature. He recommended further that the appraisal and validity in a wider scale be undertaken to meet the individual needs of the schools and he also recommended that further studies be conducted to ascertain the effectiveness of those materials in the reading levels among grade four pupils.

Another study which was related to the present study was Escuadra's (1986) "Effectiveness of Modules as an Aid in Dressmaking Instruction at Wright Vocational Schools, Wright, Samar." The study was conducted during the fourth grading period of the school year 1985-1986. She concluded that, the learning performance of the control group was substantial as indicated by the t-test which yielded results establishing significant difference between the pretest and the posttest.

Study conducted by Jamardon (1983) on the effectiveness of using modules on selected science concepts taught to first year students in St. Paul School in Nueva Ecija during the school year 1982-1983. She concluded that modular teaching can be effective means of improving learning gains of Science I students.

Azuelo (1982) in her study on the proposed self instructional laboratory guides for a biology class at

Central Mindanao University High School, Musuan, Bukidnon during the last grading period of the second semester of school year 1982-1983. He prepared laboratory guides which includes objectives, scope, activities of the course, teaching strategies and diagnostic test measures and the prepared set of experiments, illustrations of principles and process contents. Her findings showed that the nature of activities carried on in Biology classes is the laboratory integrated with the demonstration and the lecture in the class. She concluded that using self instructional guides was so effective that the test results or scores were very much higher compared to those students who were not exposed to self instructional guides.

Vista (1992: 62) in her study on "Effectiveness of Instructional Module in Garments Technology 201 at Samar State Polytechnic College, Catbalogan, Samar." Her Conclusions are the following:

1. The experimental group improve in their performance with the used of the module.

2. The control group showed improvement after the teacher used the lecture demonstration method.

3. The general conclusion was the module was just as effective as the lecture demonstration method in teaching lessons in Garments' Trades.

According to Dacula (1995), modular approach of teaching is relatively more effective than the lecture discussion method or the traditional approach of teaching. She recommended that the modular instruction should be used to students with above average intelligence as often as possible in order to maximize the learning process and output. She strongly recommended the modular approach of teaching for it helps the students to learn to be independent, responsible, self reliant and hardworking.

Roberto (1985) in his study on "Development and Validation of Modules on Students Deficiency in Math for Technology 201" found out the posttest mean scores of the control group is lower than the posttest results of the experimental group. He therefore concluded that the modular instruction can bring effective learning to students more than the traditional instruction to maximize the learning process and output. For the average and poor students, modules can be used provided that it will go hand in hand with traditional instruction.

Uy (1992) in her study on "Development and Validation of Modules in Circular Trigonometric Functions and Fundamental Identities" found out that there was significant amount of learning after the respondents were exposed to modularized instruction based on the result of the posttest. Her findings proved that the experimental group performed better than the control group in the posttest. She then concluded that the modular approach or material centered instruction was more effective than the traditional lecture discussion method. She recommended the used of modules for it serve as an effective remedial resource material for the students.

Truanda (1985) in his study on the "Construction and Validation of Modules on Selected Topics in College Trigonometry for DIT Students" revealed that the students who used modules achieved better than the students who learned through the lecture method.

Gordove (1993) conducted a study on effectiveness of self learning kits in grade four mathematics. He concluded that teaching with the use of self learning kits is more effective than lecture discussion method and he recommended that teachers should be encourage to use self learning kits in some learning areas of elementary mathematics. The developed self learning kits in Geometry should be used and evaluated in other schools to further confirm its effectivity.

Españo (1994) in her study on effectiveness of teacher made workbook in teaching basic mathematics suggested that students should be exposed to material centered instruction like text/workbook to develop in them the feeling of independence and self confidence in learning a lesson without the teacher aid.

The studies of Aguilar, Truanda, Azuelo, Vista, Dacula, Uy, Roberto, Gordove, Españo, Jamardon, Escuadra and Dacuro pointed out benefits derived from the use of modular approach. These reviewed studies have relevance to the present study because of the adoption of the phases on the preparation of the test instrument, analysis of the test results, steps in the development, try-out, and validation of the module that will be followed to effect the needed gains in the teaching of mathematics. These studies, programs and approaches designed for individual instruction simply point out shift from teacher directed group instructional strategies to an individualized instruction.

An analysis of the studies showed that there is a need to individualize instruction if we take into consideration the fact, that the learners are very tremendously dynamic and active-seeking organisms who do more than merely react to their environment but also explore and change it. This proposed study is different from the studies reviewed since the developed modules are primarily intended for enrichment of the learning experiences of the students based on their identified needs.

Chapter 3

METHODOLOGY

This chapter presents the method and research design used in the study. It describes the data required, data sources, instrumentation and statistical analysis of data.

The Research Design

This study utilized the experimental method of research using the pretest/posttest control group design. Fifty first year BSIT students were chosen as respondents/samples of the study. The students were grouped into two. One group was used as the experimental group which were subjected to the modular approach of teaching. A comparison of the performances of the two groups of respondents/samples was done by subjecting the pretest and posttest results to statistical analysis.

The respondents/samples were chosen through purposive sampling. In order to assure that the respondents had the same entry behavior, the grouping was done based on the students' grades in mathematics while they were in the fourth year high school. A pretest was given to both groups before the experiment commenced. A posttest was given after the experimentation was over. Afterwards the performance of the two groups were compared.

Instrumentation:

The research endeavor used the following instruments for gathering data: (1) tests; (2) modules; and (3) documentary analysis.

Tests. The researcher constructed three types of test namely: (a) diagnostic test; (b) pretest; and (c) posttest which is the same as the pretest but given at different time.

a. Diagnostic test. A diagnostic test composing of 70 items was constructed to establish the concepts/skills that were found too difficult by the students. The test was item analyzed and the final form consisting of 70 items was administered to the second year college students of SSPC (See Appendix J for the result of the item analysis.). The result of the diagnostic test served as the input for the researcher in the preparation of the modules. The diagnostic test was administered to six groups of students coming from the different courses and who took up algebra in their first year of study. The test covered lessons in integers, decimals, fractions, algebraic expressions, equations, solutions of linear equations and inequalities which were generally the topics covered in College Algebra.

The researcher made a table of specification to establish content validity. The diagnostic test was framed according to the table of specifications. The test was presented to the mathematics teachers of SSPC for comments and suggestions, then the test was submitted to the adviser for approval. The preliminary form of the diagnostic test was administered to students of SSPC who took College Algebra in their first year of study for further validation. The researcher observed rules in the conduct of the test. Statistical analysis was conducted on the results of the item analysis. The final form consisting of 70 items was administered to the samples/respondents.

b. Pretest. The researcher made a 30- item test that covered concepts in algebraic expressions and mathematical operations. The test was designed to determine the background knowledge of the students on the topic modularized.

A table of specification (Appendix K) was prepared based on the syllabus in Mathematics 101 (College Algebra). The purpose of constructing the table of specification was for content validity. After the test was drafted it was shown to the mathematics instructors teaching the course for comments and suggestions for the improvements of the test items. It was scrutinized by the adviser for further improvement. The improved pretest was given to the control and experimental groups. Results were tallied and interpreted.

c. Posttest. The posttest covered the same subject matter as the pretest. However, the posttest has the items rearranged to appear different in order to avoid a carryover effect of the pretest to the posttest. The posttest was used to find out the gains in learning performance of samples/respondents in both the experimental and control groups during the intervening period.

Modules. After the deficiencies of the BSIT students were identified modules were developed. Each module consisted of five parts namely: (a) Overview; (b) Objectives; (c) Input; (d) Practice Task; and (e) Feedback to the Practice Task.

a. Overview. This gives the overview of the lesson, the rationale as well as the expectations of the whole lesson.

b. Objectives. This part of the module stated the goals of the learning process for each lesson.

c. Input. This part gives the lesson proper of the module. This contains items of information needed in the acquisition of the mathematical skills and concepts.

d. Practice Task. In this part, the students were given exercises for reinforcement activities.

e. Feedback to the Practice Task. This portion of the module gives the learners the answers to the Practice Task. This serves as a way of checking whether the learner has made the correct responses in the Practice Task.

Documentary Analysis. Students' records were taken from the school registrar of Samar State Polytechnic College, Catbalogan, Samar to obtain information needed in this study. This particular technique enabled the researcher to group students by using paired matching. The grades in High School Mathematics IV were the bases of the grouping.

In addition to the mentioned records, the researcher consulted textbooks, syllabus, course outlines and guides in identifying the deficiencies of the students and in the preparation of the modules. The mentioned resources were utilized in the preparation of pretest and posttest since two tables of specification were drawn up.

Data Gathering Procedure

The data gathering procedure proceeded as outlined in the conceptual framework. The three main phases of the study are:

Phase I: Identification of Students' Deficiencies. In this phase the researcher consulted the course outlines, guides, textbooks and reference materials. A diagnostic tests was framed by the researcher to find the areas in Algebra where the students were most deficient. Scores were tallied and the test was analyzed to find out index of difficulty and index of discrimination of each item. A table was then prepared to see where the students were most deficient. The table was a basis for developing the modules.

Phase II: Preparation/Development of Modules. In this phase of the study the researcher now prepared the modules. The topics were derived from the identified deficiencies. The researcher consulted course guides in Algebra, textbooks, and other references in preparing the modules. The developed modules were presented to the adviser for comments, suggestions, and approval. The developed modules were revised and improved based on the comments and suggestions of the adviser. The modules were also shown to teachers teaching College Algebra for further improvement. Afterwards the modules were reproduced considering all their suggestions.

Phase III: Validation of the Modules. The final phase which took three months to perform was the period of experimentation. Twenty-five first year BSIT students composed the experimental group and another 25 made up the control group. The respondents' groupings were matched grouping based on High School Mathematics IV grade to ascertain that they have more or less the same entry behavior. A pretest was given to the two groups before the period of experimentation, to determine their entering capacity and ability. Afterwards the control group was subjected to lecture-discussion method while the experimental group was exposed to the modular instruction. Variables that could have affected the treatment were controlled to eliminate bias with any one method. The experimental group had their lessons from 2:30 - 4:00 P.M. every Tuesday and Thursday. The scheduled time for the control group was from 3:00 - 4:00 P.M. every Monday, Wednesday, and Friday. After all the lessons were covered, the students were subjected to a posttest. Their performances were compared. First, the performance of each group was evaluated in terms of the pretest and the posttest results. The researcher also computed the reading ease score (RES) and the human interest score (HIS) of the modules using Flesch Formula to determine the readability level of the module. The steps followed was the one used by Labro (1984) in his study. This was necessary to make certain that the modules were appropriate for the target group.

Sampling Procedure

Fifty first year BSIT students of the Samar State Polytechnic College were chosen through purposive sampling. Their grades in High School Mathematics IV were taken into consideration. They were used for testing the validity of the modules being prepared. They were paired off according to their entry behavior. The grouping was based on grades in High School Mathematics IV. Each student was paired with another in determining the experimental group and the control group. The reason is that more or less the students will have the same entry behavior and therefore reducing bias assuming that the grades are true measures of achievement as given by the teacher.

Statistical Treatment of the Data

The statistical tools used in this study were the mean, Flesch Formula, t-test for dependent samples/correlated samples, t-test for independent samples (Pooled Variance Model) and Kuder-Richardson formula.

The mean scores provided concise description of the average performance rating of each group in the pretest and posttest. It gave an accurate description of the skills and knowledge in College Algebra that were found deficient in the students. The formula used for the computation of the mean is:

$$\bar{X} = EX / N$$

Where:

EX = sum of scores

N = number of cases

The reliability of each item in the diagnostic test was determined using Kuder-Richardson Formula 20 given by Stanley and Hopkins as:

$$r = \frac{k}{k - 1} \left[1 - \frac{6 \ Epq}{(kd)^2} \right]$$

Where:

r = reliability coefficient of the test

k = number of items in the test

p = proportion of the students who answered the particular item correctly (difficulty index)

q = the proportion of the group failing the item (1 - p)

d = standard proportion of the test scores.

The interpretation of the computed r based on Garett is shown below:

Reliability Coefficient	Degree of Reliability
0.95 - 0.99	Very high, rarely found among teacher made test.
0.90 - 0.94	High, equaled by few tests.
0.80 - 0.89	Fairly high, adequate for individual measurements.
0.70 - 0.79	Rather low, adequate for group measurements but not satisfactory for individual measurement.
Below 0.70	Low entirely inadequate for individual measurement although useful for group average and school survey.

Computed r should fall from 0.70 above.

To find out the index of discrimination of the test item, the number of correct responses of the lower group was subtracted from the upper group number of correct responses for each item. The difference were divided by the maximum difference. The quotient was the index of discrimination and it was expressed in decimal fraction.

The accepted indices of discrimination range is shown below (Ebel, 1965: 374):

Index of Discrimination	**Item Evaluation**
0.40 and above	Very good items. Retain the item.
0.30 - 0.39	Reasonable good but possibly subject for improvement. Retain the item.
0.20 - 0.29	Marginal items usually needing improvements. Retain the item.
0.19 - 0	Poor items to be rejected or improved by revision.
Negative values	Reject the item.

To determine the difficulty index, item analysis was performed after retrieving, correcting and scoring the papers following the steps recommended by Stanley and Hopkins (1972: 367). The steps are shown below:

1. The scored answer sheets were arranged from the highest to the lowest score the highest at the top and the

lowest at the bottom.

2. The high group was separated by counting 27 percent (0.27 x N) of the answer sheets beginning from the top. Similarly, the low group was separated by counting 27 percent of the answer sheets starting from the bottom.

3. The total number of correct responses per item of the high group were counted and divided by the number equal to 27 percent of the total number of answer sheets. This is the proportion of the students in the high group who answered the item correctly, designated as P_H. The same was done for the answer sheets of the low group designated as P_L. Results were tabulated for easy analysis and interpretation.

$$P_H = \frac{\text{No. of correct items for the high group}}{.27 \times \text{Total no. of answer sheets}}$$

$$P_L = \frac{\text{No. of correct items for the low group}}{.27 \times \text{Total no. of answer sheets}}$$

The index of difficulty per item was computed by adding the P_H and P_L and dividing the sum by two.

$$p = \frac{P_H + P_L}{2}$$

To obtain the discrimination index per item, the P_L value was subtracted from its corresponding P_H value.

$$D = P_H - P_L$$

As to the index of difficulty, Ebel's interpretation

(1965: 376) shown below was used.

Index of Difficulty	**Item Evaluation**
0.86 - 1.00	Very easy items
0.71 - 0.85	Easy items
0.40 - 0.70	Moderately difficult items
0.15 - 0.39	Difficult items
0.10 - 0.14	Very difficult items

To determine the readability level of the constructed modules the Flesch formula was used. The computation is shown in Appendix M. The researcher computed the reading ease score (RES) and the human interest score (HIS) of the constructed module to make certain that the constructed module was appropriate for the target group. Fifty four (54) pages were randomly selected from the 270 pages and subjected to the steps in measuring the reading ease score.

1. Choosing the sample pages. The sample fifty-four pages were selected from the 270 pages of the instructional materials, representing twenty percent of the total number of pages. Practice Task, Feedback to the Practice Task, Test Your Understanding, Test Your Computational Skills and Title Page were not included of these fifty-four pages. If the sample fell on the page without reading materials, it was taken from the next page having a reading matter.

2. Counting the number of words. One hundred words were taken from each page by counting the first word of the

first paragraph up to the 100th words. In samples where there were no paragraphs, the first word of the sentence was considered. Figure captions, heading of the lessons, numbers and titles were not included in the counting.

3. Counting the number of syllables. The syllables in the 100th words in each sample were counted. The syllables were counted the way the word is pronounced.

4. Counting the number of sentences. The total number of sentences in the 100th words in each sample were counted. If the 100th word fell after more than 1/2 of the words of the sentence, it was counted as one. Otherwise, it was not counted.

5. Finding the average word length. To get the average word length, the number of syllables in all sample pages were divided by the total number of the sample pages.

6. Finding the average sentence length. To solve for the average sentence length, the number of sentences in all the sample pages were divided by the total number of pages.

7. Solving for the reading ease score (RES). The formula of the RES is:

$$\text{RES} = 206.835 - (1.015 \times \text{Average sentence length}) + (0.846 \times \text{Average word length})$$

Where:

$$\text{Ave. Sentence Length} = \frac{\text{No. of sentences in the samples}}{\text{Total number of sample pages}}$$

$$\text{Ave. Word Length} = \frac{\text{No. of syllables in all samples}}{\text{Total number of sample pages}}$$

B. Solving for the human interest score (HIS). The formula is

HIS = (% Personal words / 100 Words x 3.635) + (% Personal sentence x 0.314).

Where:

$$\text{\% Personal words} = \frac{\text{Total number of personal words in all samples}}{\text{Total number of words in all sample pages}}$$

$$\text{\% Personal sentences} = \frac{\text{Total number of personal sentences}}{\text{Total sentences in all sample pages}}$$

In testing hypothesis number 1, that is, in trying to determine whether there was a difference in the performance of the control and experimental group before and after experimentation, the t-test for independent samples (Pooled Variance Model) was used with error tolerance set at .05. The formula is:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{(N_1 - 1)SD_1^2 + (N_2 - 1)\ SD_2(1/N_1 + 1/N_2)}{N_1 + N_2 - 2}}}$$

Where:

$\bar{X}_1$ = mean of the pretest or posttest of the experimental group

$\bar{X}_2$ = mean of the pretest or posttest of the control group

N_1 = number of students in the experimental group

N_2 = number of students in the control group

SD_1 = standard deviation of the pretest or posttest of the experimental group

SD_2 = standard deviation of the pretest or posttest of the control group

To determine if there was learning or none that took place in each group, the performance of each group prior and after the experimentation was evaluated making use of the t-test for dependent samples/correlated samples. This tool was used in testing hypotheses numbers 2 and 3. The formula is as follows:

$$t = \frac{\bar{D}}{\frac{(ED) - (ED)^2 / N}{N (N - 1)}}$$

Where:

$\bar{D}$ = average difference between the posttest and pretest of the experimental group

ED = summation of the D column

$(ED)^2$ = square of the D column

N = number of students in each group.

Chapter 4

PRESENTATION, ANALYSIS AND INTERPRETATION OF DATA

This chapter discusses in length the statistical analysis made on the collated data. This includes the analysis and interpretation based on the results derived from the treatment.

Analysis of the Diagnostic Test Items Based on the Prepared Table of Specifications

Table 1 shows a table of specifications to establish content validity. The diagnostic test was framed according to the prepared table of specifications. A diagnostic test of 80 items was submitted to the adviser for approval. It was also shown to the mathematics instructors of SSPC for

Table 1

Table of Specifications
(Diagnostic Test)

Topics\Levels of Cognition	Number of items			Total
	K	C	A	
Basic Operations	5	5	6	16
Exponents and Radicals	2	6	6	14
Linear Equations	2	5	9	16
Quadratic Equations	2	2	6	10
Inequalities	2	4	18	24
Total	13	22	45	80

comments and suggestions for the improvement of the test items.

Analysis of the Diagnostic Test Items Based on the Tryout Conducted

The main instrument used in determining the deficiencies of BSIT students in College Algebra was the teacher-made test which was tried out at SSPC on June, 1992, before it was administered in its final form.

The result of the item analysis on the 80 item diagnostic test which was tried out to six groups of college students who took up College Algebra as to the index of difficulty and index of discrimination of each item is shown in Appendix J.

Based on the computed index of difficulty: 11 items were considered very difficult, 26 items were moderately difficult, 42 items were difficult and one item was considered as easy by the 132 respondents. Items of the test which were very difficult were rejected. Some items were modified and improved. The final form of the diagnostic test consisting of 70 items was administered to the samples/respondents of the study on June 1995.

Identification of Difficulties

The final form of the diagnostic test which was administered to the samples/respondents of the study was again item analyzed to determine their difficulties and deficiency. The ranking of the subtopics in College Algebra where the samples/respondents were found deficient is shown in Table 2 below.

Table 2 gives the subtopics in College Algebra which was found to be difficult by the samples/respondents based on the percentage of respondents correct responses to the test items under each subtopics in the diagnostic test. In the making of the modules particular attention was given to this topic. The topic having the greatest percentage of incorrect responses to the test items prepared is algebraic expressions with only 4.23% of the samples getting the

Table 2

Ranking of Subtopics According to Difficulty

SUBTOPICS	Percentage (%) of Respondents Correct Responses	Rank
Algebraic Expressions	4.23%	1
Inequalities	5.59%	2
Decimals	5.87%	3
Systems of Linear Equations	6.10%	4
Radicals	6.68%	5
Quadratic Equations	6.75%	6
Solutions of Linear Equations	7.22%	7
Exponents	8.70%	8
Fractions	9.50%	9
Integers	11.53%	10
Exponent Notation	12.37%	11

correct response. This is followed in descending order by the following topics: inequalities - 5.59%, decimals - 5.87%, systems of linear equations - 6.10%, radicals - 6.68%, quadratic equations - 6.75%, solutions of linear equations - 7.22%, exponents - 8.70%, fractions - 9.50%, integers - 11.53% and exponent notation having the rating of 12.37% which is considered to be easy.

Age, Sex and Grades Profile of the Samples/Respondents

Table 3 shows the age, sex and grades profile of both the control and the experimental group. The samples belong more or less to the same age bracket. The average age for the control group is 16.89 and 17.02 for the experimental group. The members of the control and experimental group were composed of 20 males and 5 females showing that the effect of gender towards interest and ability in mathematics is the same for each group. The samples were paired in terms of their grades in High School Mathematics IV. The highest grade for both groups is 89 and the lowest is 75. Again, with their grades, it can be implied that the students' mathematical ability is far from superior, hence the existing deficiencies in their mathematical skills.

Table 3

Age, Sex, and Grades of Samples

Samples	SEX		Average Grade in Math IV		AGE	
Number	CG	EG	CG	EG	CG	EG
1	F	M	89	89	17.2	15.0
2	M	F	87	87	16.0	18.5
3	M	M	85	85	17.8	16.0
4	M	M	84	84	16.0	18.2
5	M	M	84	84	15.2	16.5
6	M	M	83	83	16.0	16.0
7	M	M	83	83	17.5	18.0
8	F	F	82	82	17.0	16.0
9	M	M	82	82	15.2	17.5
10	M	M	82	82	17.2	16.0
11	M	M	81	81	15.0	18.0
12	M	F	81	81	21.4	16.0
13	M	M	81	81	15.2	16.5
14	M	M	80	80	16.5	19.8
15	M	M	80	80	16.0	17.0
16	F	M	80	80	15.0	18.0
17	M	M	79	79	16.5	17.5
18	M	M	79	79	15.0	17.2
19	M	M	79	79	16.2	18.2
20	F	M	78	78	22.2	15.5
21	M	F	78	78	19.0	16.8
22	M	M	78	78	16.5	16.0
23	M	M	77	77	18.2	19.8
24	M	F	76	76	16.2	15.5
25	F	M	75	75	18.2	16.0
Total	M =	20	2023	2023	422.20	425.50
Average	F =	5	80.92	80.92	16.89	17.02

Entry Behavior of the Experimental Group and the Control Group

Table 4 shows the entry behavior of the control and experimental group in the form of pretest scores. In the experimental group, the highest score was 20 and the lowest score was eight. The sum total of all scores was 362. When the mean was computed by dividing the sum total of 362 by 25 (the number of students in the experimental group) the result was 14.48. On the other hand, the highest score obtained by the control group was 19 and the lowest score was seven. Hence, the total of the scores reached a value of 358. The mean derived was 14.32.

In comparing the mean scores obtained by these two groups of respondents, the experimental group turned out to be higher than the control group by 0.16. To find out whether this difference is significant, t-test for independent samples (Pooled Variance Model) was applied. The computed t-value of 0.16 turned out to be lesser than the tabular t-value of 2.00 at .05 level of significance and degrees of freedom at 48.

The result of the mean scores means that the two groups had the same entering competency before the start of the experiment.

Thus, the first portion of the hypothesis which states that "There is no significant difference in the learning performance between the experimental group and the control group based on the pretest results" was accepted. The observed difference between their pretest mean scores was not significant.

Table 4

Pretest Results of Experimental and Control Group

Sample Number	PRETEST SCORES			
	Control Group		Experimental Group	
	C_1	C_1^2	E_1	E_1^2
1	13	169	17	289
2	17	289	15	225
3	18	324	13	169
4	11	121	17	289
5	13	169	8	64
6	17	289	20	400
7	19	361	15	225
8	12	144	15	225
9	15	225	17	289
10	8	64	12	144
11	16	256	19	361
12	19	361	11	121
13	7	49	10	100
14	14	196	20	400
15	13	169	14	196
16	14	196	15	225
17	19	361	11	121
18	17	289	18	324
19	12	144	16	256
20	8	64	14	196
21	17	289	16	256
22	18	324	9	81
23	12	144	10	100
24	15	225	19	361
25	14	196	11	121
Total	358	5418	362	5538
Mean	14.32		14.48	
Computed t = 0.16			Tabular t = 2.00	
Level of Significance .05			df = 48	

Performance of the Experimental Group Before and After Modular Instruction

The data found in Table 5 reveal the performance of the experimental group based on the pretest and posttest results. The result of their pretest reveal of their entering behavior. The computed mean was 14.48. A posttest was given to the same group after the session was over. The posttest results gives the mean score of 22.12. The computed mean increased from 14.48 to 22.12. The data gave the difference of 7.64.

Initially, it can be observed that there was an improvement in the performance of the experimental group after the experimentation. It is a very significant fact that minimal teaching supervision was given to this group. Subjecting the mean scores to the t-test for dependent samples the analysis revealed that the subjects improved in their performance. The computed t-value of 9.526 was very much greater than the tabular t-value of 2.064 at .05 level of significance and df set at 24.

The results led the researcher to reject null hypothesis no. 3, stating that "There is no significant difference between the pretest and posttest mean scores of the experimental group." The results mean that there was an improvement in the performance of the experimental group. Learning took place even with the minimal instruction the teacher gave to the students.

Table 5

Pretest and Posttest Results of the Experimental Group

Sample	EXPERIMENTAL GROUP					
	Posttest Scores:		Pretest Scores:		Difference$(D=E_2-E_1)$	
	E_2	E_2^2	E_1	E_1^2	D	D^2
1	28	784	17	289	11	121
2	18	324	15	225	3	9
3	24	576	13	169	11	121
4	19	361	17	289	2	4
5	21	441	8	84	13	169
6	26	676	20	400	6	36
7	28	784	15	225	13	169
8	19	361	15	225	4	16
9	24	576	17	289	7	49
10	16	256	12	144	4	16
11	28	784	19	361	9	81
12	15	225	11	121	4	16
13	21	441	10	100	11	121
14	24	576	20	400	4	16
15	20	400	14	196	6	36
16	21	441	15	225	6	36
17	25	625	11	121	14	196
18	19	361	18	324	1	1
19	28	784	16	256	12	144
20	18	324	14	196	4	16
21	30	900	16	256	14	196
22	14	196	9	81	5	25
23	21	441	10	100	11	121
24	28	784	19	361	9	81
25	18	324	11	121	7	49
Total	553	12745	362	5538	191	1845
Mean	22.12		14.48		7.64	
Computed t = 9.526			Tabular t = 2.064			
df = 24			Level of Significance = .05			

Performance of the Control Group Before and After the Use of the Lecture Method

Table 6 gives a clear idea of how the control group fared along before and after the experimentation period. Their performance before the experimentation based on the mean of their pretest results was 14.32. After the experiment they were given the posttest. There was a marked increase in their performance from mean score of 14.32 to 20.76. The difference of the posttest and the pretest mean scores is 6.44.

To test whether the numerical difference is significant, t-test for dependent sample was utilized. The computed t-value is 7.56 which is very much higher than its table value of 2.06 at 24 degree of freedom with error tolerance set at .05. The results led the researcher to reject hypothesis no. 2.

The result means that learning took place on the part of the control group. The hypothesis which states that "There is no significant difference between the pretest and the posttest mean scores of the control group" is rejected. There is a marked difference in their performance before and after the period of experimentation. Like in the case of the experimental group, the control group showed marked improvement after they were taught the lessons in College Algebra with the use of the traditional approach which is the lecture-discussion.

Table 6

Pretest and Posttest Results of the Control Group

	CONTROL GROUP					
Sample	Posttest Scores		Pretest Scores		Difference $(D=E_2-E_1)$	
	C_2	C_2^2	C_1	C_1^2	D	D^2
1	26	676	13	169	13	169
2	24	576	17	289	7	49
3	20	400	18	324	2	4
4	18	324	11	121	7	49
5	15	225	13	169	2	4
6	19	361	17	289	2	4
7	21	441	19	361	2	4
8	15	225	12	144	3	9
9	26	676	15	225	11	121
10	19	361	8	64	11	121
11	23	529	16	256	7	49
12	21	441	19	361	2	4
13	14	196	7	49	7	49
14	16	256	14	196	2	4
15	22	484	13	169	9	81
16	28	784	14	196	14	196
17	20	400	19	361	1	1
18	23	529	17	289	6	36
19	16	256	12	144	4	16
20	12	144	8	64	4	16
21	21	441	17	289	4	16
22	27	729	18	324	9	81
23	26	676	12	144	14	196
24	28	784	15	225	13	169
25	19	361	14	196	5	25
Total	519	11275	358	5418	161	1473
Mean	20.76		14.32		6.44	

Computed t = 7.56 Tabular t = 2.06

df = 24 Level of Significance = .05

Performance of the Experimental and the Control Group After the Experiment

Table 7 yields the data on the performance of the two groups after the period of experimentation was over. The experimental group was taught employing the modular instruction while the control group was taught with the use of the lecture-discussion method. When the posttest was administered to the experimental group, the group obtained the total score of 553. The mean was 22.12. On the other hand, the control group got the total score of 519 and the mean was computed to be 20.76.

Further test was employed to the two means to see if there was a significant difference in the performance of the two groups. Computation for the t-value using the t-test for independent samples (Pooled Variance Model), the t-value obtained was 1.05 which is very much lower than the tabular t-value of 2.00 at .05 level of significance at 48 degrees of freedom.

It is apparent that the performance of the two groups has no significant difference. Thus the hypothesis stating that: There is no significant difference between the posttest mean score of the experimental group and the control group" is accepted.

The findings were very significant on the light of the fact that when the modular approach was used with the exper-

Table 7

Posttest Results of Experimental and Control Group

Sample Number	POSTTEST SCORES			
	Experimental Group		Control Group	
	E_2	E_2^2	C_2	C_2^2
1	28	784	26	676
2	18	324	24	576
3	24	576	20	400
4	19	361	18	324
5	21	441	15	225
6	26	676	19	361
7	28	784	21	441
8	19	361	15	225
9	24	576	26	676
10	16	256	19	361
11	28	784	23	529
12	15	225	21	441
13	21	441	14	196
14	24	576	16	256
15	20	400	22	484
16	21	441	28	784
17	25	625	20	400
18	19	361	23	529
19	28	784	16	256
20	18	324	12	144
21	30	900	21	441
22	14	196	27	729
23	21	441	26	676
24	28	784	28	784
25	18	324	19	361
Total	553	12745	519	11275
Mean	22.12		20.76	

Computed t = 1.05 Tabular t = 2.00

df = 48 Level of Significance .05

imental group, they were very much on their own. There was a minimal teaching and supervision done by the researcher. On the other hand, she lectured, demonstrated and closely supervised the control group. Thus, the findings revealed that the modular instruction was just effective because even with the minimal guidance from the teacher, the same degree of learning took place on the students. Results could have been better if the teacher exerted the same effort as she did with the control group.

Analysis of the Readability Level of the Developed Modules

The readability level of the developed module is measured in terms of its appropriateness and how interesting it is to the users. The researcher used Flesch Formula to determine the readability level of the developed modules.

Table 8

Results of the Reading Ease Score (RES) and Human Interest Score (HIS)

Module	RES	Interpretation	HIS	Interpretation
1	50.20	Fairly Difficult	21.4	Interesting
2	50.70	Fairly Difficult	20.6	Interesting
3	50.50	Fairly Difficult	21.9	Interesting
4	49.50	Fairly Difficult	19.5	Interesting
5	48.20	Fairly Difficult	18.4	Interesting
6	50.10	Fairly Difficult	21.2	Interesting

Table 8 reflects the result of the Reading Ease Score (RES) and the Human Interest Score (HIS) obtained from the 25 students chosen as samples.

As can be gleaned from Table 8, the developed modules were fairly difficult but suited and appropriate for first year BSIT students. It is also interesting to the students to go through the materials. The findings of this study support other research findings that modular instruction is very effective as a method of learning.

Chapter 5

SUMMARY OF FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

This portion of the study gives the summary of findings, and conclusions derived from the data presented. This gives recommendations based on the conclusions and implications of the research findings.

Summary of Findings

The study focused on the effectiveness of instructional materials in Mathematics 101 (College Algebra). The instructional materials were intended for BSIT students. Modules were prepared for the experimental group as a strategy in teaching while the lecture-discussion method was given to the control group. The responses of the two groups were noted before and after the experimentation was conducted. After the experimentation, their performances were compared.

The data were tallied and computed and the results analyzed and interpreted. The following were the findings of the experiment based on the questions posed in this study:

1. As to age and grade profile of the samples, the control group's average was 16.89 and the experimental group had 17.02 as the average age. Both groups had the same entry behavior as both groups had students whose aver-

age grades in High School Mathematics IV ranged from 89 as the highest and 75 as the lowest.

2. The students' deficiencies as revealed by diagnostic test were on the following topics: (1) algebraic expressions; (2) inequalities; (3) decimals; (4) systems of linear equations; (5) radicals; (6) quadratic equations; (7) solutions of linear equations; (8) exponents; (9) fractions; (10) integers; and (11) exponent notation.

3. The control group registered the pretest mean score of 14.32 and the experimental group 14.48 in the pretest. Both groups improved in their performance after the posttest with the experimental group having the mean score of 22.12 and the control group 20.76. The higher mean for the experimental group is attributed to the modules.

4. When the mean scores for the pretest of both groups were subjected to the t-test (Pooled Variance Model) the result was the computed t of 0.16 was very much lower than the tabular t-value of 2.02. This means that there was no significant difference in the entry behavior of the respondents. After computing the t-test for the posttest mean scores of both groups, the data showed that there was no significant difference. The tabular t-value was 2.02 while the computed t was 1.05. Both groups learned the lessons but in the control group the teacher dominated the teaching-learning situations.

5. The study revealed that learning took place with the control group as their posttest result was higher than the pretest. The posttest mean scores was higher than the pretest. The pretest mean scores was 14.32 and the posttest mean scores was 20.76. Using the t-test for independent samples, the computed t was very much higher than the tabular t. This means that there is a significant difference in their results. Thus the data implied that learning took place in this group.

6. The experimental group had a marked improvement in their performance as shown in the result of the posttest. The posttest mean scores was 22.12 as compared against the pretest mean scores of 14.48. The computed t of 9.526 was very much higher than the tabular t of 2.06. Again it can be implied that there was learning that took place.

Based on the results, hypothesis no. 1 is accepted and hypothesis no. 2 and 3 are rejected.

Conclusions

From the findings of the study, the following conclusions are hereby drawn:

1. Learning took place in the control group as shown in the higher mean scores of the posttest.

2. Learning was also facilitated with the experimental group as shown in the posttest result.

3. The findings that the students using modules

learned just as well as the students who were taught with close guidance and supervision by the teacher led the researcher to conclude that modular instruction is effective. If the students learned with the minimum effort from the teacher how much more if the teacher would closely supervise and supplement them with lectures and discussions.

Although the conclusions derived from the study that the modular instruction is just as effective as the lecture-discussion method, the modular instruction had the following advantages:

1) The teacher exerts minimum efforts in teaching with the use of the modules. Learning is a student activity rather than teacher activity.

2) The students learn at their own pace. They do not need to cope with the pace of the class in learning the lessons. Their progress depends on their own and they learn at their time.

3) It has a higher standard when it comes to performance because the students' performance is not evaluated against the accomplishments of his classmates but against the standard set forth by the modules.

4) It is very personalized or individualized instruction catered to the needs of the students.

5) It can accommodate more students.

4. Because of the improved performance of the

experimental group it can be inferred that the developed modules were appropriate for the target group.

Recommendations

In view of the foregoing findings and conclusions the following proposals are hereby recommended.

1. Mathematics teachers are encouraged to use modular instruction in the classroom.

2. Mathematics teachers are encouraged to use both traditional method and modular instruction to reinforce the former.

3. Teachers should develop modules to increase the dearth supply of instructional materials in mathematics particularly in College Algebra.

4. Similar studies should be conducted using other content topic and experimental design.

Chapter 6

The Module

This chapter presents the module developed in this study.

The module developed in this study is on algebraic expressions. It consists of the following:

1. A pretest/posttest constructed and suited to achieve the objectives of the module which should be taken before and after going over the module in order to determine the extent to which the students have learned the topics modularized and find out the extent to which the objectives of the module were attained.

2. The different lessons containing an Overview, Specific Objectives, the Input, Practice Tasks and Feedback to the Practice Tasks.

3. An instruction on the use of the module.

This instructional material consists of two parts. Part I is Basic Definitions and Part II are the Algebraic Expression Concepts. It is composed of six modules namely:

Module 1

Algebraic Expression I	Let Us Start at the Beginning

Module 2	
Algebraic Expression II	Addition of Polynomials
Module 3	
Algebraic Expression III	Subtraction of Polynomials
Module 4	
Algebraic Expression IV	Grouping Symbols
Module 5	
Algebraic Expression V	Multiplication of Polynomials
Module 6	
Algebraic Expression VI	Division of Polynomials

Objectives of the Modules:

After reading these modules the students should be able to:

1. define and give examples of algebraic expressions.
2. identify given expressions as monomials, binomials, trinomials, polynomials.
3. operate on algebraic expressions.
4. evaluate algebraic expressions.
5. solve problems involving algebraic expressions.

Samar State Polytechnic College

Catbalogan, Samar

Pretest/Posttest

MULTIPLE CHOICE: Write the letter of the correct answer on the answer sheet.

1. An example of algebraic expression is
   - a. $y = 2 + x$
   - b. $\sqrt{625}$
   - c. 28/4
   - d. $6x - 3ab + 2c$
   - e. none of the above
2. A term of an expression is that part of it which is connected to the other parts of the expressions by either a plus or minus sign. In the expression $2x + 3$ is
   - a. monomial
   - b. binomial
   - c. trinomial
   - d. polynomial
   - e. none of the above
3. One or two or more numbers that are multiplied together like $3 \times 7 = 21$, 3 and 7 are
   - a. factors
   - b. addends
   - c. minuends
   - d. quotients
   - e. none of the above
4. The sum of $2(x + y)$, $-3(x + y)$ and $-4(x + y)$ equals
   - a. $-9(x + y)$
   - b. $-5 (x + y)$
   - c. $-2(x + y)$

d. -8(x + y)

e. none of the above

5. 3a is an expression of

a. 1 term

b. 2 terms

c. 3 terms

d. 4 terms

e. none of the above

6. The quantity a^3 is read as

a. a square

b. a cube

c. a to the fourth power

d. a factor

e. none of the above

7. The algebraic sum of -(11 + 4) equals

a. +18

b. -16

c. -12

d. -15

e. none of the above

8. Simplify the expression 3 - (4x +6y -7z)

a. 3 + 4x + 6y -7z

b. 3 - 4x -6y +7z

c. 3 - 4x +6y -7z

d. -3 +4x +6y +6z

e. none of the above

9. The expression x - {6x - [3x - (x - y) - 4y]} is equal to

a. -3x -3y

b. 4x +4y

c. -5x -5y

d. $6x +6y$

e. none of the above

10. The sum of $2x^2 +xy +y^2 +2xy -2xy^2 +x^2 +3xy + 7y^2$ is equal to:

a. $6xy + 4y^2$

b. $5xy + 3y^2$

c. $4xy + 2y^2$

d. $3xy + 2y^2$

e. none of the above

11. The expression (-4x) -(2x) -(6x) is equal to

a. 3

b. -4

c. 0

d. 5

e. none of the above

12. (4y) -(6y) -(3y) is equal to

a. 10y

b. 2y

c. 8y

d. 7y

e. none of the above

13. Simplify the following expressions $(x^2 -4x -5) + (3x^2 +4x -6) - (2x -3)$

a. $3x^3 -x -3$

b. $4x^2 -2x -8$

c. $2x^2 -5x -5$

d. $x^2 -5x -8$

e. none of the above

14. The sum of $(2x^2 +2xy +3y^2) + (-3x^2 -2xy +2y^2) + (-2xy +3xy -2y)$ is

a. $-3x^2 -3xy +3y^2$

b. $4x^2 -4xy -4y^2$

c. $5x^2 -5xy +5y^2$

d. $6x^2 -6xy +6y^2$

e. none of the above

15. Subtract 3y from 4x

a. 9x -8y

b. 8x -7y

c. 7x -6y

d. 4x -3y

e. none of the above

16. The difference of c^2y^3 and $-2c^2y^3$ is equal to

a. $5c^2y^3$

b. $3c^2y^3$

c. $4c^2y^3$

d. $9c^2y^3$

e. none of the above

17. (-4abx) -(-6abx) is equal to

a. 5abx

b. 2abx

c. 9abx

d. 10abx

e. none of the above

18. Find the difference between 2a -3b +6c and 3a -4b -2c.

a. -3a +b +8c

b. 2a +3b -8c

c. 9abc

d. 3a +b -5c

e. none of the above

19. Subtract $3x^2 -4x -6$ from $6x^2 -5x +2$

a. $5x^2 -3x +15$

b. $8x^2 +3x -3$

c. $3x^2 -x +8$

d. x^2 -2x -6

e. none of the above

20. A small number written to the right and a little above a quantity to show how many times the quantity is used

a. power

b. exponent

c. factor

d. term

e. none of the above

21. The product obtained when the number is multiplied by itself one or more times is

a. unlike terms

b. base

c. power

d. exponent

e. none of the above

22. An algebraic expressions of three terms

a. trinomial

b. binomial

c. monomial

d. exponent

e. none of the above

23. The expression $(4a^2x)$ $(-2ax)$ $(-3ax^2)$ is equal to

a. $16a^2x$

b. $24a^4x^4$

c. adc^2

d. cde3

e. none of the above

24. Simplify the following expression $3x^2y(x^2 - 3y^2)$ equals

a. $5x^4y + 3xy + 2y^2$

b. $3x^2 +6x^3y^2 +9x^2y^3$

c. 24x4 +2xy +y2

d. 5xy

e. none of the above

25. The product of (x + 6) and (x - 2) is equal to

a. $x^2 +4x +12$

b. $2x^2 -3x +15$

c. $5x^2 +3x +15$

d. $2x^2 +3x -4$

e. none of the above

26. Dividend equals quotient times divisor plus what?

a. subtrahend

b. minuend

c. remainder

d. difference

e. none of the above

7. The quotient of this expression $\dfrac{2x + 3}{-6x^2 +3x +18}$ is equal to

a. 9x +7

b. 4x -3

c. 5x +8

d. -3x +6

e. none of the above

28. Divide $6a^4 -41a^2 +3a +6$ by $2a^2 -4a -3$

a. 2x +5

b. $3a^2 +6a -4$

c. $5a^2 +5a -2$

d. 5a -6

e. none of the above

29. The product of $3x - 4y$ and $2x + 5y$ is

    a. $2x^2 + 3xy - 4y^2$
    b. $x^2 + xy - y^2$
    c. $6x^2 + 7xy - 20y^2$
    d. $2x^2 + 5xy - y^2$
    d. $2x^2 + 5xy - y^2$
    e. none of the above

30. $(x - y)^2$ equals

    a. $x^2 - 2xy + y^2$
    b. $2x^2 - xy + y^2$
    c. $x^2 - xy + y^2$
    d. $2x^2 + xy - y^2$
    e. none of the above

The Module

To the Student

This instructional material is designed for use for first year students enrolled in Bachelor of Science in Industrial Technology (BSIT). It contains lessons that are simplified for easy understanding. Each lessons begins with an **Overview** which presents the topic/topics to be developed. **Specific Objectives** are also listed to give you an insight of the concepts and skills that you must develop at the end of the lesson.

After the presentation of the objectives comes the **Input** which explains to you all the concepts involved in understanding the lesson presented. It also contains exercises in the form of **Test Your Understanding** and **Test Your Computational Skills** as means of checking the understanding of the concepts presented. It is of utmost importance that you read first the Input and answer the exercises. The **Practice Tasks** which comes after the Input consists of exercises to reinforce what one have learned. They are of varied levels of difficulty to suit your individual styles and needs. The **Feedback to the Practice Tasks** gives you the correct answer to the Practice Tasks. After the feedback comes the **Answers to Test Your Understanding** and **Test Your Computational Skills**. You can check your answers to the exercises by comparing your answers with the printed answers.

Module 1
Lesson 1

Algebraic Expression 1

(Let us start at the beginning)

What are algebraic expressions?

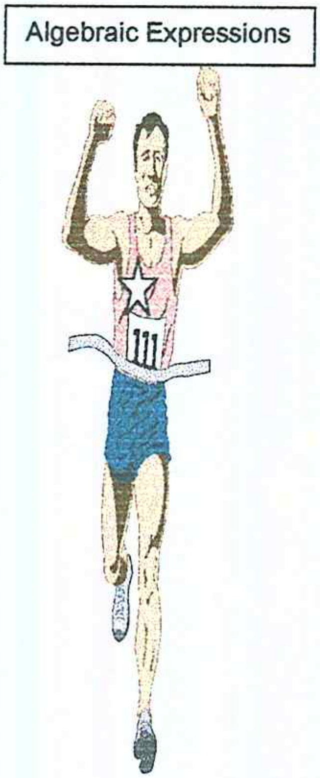
Overview:

The concept of algebraic expression is a fundamental concept in algebra. In order to understand the concept, a student needs a good background concerning the operations of signed numbers. Field axioms and theorems furnish the basis for these operations.

Moreover there are algebraic terms frequently mentioned that catch our attention such as monomials, polynomials, variables etc. The key to understanding the concepts is knowing the meaning of some of the terms and applying the knowledge of fundamental operations in algebra with the help of the field axioms and theorems.

Let us start with some of the basic things that we should know about algebraic expressions, terms used and their definitions.

What are algebraic expressions?



Can you identify algebraic expressions from several given mathematical expressions?

There is no need to put them under a high powered microscope. Just read Lesson 1 of Module 1, do the exercise, and bingo!

You will be able to identify them.

Objective: **At the end of this lesson you should be able to identify algebraic expressions.**

ALGEBRAIC EXPRESSIONS

Input

Algebraic expressions are made up of four different kinds of symbols ... numbers, variables, operation signs and grouping symbols (Not all of these need be present).

These collection of symbols represent a **specific real number or unspecific real number.**

$2x^2 +3x -1$ is an algebraic expression for if the variable x is substituted by a specific real number the result is a real number in view of the closure property of real numbers.

$3/4\ x^3 +x/4 +y^3$ is an algebraic expression for if the variables x and y are substituted by a specific real number the result is a real number.

$3x^2/y +3x +6$ is an algebraic expression if $y \neq 0$. Since division by zero is not defined.

Some examples of algebraic expressions are $3x -y +3z$, $a -2b +c$, $2x5y2 -7$, $at^2 +bt +c$, $3y2 -(x +2y) +6$, $(x^3 -2y -8)$ and etc.

Let us look at some examples. Okay!

$2x^4 - 2x + \sqrt{\ } - 4$ is not an algebraic expression because the symbol $\sqrt{\ }$ does not represent a real number (specific or unspecific).

$3x^2 \sqrt{y} + 2$ is not an algebraic expression if y is negative since the $\sqrt{y}$ is imaginary, hence not real.

$2x/y - 3x^2 - 7$ is not an algebraic expression if y is zero since division by zero is not defined.

Important: For a group of symbols consisting of numbers, letters, operation signs and grouping symbols to form an algebraic expression it must represent a real number (*specific or unspecific*).

Test Your Understanding 1.1

Which group of symbols (always, sometimes and never) represent an algebraic expression?

1. $5x - 1/3$
2. $y^3 + 2y - \sqrt{\;}$
3. $3x^2 + x - 7$
4. $4x^3 - 3x^2 + 9x + 8$
5. $3/4\, y^3 + \sqrt{y} - xz$
6. $a^2 - bc - (a - 3) + 4$
7. $ax - by/z$
8. $(a + b)(a - b)$
9. $x + iy$
10. $\frac{-y + (1 - 4x)}{3 - x}$
11. $at^2 + bt + c$
12. $x/a - y/b + z/c$
13. $(x + y)(x - y)$
14. $(x + iy)^2$
15. $\frac{(x - y)(y - x)}{y - x}$

Module 1
Lesson 2

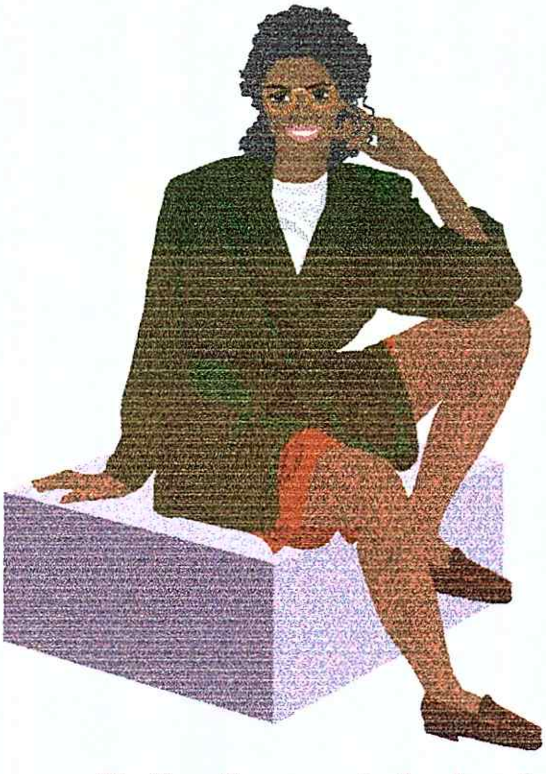
Algebraic Expression 1

(Let us start at the beginning)

What is the numerical value of a given algebraic expression?

Do you know how to find the numerical value of a given algebraic expression?

What do you call the process of calculating the numerical value of algebraic expressions?



Finding the numerical value of a given algebraic expression is very easy. It is no big deal especially with MR. CALCULATOR.

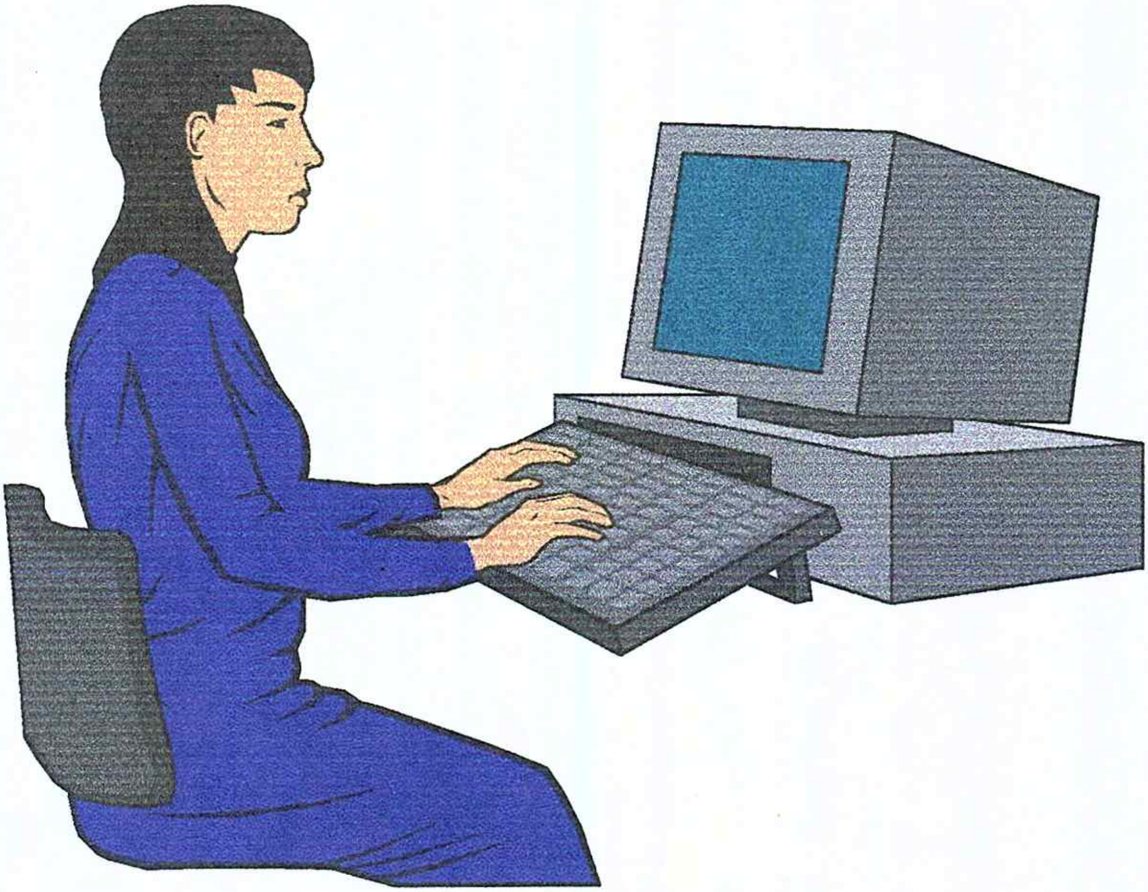
Objective:

At the end of this lesson you should be able to *evaluate* or find the numerical values of given algebraic expressions.

Input

Algebraic expressions represent real numbers.

Let us try to find these real numbers.



How will I find this real number which is the value of the given algebraic expression?

The real number obtained, when variables present in the expression are substituted with specific real numbers, is the numerical value of the expression.

The numerical value of an algebraic expression can be calculated when each literal number in the expression is given specific value.

The process of calculating the numerical value of algebraic expressions is called **evaluation.**

To find the value of an expression having variables and numbers

1. Replace each variable by its number value.
2. Carry out all arithmetic operations using the correct order of operations.

Example 1: Find the value of $3x - 5y$ if $x = 10$ and $y = 4$.

Solution: $3x - 5y = 3 \cdot x - 5 \cdot y = 3(10) - 5(4) = 30 - 20 = 10$

Notice that we simply replace each variable by its number value, then carry out the arithmetic operations as we have done before.

CAUTION: When replacing a variable by a number, enclose the number in parenthesis to avoid the following common errors.

Evaluate : $3x$ when $x = -2$

CORRECT	**COMMON ERROR**
$3x = 3(-2) = -6$	$3x \neq 3 -2 = 1$

Evaluate $4x^2$ when $x = -3$.

CORRECT	**COMMON ERROR**
$4x^2 = 4(-3)^2 = 4(9) = 36$	$4x^2 \neq 4 -3^2 = 4 - 9 = -5$ or $4x^2 \neq 4 -3^2 = 4 + 9 = 13$

Example 2. Find the value of $2a - [b - (3x - 4y)]$ for $a = 3$, $b = 4$, $x = -5$, and $y = 2$.

Solution: $2a - [b - (3x - 4y)]$

$= 2(-3) - [4 - \{3(-5) - 4(2)]$ Notice { } were used in place of ( ) to clarify the grouping.

$= 2(-3) - [4 - \{-15 - 8\}]$

$= 2(-3) - [4 - \{-23\}]$

$= 2(-3) - [4 + 23]$

$= 2(-3) - [27]$

$= -6 - 27$

$= -33$

Example 3. Evaluate $b - \sqrt{b^2 - 4ac}$ when $a = 3$, $b = -7$, and $c = 2$.

Solution : $b - \sqrt{b^2 - 4ac}$

$= -7 - \sqrt{(-7)^2 - 4(3)(2)}$

$= -7 - \sqrt{49 - 24}$

$= -7 - \sqrt{25}$

$= -7 - 5$

$= -12$

CAUTION: A common error often made is to mistake $(-3)^2$ for -3^2.

$(-3)^2 = (-3)(-3) = 9$ The exponent 2 applies to -3.

$-3^2 = -(3)(3) = -9$ The exponent 2 applies to 3.

Let us find the numerical value of algebraic expressions in the following examples:

Let $x = 1$, $y = 2$, and $z = 4$. Find the numerical value of the given algebraic expressions.

1. $2x^2 + 3x - 1$
2. $3/4x^3 + x/4 + y^3$
3. $3x^2/y + 2x + z$

Solution: To find the numerical value of the given algebraic expressions substitute the specific value for each letter and simplify.

1. $2x^2 + 3x - 1$

$= 2(1)^2 + 3(1) - 1$

$= 2(1) + 3(1) - 1$

$= 2 + 3 - 1 = 4$

2. $3/4x^3 \quad + x/4 + y^3$

$= 3/4(1)^3 \quad +1/4 + (2)^3$

$= 3/4(1) \quad +1/4 + 8$

$= 4/4 + 8$

$= 1 + 8$

$= 9$

3. $3x^2/y \quad +2x \quad +z$

$= 3(1)^2/(2) \quad +2(1) \quad +(4)$

$= 3(1)/(2) \quad +2(1) \quad +(4)$

$= 3/2 \quad +2 \quad +4$

$= 3/2 + 6$

$= 1\ 1/2 + 6$

$= 7\ 1/2$

Let us solve some problems about the evaluation of expressions. But we should take note that **in evaluating algebraic expressions the specific real number representing a particular variable should be the same throughout the whole process.**

The content of the statement can be explained using an example.

The numerical value of the expression $2x^2 + 3xy + y^2$ is 15 when $x = 2$, and $y = 1$. The expression is a sum of three terms $2x^2$, $3xy$, and y^2. There is an x in the first term of the expression so we have to substitute it with the real number 2. An x also appears in the second term so we have to substitute the real number 2 again. It is not correct practice if another real number is substituted to x in the second term since the second term is part of the given algebraic expression. In the same manner with the variable y, if 1 is substituted to y in the second term, it must be the same number that will be substituted to y in the third term of the expression not any other number. However for these algebraic expressions such as, $x -2y$, $2xy - 4$, $y^2 -x^2$ and a lot of others we can substitute other real numbers for the variables x and y unless otherwise stated.

Remember:

To evaluate an algebraic expression is to find the numerical value of the given expression.

Note:

Specific value given to a letter varies from one problem to the next, but it remains the same for that letter throughout any one problem.

Now suppose you are given the numerical value of an algebraic expression and you are asked to find the specific real number which was substituted to the variable, how will you find it?

Let us study the example presented.

If the value of the algebraic expression $3x^2 +2x +1$ is 6 what real number was substituted to the variable x?

We can restate the given problem as $3x^2 +2x +1 = 6$. What is x?

Observed that the numerical coefficient of the first term is 3, and that of the second term is 2. The last term is the constant 1. If we add the numbers 3, 2, and 1, the sum is 6. Hence $x = 1$, since $3(1)^2 +2(1) +1 = 6$. By inspection we were able to guess correctly the answer to the problem.

Let us find the value of x in the given problem by using completing the square. Completing the square is but one of the many different ways of finding the solution to the given problem.

$$\begin{aligned}
\mathbf{3x^2 +2x +1} &= \mathbf{6} \\
3x^2 +2x +1 -1 &= 6 -1 \\
\frac{3x^2}{3} \quad \frac{+2x}{3} &= \frac{5}{3} \\
x^2 +\frac{2x}{3} &= \frac{5}{3} \\
x^2 +\frac{2x}{3} + \frac{1}{9} &= \frac{5}{3} + \frac{1}{9} \\
(x + 1/3)^2 &= (4/3)^2
\end{aligned}$$

$(x + 1/3) = \pm 4/3$

$x = -1/3 + 4/3 ; \quad x = -1/3 - 4/3$

$x = 3/3 = 1; \quad x = -5/3$

It is just very easy to find the numerical value of an algebraic expression. We only need specific real numbers to be substituted to the variables in the expressions.

When you already have the specific real numbers to be substituted to the variables the next steps are substitution and computation.

Calculations are easier and the likelihood of an error is reduced when the specific value for each letter is substituted using parenthesis in a distinct step before the operations are performed.

After we substitute the specific value for each letter in the expression, the resulting numerical expression can be simplified by carrying out the operations according to the following rules.

ORDER OF OPERATIONS

1. If there are any parentheses in the expression, that part of the expression within a pair of parentheses is evaluated first. Then the entire expression is evaluated.
2. Any evaluation always proceeds in three steps:
   First: Powers and roots are done in any order.
   Second: Multiplication and division are done in order *from left to right.*
   Third: Addition and subtraction are done *from left to right.*

The order of operations rules provide us with specific order in which an expression with several operations can be evaluated to insure a correct answer. Since the commutative and associative properties make it possible to evaluate addition and multiplication in more than one order yet get the same result, it may be possible to perform the operations in an expression in more than one order yet get the same answer.

For example it is important to realize that an expression such as

$$8 -6 -4 +7$$

is evaluated by doing the addition and subtraction in order from left to right (because subtraction is not commutative or associative).

The same expression may be considered as a sum:

$$(8) + (-6) + (-4) + (7)$$

If the expression is considered as a sum, then the terms can be added in any order (because addition is commutative and associative).

Evaluated only left to right	***Added in any order***
$8 -6 -4 +7$	$(8) + (-6) + (-4) +(7)$
$= 2 -4 +7$	$= (8) + (7) + (-6) + (-4)$
$= -2 +7$	$= 15 + (-10)$
$= 5$	$= 5$

When only division and multiplication are to be done, as in

$$75 \div 5 \cdot 3.$$

It is equally important that the left-to-right order be followed (because division is neither associative nor commutative). Thus we have

$$75 \div 5 \cdot 3$$
$$= 15 \cdot 3 = 45$$

Example 1. Using order of operations rules to evaluate expressions:

(a) $(7+3)\cdot 5$ — **We do the part in parenthesis first**

$= 10 \cdot 5 = 50$

(b) $7 + 3 \cdot 5$ — **Multiplication is done before addition**

$= 7 + 15 = 22$

(c) $4^2 + \sqrt{25} - 6$ — **Powers and roots are done first**

$= 16 + 5 - 6$

$= 21 - 6$

$= 15$

(d) $16 \div 2 \cdot 4$ — **Here division is done first because in**

$= 2 \cdot 4$ — **reading from left to right the division**

$= 8$ — **comes first.**

(e) $\sqrt{16} - 4(2 \cdot 3^2 - 12 \div 2)$ — **First evaluate the expression inside**

$= \sqrt{16} - 4(2 \cdot 9 - 12 \div 2)$ — **Do the powers inside the parenthesis**

$= \sqrt{16} - 4(18 - 6)$ — **Do the x and ÷ inside ( )**

$= \sqrt{16} - 4(12)$ — **Do the inside ( )**

$= 4 - 48 = -44$ — **Next find the root**

(f) $(-8\ 1/2) \div 2 - (-4\ 1/2)$ **Division is done before subtraction**

$= (-17/2 \div 2 - (-9/2)$

$= (-17/2) \cdot 1/2 + 9/2$

$= -17/4 + 18/4 = 1/4$

Test Your Computational Skills

Evaluate the given algebraic expressions when $x = 1$, $y = 2$ and $z = 4$.

1. $2x^2y^3 - 3xy^2 + (x - 3y)^2$
2. $(3x + 2y - z)^2 - x^2y^3z^2$
3. $2x^2y^3z^2 - xyz + 7$
4. $(2x - y)^2 + (x - 3y)^3$
5. $x^2 - 4y/3 + 3x$
6. $3x^2y^2 - [xy^2 + (x - 3y)]^2$
7. $(3x + 2y - z^2 - x^2y^2z)^2$
8. $2(x^2y^3z)^2 - xyz + 7$
9. $[(2x - y)^2 + (x - 3y)^3]^2$
10. $x^2/3 - 4y/5 + 3x/2$
11. $(x^2)^3 - (3xy)^2 + (x - 3y)^2$
12. $[(x + y - z)^2]^2 - [x^2 y^3z]^2$
13. $[x^2y^3z^2 - 7]^2$
14. $[(2x - y)^2 + (x - 3y)^2]^2$
15. $\dfrac{x^2 - 4y^2 + 3z}{4}$

Module 1

Lesson 3

Algebraic Expression 1

(Let us start at the beginning)

What is a term in an algebraic expression?

This scene is very ordinary during the Christmas season.

Look at the delivery boy as he carries all your purchases.

How many boxes are in the pile?

Finding the number of terms in an algebraic expression is like counting the number of boxes in the picture.

Read Lesson 3 of Module 1 and Bull's eye! You'll hit the jackpot.

Objective:

At the end of the lesson you should be able to determine the number of terms in an algebraic expression.

Input

A **term** can be a number or a product of number and symbols. When a term is composed of a product of number and symbols each number and symbols is called a factor of the term.

5, x^2, and y^2 are factors of the term $5x^2y^2$. 2 and (x -3y) are factors of the term 2(x -3y). Also 2, a, b, and c are the factors of 2abc, and 5 and x are the factors of 5x.

An algebraic expression is separated into distinct parts called terms by a **plus** or **minus** sign.

Example: Terms of an algebraic expression.

(a) In the algebraic expression $3x^2y \;\; \underset{\wedge}{-} \; 5xy^3 \;\; \underset{\wedge}{+} \; 7x$

The - and + sign separate the _____⌋_____⌋
algebraic expression into three terms.

$3x^2y$	$-5xy^3$	$+7x$
First Term	Second Term	Third Term

(b) In the algebraic expression $3x^2$ -9x(2y -x)

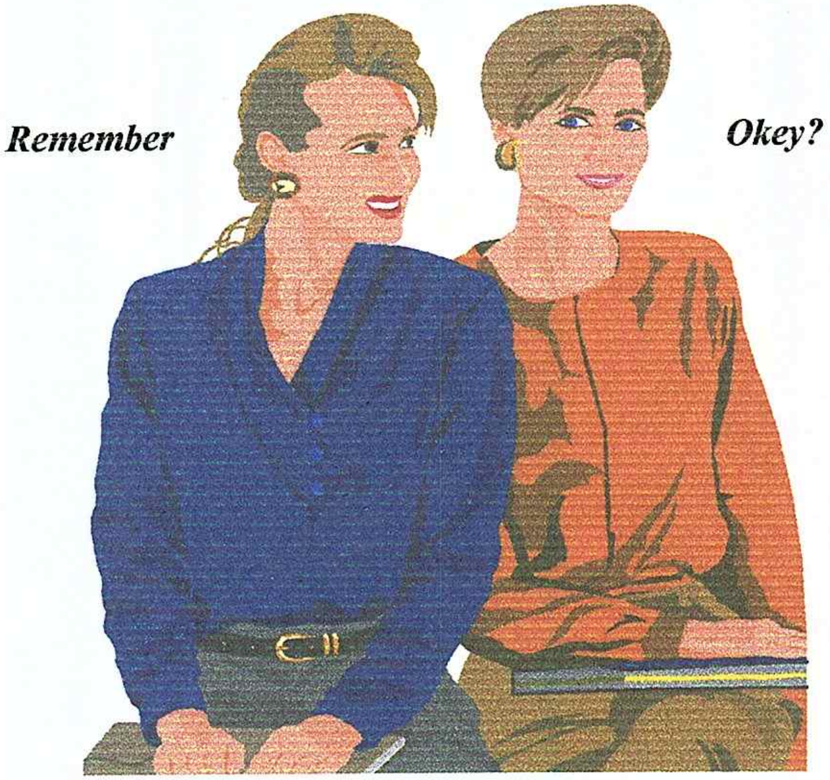
$3x^2$	-9x(2y -x)
First Term	Second Term

(c) In the algebraic expression $\frac{2 - x}{xy} \qquad + 5(2x - y)$

$$\frac{2 - x}{xy} \qquad +5(2x - y)$$

First Term   Second Term

The number of + and - signs indicate the number of terms in an algebraic expression.



Each + or - sign is part of the term that precedes it.

How many terms are there in the given algebraic expressions?

1. $3 + (2x - 1) + x^2$
2. $(2x + 3y + 4) - (8 - 5y)$
3. $(3x^2 + 2x + 3y - 8)$

Expression 1 has three terms, 2 has two terms and 3 has only one term.

Then you may ask, have you counted the number of + and - sign correctly? If you have counted, then the answers to the problems above are wrong. But look at the given expressions, you'll notice that some symbols consisting of numbers and variables are within grouping symbols and must therefore be exception to the rule.

Exception:

An expression within grouping symbols is considered a single piece even though it may contain a + or - sign.

Remember:

When a term has no sign preceding it, as in x^3 the plus sign (+) is implied.

x^3 is understood to be $+x^3$.

No sign preceding a term means that the term is carrying a positive (+) sign

In listing, naming, or identifying the terms of an algebraic expression the plus + sign and minus (-) sign preceding the term is part of the term.

Below are some examples of terms that will offer a better understanding of what terms are.

$x^2 + 5x^2y^3 + 3(x - 3y) + z$ has four terms.

$a - 2b + c$ has three terms.

$(at^3 + bt^2 + ct + d)$ has one term only.

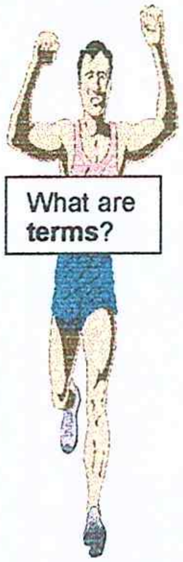
From the examples given we now have an idea of what terms are.

What are terms?

Review:

Terms of an algebraic expression are the parts that are being added.

A term is a group of symbols and or numbers within an algebraic expression that is separated from other groups by a plus (+) or minus (-) sign.



Test Your Understanding 1.2

Determine the number of terms in the given expressions and identify the second term if possible.

1. $a + 2b - c$
2. $3x^2 + (2 - y) + z^4 - 6$
3. $1/2\,xyz + 7$
4. $ct^2 - bt + c$
5. $2xyz / 7$
6. $(3x^3 - 5z) + (z - 6y + x) - 6$
7. $x/y + y^3 - 5$
8. $x - 9y + (z - y) + (x - 3)$
9. $1/2 + t^2 - t/3 + 2$
10. $\dfrac{3 + 4x - z}{2x} - \dfrac{3/8 + z}{y}$

Module 1
Lesson 4

Algebraic Expression 1

(Let us start at the beginning)

What are single factors of a given term?

What are the single factors of a given term?

Can you list down or name the single factors of a given term?

There is no need to put them under a high powered microscope.

Just read Lesson 4 of Module 1, do the exercises, and bingo!

You will be able to name them.

Objective: **At the end of the lesson you should be able to give the single factors of a term.**

Factors are numbers and letters that are multiplied together to give a **product.** The number **one (1)** is a factor of any number. Even though 1 is a factor of every number it is often times omitted.

To give you a clear idea of factors we have the following example:

Example: Factors of a term.

(a) $(3)(5) = 15$
$\uparrow\ \uparrow$ ________________ are factors of 15

(b) $(5)(x) = 5x$
$\uparrow\ \uparrow$ ________________ are factors of 5x

(c) $(7)(x)(y)(z) = 7xyz$
$\uparrow\ \uparrow\ \uparrow\ \uparrow$ ____________ are factors of 7xyz

________________ are terms of the expression $3x^2 + 5x$
$\downarrow \qquad \downarrow$

(d) $3x^2 \quad +5x$
$\uparrow\uparrow \qquad \uparrow\uparrow$ ________ are factors of 5x
$\sqcup$ ________________ are factors of $3x^2$

(e) $(3)(x^2)(y^3)(z) = 7x^2y^3z$
$\uparrow\ \uparrow\ \uparrow\ \uparrow$ ________ are factors of $7x^2y^3z$

(f) $(-3)(x)(y)(z) = -3xyz$
$\uparrow\ \uparrow\ \uparrow\ \uparrow$ ________ are factors of -3xyz

Remember:

***Factors** are **numbers** and **letters** that are multiplied together to give the term.*

Let us test your understanding of factors.

TEST YOUR UNDERSTANDING 1.3

Give the single factors of the given term.

1. ab/cd
2. x^4yz
3. $2xyz^3$
4. $a^n b$
5. $-3x^4$
6. $(-3x)^4$
7. x
8. $(a + b)^2$
9. $a^2 - b^2$
10. $4x/8$

Let us take a look at base and exponents..... next page.

Module 1

Lesson 5

Algebraic Expression 1

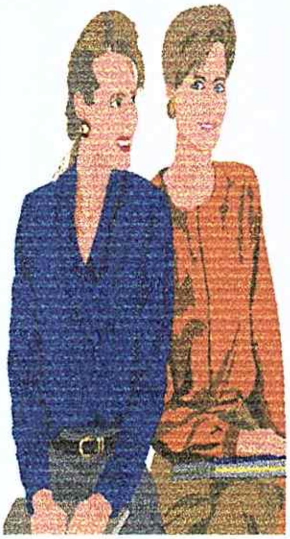
(Let us start at the beginning)

What is a base? What is an exponent?

What instrument is used to see distant stars at night?

You will not need a telescope to be able to tell a base from an exponent in a term.

Read Lesson 5 of Module 1 and forget about the telescope.



Objective:

At the end of this lesson you should be able to identify the *base* and the *exponent* from the given examples.

Input

Look at 3 and x^2 the factors of $3x^2$. The variable x is raised to the second power. We call x the base and 2 the exponent.

If **a** is a real number and **n** is a natural number, the symbol $\mathbf{a}^n$ denotes the nth powerof a.

$$a^n = \underbrace{a \cdot a \cdot a \cdot a \cdot a \ \ldots \ a}_{\text{n factors}}$$

In the symbol $\mathbf{a}^n$ we call **a** the **base** and **n** the **exponent**.

Example 1: Identify the base and exponent and find its value if possible.

(a) 3^4 The base is 3 and the exponent is 4.

$3^4 = (3)(3)(3)(3) = 81$

(b) $-3x^4$ The expression $-3x^4$ is the product of -3 a numerical coefficient and a power, x^4. The base is x and the exponent is 4.

(c) $(-3x)^4$ The base is -3x and the exponent is 4.

$(-3x)(-3x)(-3x)(-3x) = 81x^4$

(d) $-3x^4$ The base is x and the exponent is 4.

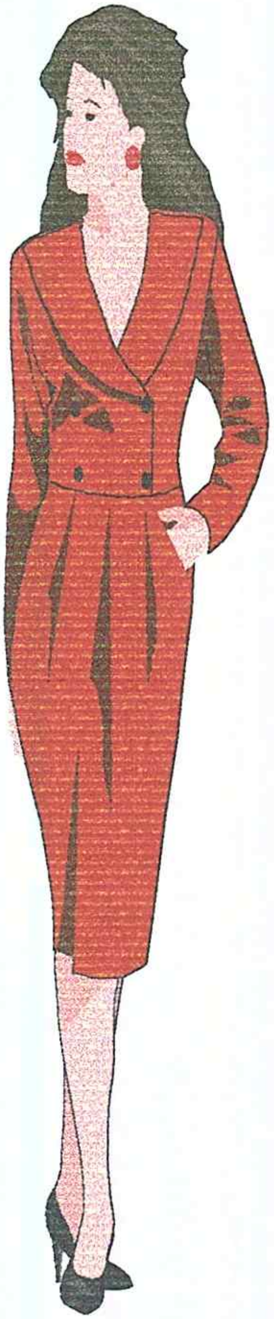
The expression is a product of -3 and x^4.

$(-3)(x)(x)(x)(x) = -3x^4$.

Ops! take note, that the exponent acts on the symbol immediately preceding it. Okey?

Also, when numbers and symbols are enclosed in parenthesis or any other grouping symbols, the exponent acts on the whole term enclosed in parenthesis.

When no exponent is written it is understood that the exponent of the term is one. Okey?



Test Your Understanding 1.4

Identify the base and exponent.

1. $-x$
2. a^m
3. $3z^4$
4. $(-3xyz)^{-2}$
5. $xyzw$
6. $(-1.2)^2$
7. $(a-b)^2$
8. $+4.3$
9. $(x^2 - y^2)^2$
10. $(-abc)^0$
11. $z^{-5/4}$
12. $(b \ -4ac)$
13. $(at^2 +bt +c)^3$
14. p^q
15. $(-1)^{m-n}$
16. 1^{k+j}
17. $(-4)^p$
18. e^x
19. a^x
20. $(\ln)^e$

Module 1

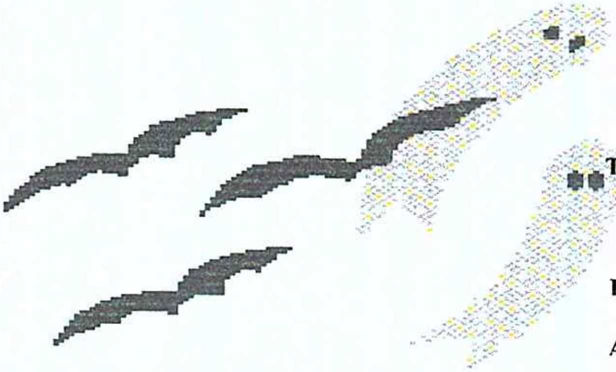
Lesson 6

Algebraic Expression 1

(Let us start at the beginning)

What are similar terms?

What are dissimilar terms?



Time is gold.

Learn to use your time wisely.

A minute wasted is a minute gone.

Tomorrow is another day.

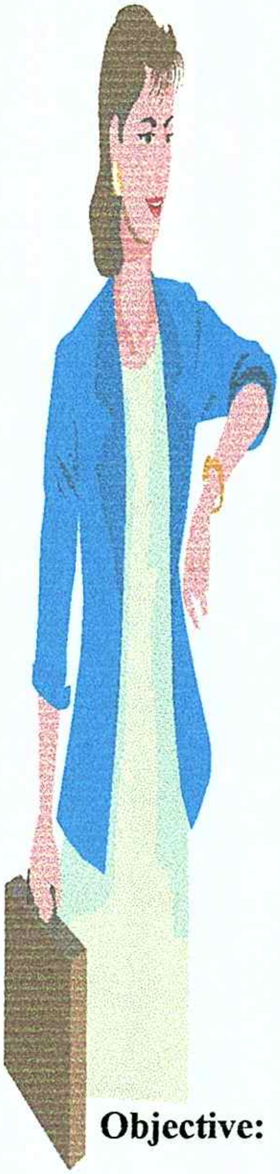
Do you know that because of our limited time in this world we have to race against our time?

Saving time gives birth to some major inventions and discoveries;

If you can identify similar terms from dissimilar terms in a blink of an eye, that is time saving.

Okey?

Read Lesson 6 of Module 1 and save your time.



Objective:

At the end of this lesson you should be able to identify similar terms from dissimilar terms from given examples of terms.

Input

Algebraic expressions can be simplified by combining **like** or **similar terms** within the expression.

Like or similar terms are combined by adding the numerical coefficients. This is possible by the **distributive axiom**.

What are **like** or **similar terms?** **Like** or **similar terms** are two or more terms that have identical literal part or that differ only in their numerical coefficients.

2ab, -8ab, 4ab, 1.2ab, 1/2ab are examples of **similar terms**.

3xyz, -4xyz, and **3/8xyz** are **similar terms** or **like terms.**

All constant terms such as, 4, -8, 1.2, -1/2 , 5/8, etc. involve no variables, hence are like terms.

Some examples of **like** or **similar terms** are given below.

(a) $3x^2$, $-2x^2$, $+12x^2$, $2/3x^2$, $-0.2x^2$

(b) a, 2a, 4/5a, -a, 0.4a

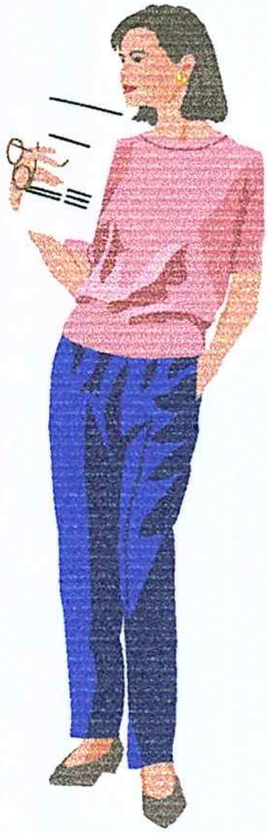
(c) $2x^2y^3$, $-x^2y^3$, $-72x^2y^3$, $1/3x^2y^3$

(d) 2a/bc, -6a/bc, 2a/3bc, a/bc

(e) 2xyz, -5xyz, 2/5xyz, -1.9xyz

(f) -1, 2.7, 2 / 7, -4.78

What did you observed from the given examples?



Notice:

1. Similar terms differ only in their numerical coefficients.
2. Literal parts are identical - same variables with the same exponents.
3. Similar terms involve two or more terms.
4. All constants are like terms.

REMEMBER:

Like terms are terms which differ only in their numerical coefficients but they have exactly the same literal coefficients. The variables (letters) involved are exactly the same and they have the same powers or exponents.

If we have like or similar terms there are also terms which are unlike or dissimilar.

What are dissimilar or unlike terms?

Unlike terms or **dissimilar terms** are two or more terms which have different literal factors.

Take a look at some dissimilar terms.

$3x^4y^2$ and $-4x^2y^3$ are **not** like terms or similar terms. The literal part for both terms is xy but the same variable in one of the terms do not have the same power in the other term. In $3x^4y^2$ the variable x is raised to the 4th power while in $-4x^2y^3$ it is raised to the second power. Also, the variable y is having a second power in $3x^4y^2$ but it has a third power in $-4x^2y^3$.

Some examples of dissimilar terms are listed below:

Example 1.6 (B). Dissimilar Terms

(a) $a,\ 2b,\ -c,\ ab,\ ac,\ e$ (All terms are unlike or dissimilar.)

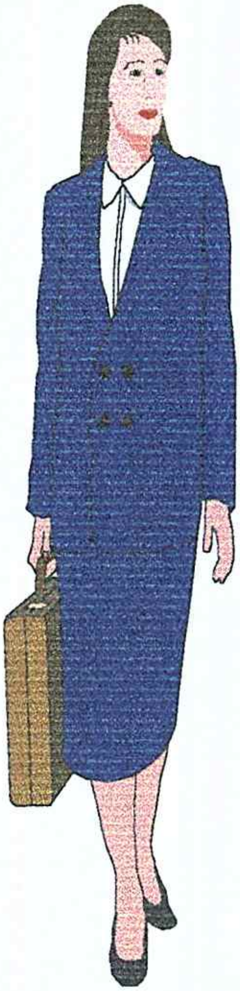
(b) $4x$ and $9y$ (In 4x, x is the literal part and y is the literal part in 9y.)

(c) $3xy,\ -5xy,\ 3x^2y^2,\ +5xy^3$ (All terms have the same literal parts but the literal parts have different powers or exponents.)

(d) $(3a - b),\ (3a^2 - b^2)$ (Both terms have the same literal parts but the powers or exponent of the variables involved are not the same.)

(e) $-4xyz,\ -4x^2y^2z^2,\ -4xyz^4$ (All terms have the same literal parts but the powers or exponents of the variables involved are not the same..)

How do we know that the terms have different literal coefficients?



Different literal coefficients mean ...

1. The literal factors of the terms involve are different.

   Examples: $2x$ and $3y$

2. The literal factors are the same but with different exponents.

   Examples: $2x^2y$ and $3xy^2$

Let us test your knowledge of similar and dissimilar terms. Answer the exercise next page.

Test Your Understanding 1.5

I. List the terms that are like or similar for each number.

1. $2ac$, $3bc$, $-3ac$, $1/2ac$, $-bd$

2. $2xy$, $9xyz$, $1.3xzy$, $6yz$, $5zyx$

3. $2x^3$, $4x^3y$, $-3yx^3$, $-1/3\ zx^3$

4. $2a/b$, $-3a/b$, $2b/a$, bc

5. $2(x-2y)$, $-4(-2y+x)$, $3(-x+2y)$, $4(2x-2y)$

II. List all the terms at least no two of which are similar.

6. $2x$, 2, $3y$, $-4y$, $1/4y$, x/y

7. $3x^2y^3$, $-5x^3y^2$, $12xy$, $1.5\ yx$, $2xy^3$

8. $2a$, $3/a$, $-5/a$, ab, b

9. $2a/3b$, $a/2b$, $-4a/b$, a, b

10. abc, $3zyx$, $-abc$, $1/3cba$, $5x$

Module 1
Lesson 7

Algebraic Expression 1

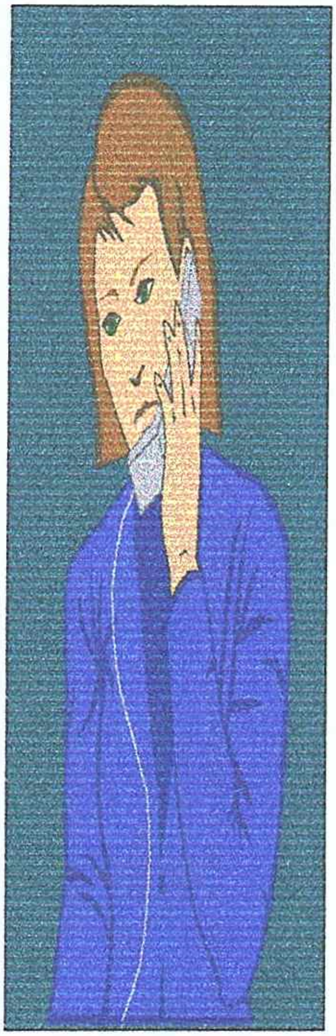
(Let us start at the beginning)

What are coefficients of terms?

What is the numerical coefficient of a term?

What are literal coefficients?

Can you tell a fake from a collector's item in a treasure chest?



You need to be a collector to identify numerical coefficients and literal coef-ficients from a chest of terms.

Read Lesson 7 of Module 1 and become an expert overnight.

Objective:

At the end of this lesson you should be able to identify the coefficients of a given term.

Input

Now back to terms of algebraic expressions.

A term is usually expressed as a product of two or more factors. These factors can be a number (constant) and variables (letters).

In a term having only two factors, the coefficient of one factor is the other factor. In a term having more than two factors, the coeffient of each factor is the product of all the other factors in that term.

Example: Coefficients

(a) 3/4 y

| | is the coefficient of 3/4

| is the coefficient of y

(b) 3xy = 3 (xy) = (3x)y = (3y) x

| | | | | | is the coefficient of 3y

| | | | | is the coefficient of x

| | | | is the coefficient of 3x

| | | is the coefficient of y

| | is the coefficient of 3

| is the coefficient of xy

NOTICE: The coefficient of a term may be a single factor of the term or the product of two or more single factors.

(c) 3xyz = 3(xyz) = (3xz)y = (3yz)x = (3xy)z

is the coefficient of 3xy

is the coefficient of z

is the coefficient of 3yz

is the coefficient of x

is the coefficient of 3xz

is the coefficient of y

is the coefficient of 3

is the coefficient of xyz

In example (c) the single factors of 3xyz are 3, x, y and z. 3 is the specific real number in the given term. 3 is called the **numerical coefficient.**

The numerical coefficient is the **constant** (number) that is present as a factor in a term. 3, 2, and 1 are the numerical coefficients of 3xy, 2x and abc.

When there is no specified number in a term the numerical coefficient is understood to be the number 1.

The numerical coefficient of x^2, abc, xyz is 1. The numerical coefficient of x is understood to be 1, since x = (1)(x) and the numerical coefficient of -x is -1, since -x = (-1)(x).

Example: Literal and Numerical Coefficients

(a) 6w

Literal coefficient of 6.

Numerical coefficient of w.

(b) $12xy^2$

| | xy^2 is the literal coefficient of 12

| 12 is the numerical coefficient of xy^2

(c) $$\frac{3xy}{4} = \frac{3}{4}\,xy$$

| | is the coefficient of 3/4

———————— is the coefficient of xy

(d) $-a^2$

| -1 is the coefficient of a^2.

The numerical coefficient of a term may be associated with any literal factor of the term not just the first literal factor.

Example: 3xyz

3 is the numerical coefficient of xyz

3 is the numerical coefficient of x

3 is the numerical coefficient of y

3 is the numerical coefficient of z

3 is the numerical coefficient of xy

3 is the numerical coefficient of yz

3 is the numerical coefficient of xz

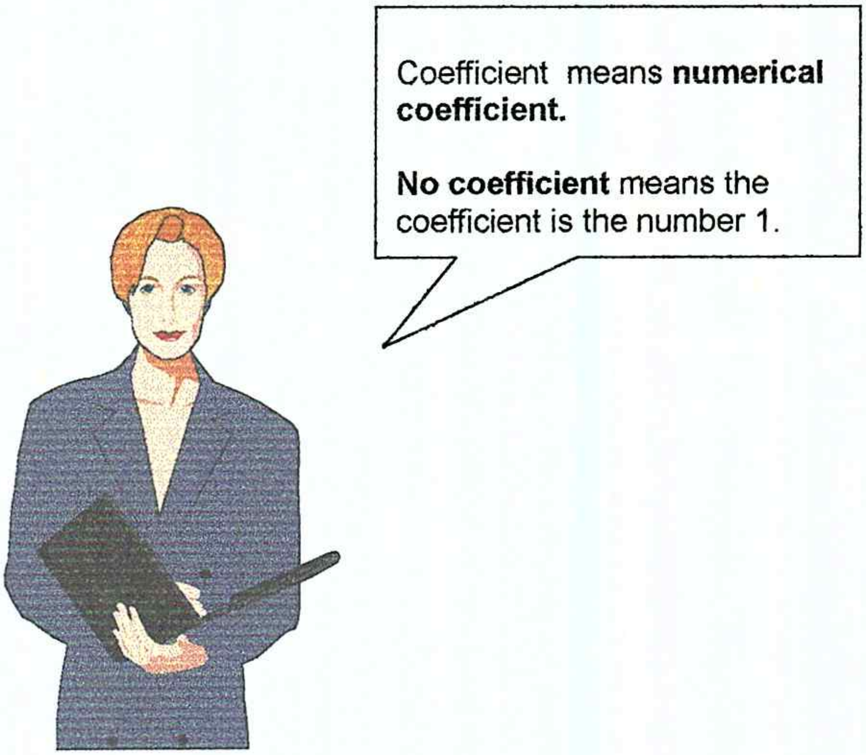
For terms within grouping symbol the numerical coefficient is understood to be 1. When there is no specific number preceding a term enclosed in grouping symbols, it is understood to be preceded by 1.

The numerical coefficient of (x -2y) is 1, since (x -2y) = (1) (x - 2y). The numerical coefficient of -(x - 2y) is -1, since -(x -2y) = (-1)(x -2y).

Numerical coefficients are always associated with literal factors of the term. The **letters** present as factors of the term are the literal coefficients of the term. When a term consist only of a number such as the last term of the polynomial $3x^2 + 2x + 5$, we do not say 5 is the numerical coefficient of 5. When we say "**coefficient**" of a term what we mean is the "**numerical coefficient**" of the term.

In 4xyz, the literal coefficient is xyz. In the term abc the literal coefficient of 1 is abc. Also the literal coefficient of $2x^2y^3$ is x^2y^3.

In the expression 2(x -4y), the numerical coefficient of the first term (2x) is 2 and the second term (-8y) is -8.



Coefficient means **numerical coefficient.**

No coefficient means the coefficient is the number 1.

Test Your Understanding

---

1. List the coefficients of abc.

2. Give the numerical coefficients of the following terms:

   (a) $4xy$
   (b) $-2x^3y^4$
   (c) wlm
   (d) $10ab/17$
   (e) $-xyz/2$
   (f) $-b/3c$
   (g) $+0.1x$
   (h) pq
   (I) jk^2
   (j) n

3. Give the literal coefficients for the terms in number 2.

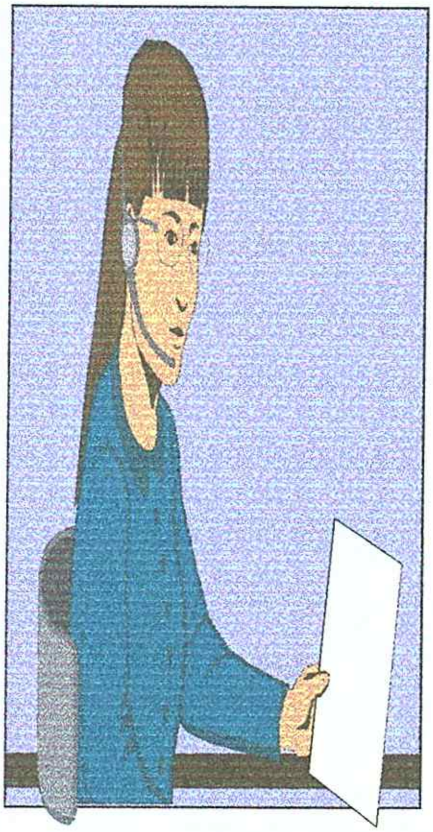
Module 1
Lesson 8

Algebraic Expression 1

(Let us start at the beginning)

What are constants? What are variables?

Can you tell a variable from a constant in a given term or in a given expression?



You'll hit your target if you read Lesson 7 of Module 1 even if blindfolded, you'll be able to identify them.

Objective:

At the end of this lesson you should be able to identify constants and variables from given expressions.

Input

What are constants?

Constants are symbols, usually **numbers** or **letters**, that have fixed or specified value. All numbers are constants since they have fixed value (not changing). The beginning letters of the alphabet are usually used as **constants**.

In the polynomial expression $\mathbf{at^2 + bt + c}$, the letters **a, b,** and **c** are constants.

We also have some known nature's constants such as the speed of light which is usually denoted by the letter "**c**" the gravitational constant denoted by "**g**" the charge of an electron denoted by "**q**" etc.

What other symbols represent constants?

In mathematics we use several symbols in solving problems such as π, ∞, e, ι, μ, and others. These symbols are constants since they represent real numbers.

Are these symbols constants? variables?

A, l, w, C, v, s, d, I, p, r, t, V are used in the solution of science and mathematics problems. These symbols represent numbers but we don't know the exact value since its value will defend on the value of other numbers and symbols, hence are called variables.

Examples: $I = prt$, $C = \pi d$, $V = lwh$, and etc.

Variables are symbols that represent arbitrary (not fixed) element of a set. Also these are symbols that may change their values in a particular problem or discussion.

In algebra we usually use the last letters of the English alphabet as variables. In the algebraic expression $at^2 +bt +c$ the variable is the letter "**t**" and the constants are a, b, and c. In the expression $2x^2 + 3x + 4$, the variable is x, the constants are the numbers, 2, 3, and 4.

Example: Constants and Variables

(a) 2xy Constant - 2; Variables - x and y

(b) c Constant - 1; Variable - c

(c) 2lw Constant - 2; Variable - l and w

(d) πr^2 Constant - 2, π; Variable - r

Let us test your understanding of constants and variables by answering the exercise next page. Okey?

Test Your Understanding

Identify the constants and the variables in the given examples.

1. x
2. xy
3. $4a$
4. at
5. r^2
6. prt
7. $2(l + w)$
8. s/t
9. xyz
10. p/q
11. $KE = mv^2/2$
12. $V = lwh$
13. $I = prt$
14. $d = 2r$
15. $C = \pi d$
16. $A = s^2$
17. $A = a + b + c$
18. $P = F/A$
19. $F = GM_1M_2/d^2$
20. $V = s^3$

Module 1

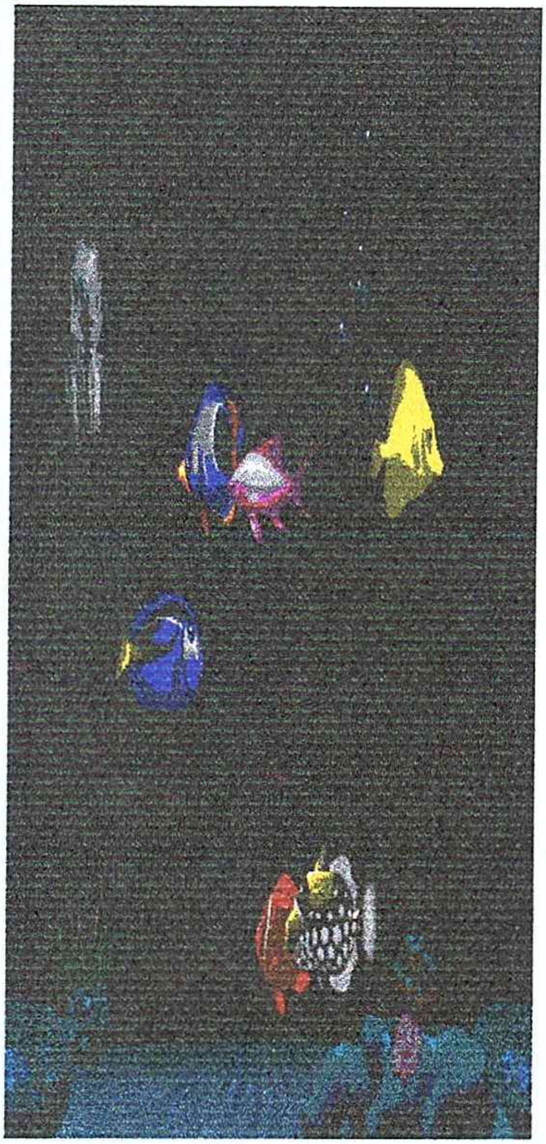
Lesson 9

Algebraic Expression 1

(Let us start at the beginning)

What are monomials?

What is the degree of a given monomial?



Objective:

At the end of this lesson you should be able to identify monomials, binomials, trinomials, multinomials, integral rational terms, polynomials, and real polynomials.

Input

What are monomials?

A **monomial** is either a number or the product of a number and one or more variables with whole number exponents.

5, 2a, $3x^2y^3$ are monomials since all the exponents of the variables involved are **non-negative** integers.

$x^2y^3/2$, $3xyz/4$, 5, abc are examples of monomials. The variables involved in the given expressions have whole number exponents or the exponents of the variables are non -negative integers.

$x^{1/2}y^{1/2}$, x^2/y^2 and $1/x$ are not monomials. In $x^{1/2}y^{1/2}$, the variables x and y have **fractional exponents,** hence is not a monomials. Also, the expression x^2/y^2 can be written as x^2y^{-2}, hence the variable y has **negative exponent** · so is not a monomial. $1/x$ can be written as x^{-1}, hence the variable x has **negative exponent,** therefore, it is not a monomial.

Example: Monomials

(a) $3x^2y^2z/5$
(b) $1/5\ xy^3z^4$
(c) at^2
(d) $pt/\sqrt{25}$
(e) $xyz/9$

RECALL:

1. **The set of non-negative integers is not the same set as the set of positive integers.**
2. **The set of non-negative integers contains the positive integers and zero (set of whole numbers).**
3. **Variables should not appear in the denominator, since the exponent of the variables will be negative.**
4. **The variables should not be within the radical expression since the exponent of the variables will be fractional.**

What is the degree of a monomial?

The **degree of a monomial** is the sum of the exponents of its variables.

If the term has only one variable the degree of the term (monomial) is the power of the variable.

The degree of $3x^4/4$ is 4. The degree of x is 1, since $x = x^1$.

The degree of $4a^2b^3c$ is 6. Since the variables present in the term are a, b, and c and the exponents of the variables are 2, 3, and 1 respectively.

The degree of all non - zero constants such as 12, -1, 1/2, 100, etc. is zero (0). Since, all non-zero constants can be written as the number times any variable raised to the zero power.

The number zero has no degree.

Example: Degree of Monomials

(a)	$-4x^4y^2z^2$	Degree 8
(b)	$-pqrs$	Degree 4
(c)	$abcd^2$	Degree 5
(d)	$3xy/4$	Degree 2
(e)	$x/3$	Degree 1
(f)	$3c/4$	Degree 1
(g)	$-4z^5$	Degree 5
(h)	a^2b^3cdx	Degree 8
(I)	x^5y^3	Degree 8

Test Your Understanding 1.8

Identify if the given term is a monomial and give its degree.

1. $a^2b/4$
2. $5/xyz$
3. $3a^2b/c$
4. $3a^{1/2}bc^{1/2}$
5. $a^{4/2}bc$
6. $2^{-2}abc$
7. $3^{2/3}xyz$
8. $2/5\ xy^3$
9. $1/4^{1/4}$
10. x/y
11. $2/xy$
12. 2^xy
13. $1/x^2$
14. $-x^{-2}y$
15. 28

Module 1

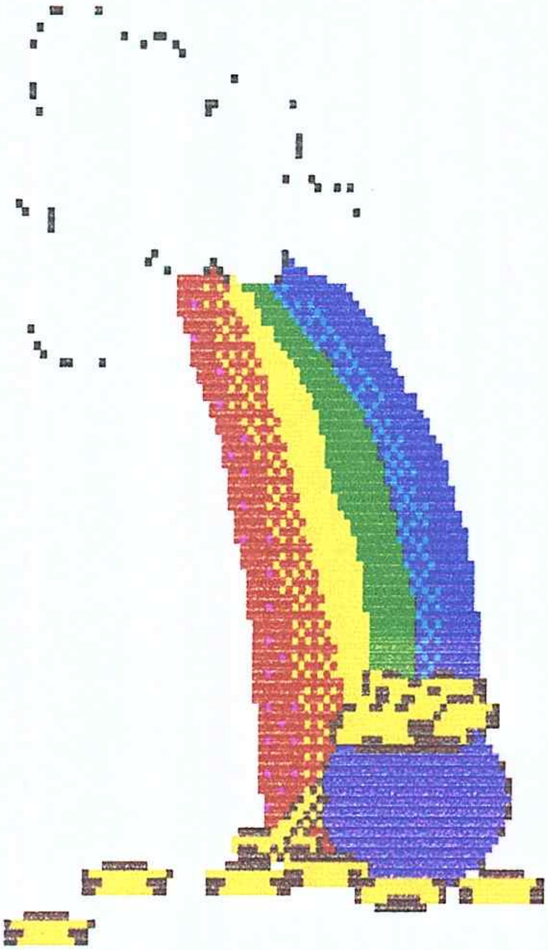
Lesson 10

Algebraic Expression 1

(Let us start at the beginning)

What are integral rational terms?

Is there a pot of gold at the end of a rainbow?



If you can identify integral rational terms from a set of terms and expressions

Now, that is an achievement.

.... like having your pot of gold

Objective:

At the end of this lesson you should be able to identify integral rational terms.

Input

What are integral rational terms?

An **integral rational** term is a term whose factors are **real numbers** and **non-negative integral powers of variables.**

$3x^2y^3$ is an example of an integral rational term. Some examples of integral rational terms are $1/4abc$, $4xyz$, at^2y, $-2x^2y^3$.

The following terms are **not** integral rational.

$\sqrt{-2}\,x^2y$ is **not** an integral rational term since the factors of $\sqrt{-2}$ are not real numbers even if the exponent of the variables are whole numbers.

$3x^2y^{1/3}$ is **not** an integral rational term since the variable y has fractional exponent.

Example: Integral Rational Terms

(a) $1/4\ xyz$
(b) $\sqrt{9}abc^{4/2}$
(c) $-3/5\,x^3y^4$
(d) $-yz/5$
(e) $5pqr/\sqrt{81}$
(f) $abc/6$
(g) $4rt/8$
(h) $-3wxy/8$
(I) x
(j) $4ijk$

Test Your Understanding 1.9

Identify the integral rational terms.

1. $3x/y$
2. $-2y^2$
3. $2xy/x$
4. $6yz^2/7$
5. $\sqrt{}\, xyz$
6. $2\sqrt{25}\ xyz$
7. $2xy/z$
8. $xy/7z$
9. $6x^2y^2z$
10. $3ab/4c$
11. $xy / 6x^2y$
12. $4x^3y^2 / z^4$
13. $x^{1/2}y^{-4}$
14. ab/cd
15. $x/2^2$

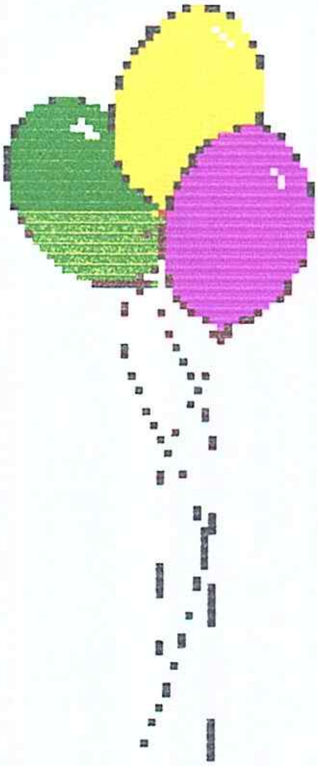
Module 1
Lesson 11

Algebraic Expression 1

(Let us start at the beginning)

What are polynomials and their degrees?

Do you know why balloons goes up when you release it?



An object that goes up is lighter than air.

If you want to raise the level of your achievement in mathematics...

like the balloons ...

be able to identify, described, & define polynomials and their degree.

Objective:

At the end of this lesson you should be able to define polynomials and give their degree.

Input

An algebraic expression which is expressed as a **sum of monomials** is called a **polynomial.**

Monomials are **polynomials** consisting of a single term. A polynomial is any monomial or multinomial in which each term is integral rational.

$a + b + c$ and $3x^3y^2$ are polynomials. Some examples of polynomials are: $at^2 + bt + c$, $a^0x^n + a^1x^{n-1} + a^2x^{n-2} + a^mx^0$, $z^2 - z + 4$ and etc.

A polynomial may be formed from single variable such as the following examples:

$\mathbf{x^4 - 3x^3 + x}$ is a polynomial in **x.** It is an algebraic expression having only terms of the form ax^n, where a is any real number and n is a whole number.

$\mathbf{y^2 + 3}$ is a polynomial in **y.** It is an algebraic expression having only terms of the form ay^n, where a is any real number and n is a whole number.

$\mathbf{2t^3 + 5t - t + 2}$ is a polynomial in **t.** It is an algebraic expression having only terms of the form at^n, where a is any real number and n is a whole number.

A polynomial may contain more than one variables, such as the following:

$\mathbf{x^2 + 2xy + y^2}$ is a polynomial in the variables x and y.

$\mathbf{3x^2y + 3xy^4}$ is a polynomial in x and y because each variable has positive integer as exponent.

Some examples of polynomials in two variables are: $a^2 + ab + b^2$, $z^2 + z + y$, $xy + y^2$, $x^2y/3$, $3m + n$ and $p^2 + q$.

Polynomials maybe formed from more than two variables. Some examples of polynomials with more than two variables are: abc, 3xyz, $4a^2b^3c^4d$, $2x^2y^2z^2$, $m^3 + 3m^2np + 3mn^2 + np$, zywx and others.

What are not polynomials?

Let us look at some examples.

$\mathbf{3x/y + 2\sqrt{x}}$ is **not** a polynomial for two reasons:

$3x/y$ --- The first term is **not integral rational because of y in the denominator.**

$2\sqrt{x}$ --- The second term is not integral rational because $\sqrt{x}$ is not a non-negative integral power of x.

$x^{1/3} + x^{1/2}$ and $x^{-2} + 5x + 1$ are not polynomials because of the fractional and negative exponents of the variables involved.

Now look!

Are the given algebraic expressions polynomials?

1. $\frac{2x + 1}{3x^2 - 5x + 7}$

2. $3^x - x^3 - 2$

3. $x + \sqrt{x}$

4. $|2x^3 - 5|$

Reminders:

In a polynomial, variables should NOT appear

1. in the denominator
2. as an exponent
3. within a radical and
4. within absolute –value bars.

A polynomial in x is an algebraic expression having only terms of the form ax^n, where a is any real number and n is a whole number.

Note: Because every term of a polynomial in x has the form ax^n (n a whole number), polynomials never have variables in the denominator.

1. $\frac{2x + 1}{3x^2 - 5x + 7}$

We can write the given expression as $(2x + 1) \left[\frac{1}{3x^2 - 5x + 7} \right]$

$(2x + 1)(3x^2 - 5x + 7)^{-1}$

$(2x + 1)(3^{-1}x^{-2} - 5^{-1}x^{-1} + 7^{-1})$

Thus we have variables with negative exponents.

2. A variable must not be treated as exponent.
   Example: $3^x - x^3 - 2$

If a variable appears in the exponent then the factors of the term may not all be real numbers. So, each term of the polynomial will not be integral rational.

3. A variable should not be placed within a radical.
   Example: $x + \sqrt{x}$

   We can write the given expression $x + \sqrt{x}$ as $x + x^{1/2}$. Here we have a variable with fractional exponent.

4. A variable cannot appear within absolute value bars.
   Example: $|2x^3 - 5|$

   If a variable appears within absolute –value bars for two different values of the variable the numerical value of the expression will be equal.
   Also, from our definition of terms, in a polynomial it must be a product of a real number and variable with whole number exponent. We cannot express the expression within absolute – value bars as product of such factors.

Suppose we want to determine the number of terms in a polynomial, What are we going to do?

Recall:

In a polynomial, each of the monomials in the sum is called the terms of the polynomial.

A polynomial may contain only one term such as:

$$4,\ 3,\ x^2,\ x^2yz,\ \tfrac{1}{2}abc, \text{ and } x^2y^3z^4/8$$

A polynomial may contain two terms as in the following examples:

$$a+b,\ x^2 - y^2,\ x-2,\ y+6,\ 3xyz + 24$$

A polynomial may be made of more than two terms such as:

$at^2 + bt + c$, $a + b - c$, $x^2 + 2xy + y^2$, $7w + 3x - 2y + 6z$, $3xy + 6xz - 4wx + 9yz - 3xyz + 24$ and others.

Example 8: Examples of Polynomials:

(a) $3x$ A polynomial of one term is called a **monomial**.

(b) $4x^4 - 3x^3$ A polynomial of two unlike terms is called a **binomial**.

(c) $7x^2 + 4x - 8$ A polynomial of three unlike terms is called a **trinomial**.

Is it possible for a polynomial to have 100 terms?

For your answer, recall that a polynomial is a finite sum of monomials.

The answer is very easy if you understand the meaning of "finite".

What are the names given to some polynomials?

Monomials one term polynomials

Binomials two terms polynomials

Trinomials three terms polynomials

Multinomials are polynomials with more than three terms. Examples; $2a - b - 5c - d + c$, $2x + 3z + d - 7$, $-x + y + 3z - w + 3$

A polynomial with only one term is a monomial. Examples: 3, abc, $\frac{1}{2}xyz$

A polynomial with with two terms is a binomial. Examples: $a + b$, $x - y$, $3x + 4$

A polynomial with three terms is a trinomial. Examples: $a + b - c$, $2x + yz - 6$

What is the degree of the polynomial?

Recall: The degree of a polynomial is the degree of the term in the polynomial with the highest degree.

Examples:

$3x^2y^3 + 5xy^2 + 7$ is of degree 5.
$3xy + 5x^2y + 8$ is of degree 3.
$3x + 5y - 2z$ is of degree 1
7 is of degree 0.

A nonzero constant like 7 is classified as a polynomial of degree zero, since $7 = 7x^0$.

The number zero, as a term or as a polynomial, is not given a degree.

A **real polynomial** is a polynomial in which all the numerical coefficients are real numbers and the value of the variables are restricted to real numbers.

Examples: $-3x^3 + 2x + \sqrt{6}$ when $x = 2$ is a real polynomial.

The coefficients of x^3 is a real number. Also the variable x is given a value of 2 which is a real number.

Real polynomials represent real numbers when a real number is substituted for the variables. Hence, real polynomials can be added, subtracted and multiplied.

$\sqrt{-3}x - \sqrt{-49} - 6$ is not a real polynomial since the numerical coefficient of x is not a real number. Also $3 - \sqrt{-y}$ is not a real polynomial since the coefficient of y is not real.

Some examples of real polynomials are:

$at^2 + bt + c$ when $a = 2$, $b = 3$, $c = 4$, $t = -1$

$x^2 - y^2$ when $x = -5$, $y = 4$

$-5y^2 + y + 6$ when $y = -1$.

Let us test your understanding of polynomials, degree of a polynomial, and real polynomials by answering the test given next page.

Test Your Understanding 1.11

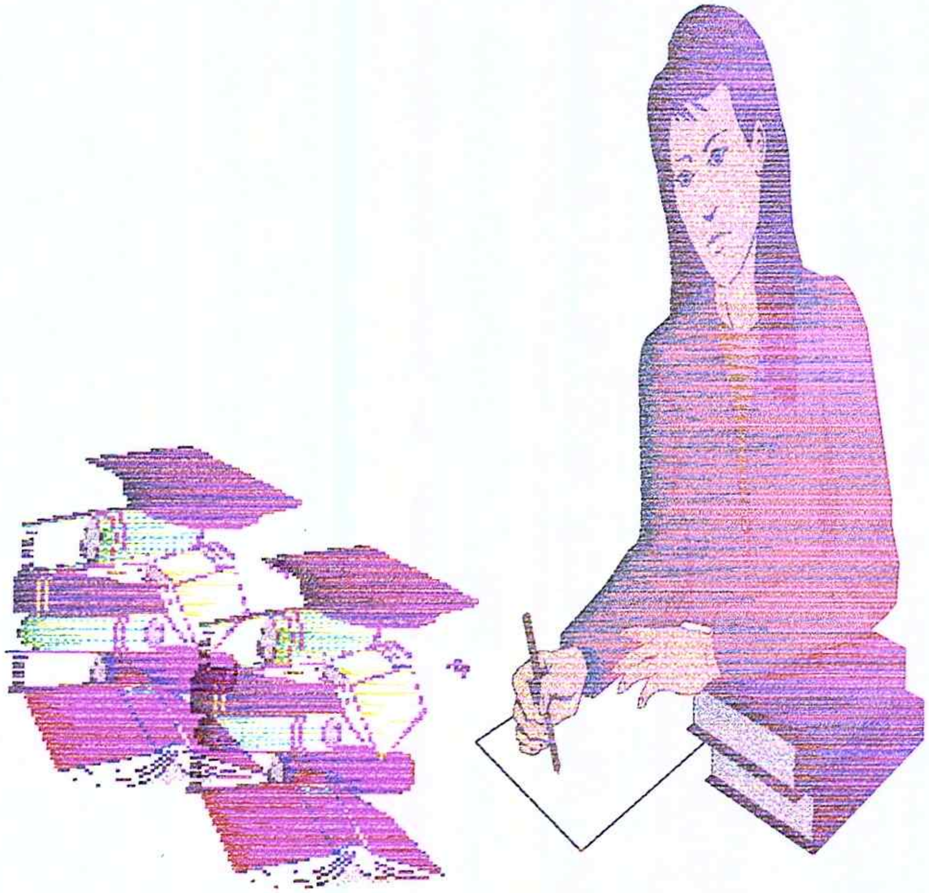
Identify if the given examples are polynomials, if it is give the degree.

1. $3x^2 y + 4xy + y$
2. $t - 2t^2$
3. $a/4 + b/c - 7$
4. $y + 7y + 10$
5. ½
6. $ab + bc + cd$
7. $\frac{a + b - c + d}{7}$
8. $8a/5 + 9b/2 + 2/5$
9. $\frac{7}{x + y - 3xy}$
10. $1/x^{-4} + y^x$
11. $x^{1/2} + y - 6$
12. $| x - 3y |$

Identify if the given polynomial is real.

13. $4xy - 5$ when $x = -1$, $y = 8$
14. $2x - 4y^2 + z$ when $x = y = z = 2$
15. $x^2 y/4 + xy/3$ when $x = 0$, $y = 8$.

CONGRATULATIONS



You have just finished ALGEBRAIC EXPRESSION I
(Let us start at the beginning).
You are now ready to take the PRACTICE TASK
next page.
Please check your answer at the
FEEDBACK TO THE PRACTICE TASK.
It is important.

Practice Task

1. In the expression 3xyz,
   (a) What is the numerical coefficient of the term?
   (b) What is the coefficient of 3?
   (c) What is the coefficient of xy?

2. In the expression 12abc,
   (a) What is the numerical coefficient of the term?
   (b) What is the coefficient of c?
   (c) What is the coefficient of ab?

In exercises 3-5 list (a) the constants (b) the variables.

3. $3x - 5y$
4. $\dfrac{5x - 2y^3}{4x^6}$
5. $2x - 4y + 4$

In exercises 6 – 8 (a) determine the number of terms; (b) write the second term if there is one.

6. $[x^2 - (x + y)]$
7. $5 - (x + y)$
8. $3x^2y + \dfrac{2x + y}{3xy} + 4(3x^2 - y)$

In exercises 9 – 12, write each polynomial in descending powers of the indicated letter.

9. $7x^3 - 4x - 5 + 8x^5$ power of x
10. $10 - 3y^5 + 4y^2 - 2y^3$ power of y
11. $3x^3y + x^4y^2 - 3xy^3$ power of x
12. $8xy^2 + xy^3 - 4x^2y$ power of y

13. The expression $2x + \dfrac{4x + 5y}{3} + 7$ has:
   (a) how many terms?
   (b) how many different variables?
   (c) how many different constants?

14. In the polynomial $x^2 - 4xy^2 + 5$ find:
   (a) the degree of the first term
   (b) the degree of the polynomial
   (c) the numerical coefficient of the second term.

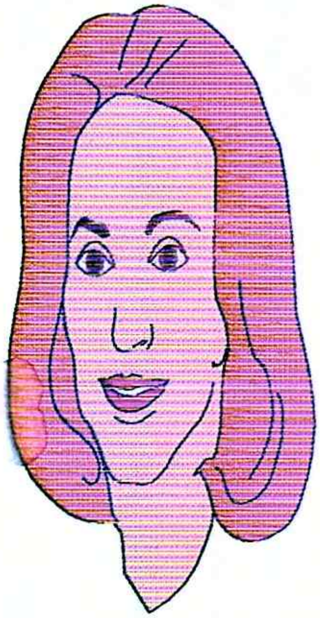
In problems 15 – 17 find the value of each expression when $a = -2$, $b = 4$, $c = -3$, $x = 5$, and $y = -6$.

15. $3a + bx - cy$
16. $4x - [a - (3c - b)]$
17. $[c/y]^2 + [x/b]^2$

In problems 18 – 20 evaluate each expression

18. $17 - 9 - 6 + 11$
19. $(2) \bullet 3^2 - 4$
20. $48 + (-4)^2 - 3[10 - 4]$

You must score 16
or higher of the
PRACTICE TASK.
If you score 10 or less,
please go over
MODULE 1 again.
It is important.



Answers to **Test Your Understanding** and **Test Your Computational Skills** are also provided on separate sheets placed after the Feedback to the Practice Task.

Please check your answers. Okey?

Feedback to the Practice Task

1. (a) 3
   (b) xyz
   (c) 3z

2. (a) 12
   (b) 12ab
   (c) 12c

3. (a) 3, − 5
   (b) x, y

4. (a) -2, 3, 4, 5, 6
   (b) x, y

5. (a) -4, 2, 4
   (b) x, y

6. (a) 1
   (b) No second term

7. (a) 2
   (b) $-(x+y)$

8. (a) 3
   (b) $\frac{2x+y}{3xy}$

9. $+8x^5 \ +7x^3 -4x \ -5$

10. $-3y^5 \ -2y^3 \ +4y^2 \ +10$

11. $+x^4y^2 \ +3x^3y \ -3xy^3$

12. $+xy^3 \ +8xy^2 -4x^2y$

13. (a) 3
    (b) 2
    (c) 5

14. (a) 2
    (b) 3
    (c) -4

15. −4
16. 9
17. 29/16
18. 13
19. 14
20. 46

Test Your Computational Skills 1.1

1. 29
2. –87
3. 248
4. –125
5. 4/3
6. 11
7. 1681
8. 2047
9. 15,625
10. 7/30
11. –10
12. –1023
13. 14641
14. 625
15. –3/4

Test Your Understanding 1.1

1. Always
2. Never
3. Always
4. Always
5. Sometimes
6. Always
7. Sometimes
8. Always
9. Never
10. Sometimes
11. Always
12. Always
13. Always
14. Never
15. Sometimes

Test Your Understanding 1.2

1. 3, 2b
2. 4, (2 – y)
3. 2, 7
4. 3, -6t
5. 1
6. 6, (z –6y +x)
7. 3, y^3
8. 4, -9y
9. 4, t^2
10. 3, -3/8

Test Your Understanding 1.3

1. a, b, 1/c, 1/d
2. x^4, y, z
3. 2, x, y, z^3
4. a^n, b
5. –3, x^4
6. -3^4, x^4
7. x
8. $(a + b)^2$
9. (a + b) (a – b)
10. ½, x

Test Your Understanding 1.4

1. x - base, 1 – exponent
2. 4 - base, m - exponent
3. z - base, 4 – exponent
4. (-2xyz) - base, -2 - exponent
5. 1.2 - base, 4 – exponent

Test Your Understanding 1.5

1. 2ac, 3ac, 1/2ac
2. 2xyz, 9xyz, 5xyz
3. $4x^3y$, $-3yx^3$
4. 2a/b, -3a/4b
5. 2(x –2y), -4(2y –x)

Test Your Understanding 1.6

1. $3y$, $-4y^2$, $5y^3$
2. $-6x^3y^2$, $1.5xy$, $2xy^3$
3. $2a$, $-3a^2$, $9b$
4. ac/bcd, $2ac/bd$
5. $3xyz$

Test Your Understanding 1.7

1. The Coefficients of 2abc are
   2 is the coefficient of abc
   a is the coefficient of 2bc
   b is the coefficient of 2ac
   c is the coefficient of 2ab
   2a is the coefficient of bc
   ab is the coefficient of 2c
   2c is the coefficient of ab
   bc is the coefficient of 2a

2. The numerical coefficient of
   $4xy$ is 4
   $-2x^2y$ is -2
   wlm is 1
   $10ab/11$ is $10/11$
   $-xyz/7$ is $-1/7$

3. The literal coefficient of
   $4xy$ is xy
   $-2x^2y$ is x^2y
   wlm is wlm
   $10ab/11$ is ab
   $-xyz/7$ is xyz

Test Your Understanding 1.8

1. constant -2; variables x, y
2. constant 1; variable c
3. constant 1/3; variables a, b, c
4. constant 1.2; variables a, t
5. constant 11, 2; variables A, r

Test Your Understanding 1.9

1. monomial, degree 3
2. not monomial
3. monomial, degree 5
4. not monomial
5. monomial, degree 4
6. not monomial
7. monomial, degree 3
8. monomial, degree 3
9. monomial, degree 0
10. not monomial

Test Your Understanding 1.10

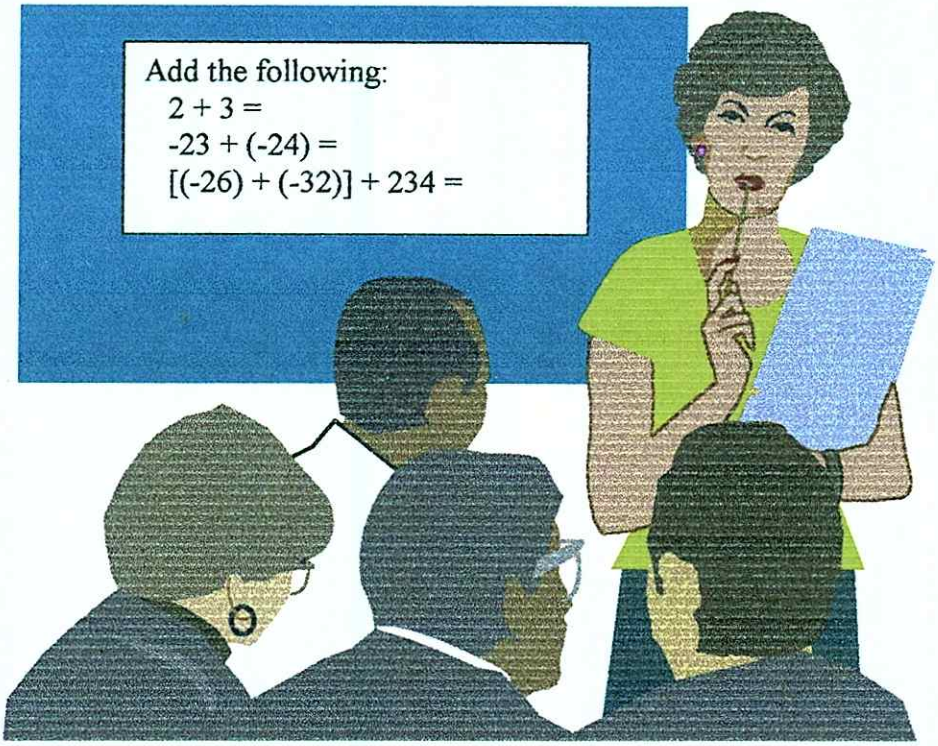
1. not
2. integral rational term
3. integral rational term
4. integral rational term
5. not

Test Your Understanding 1.11

1. polynomial; degree 3
2. polynomial; degree 2
3. not
4. polynomial; degree 2
5. polynomial; degree 0
6. polynomial; degree 1
7. polynomial; degree 1
8. polynomial; degree 1
9. not
10. not
11. not
12. not
13. real polynomial
14. real polynomial
15. real polynomial

Module 2
Lesson 1

Addition of Integers



Add the following:
2 + 3 =
-23 + (-24) =
[(-26) + (-32)] + 234 =

How do we add integers?

Overview:

Algebraic expressions represent real numbers (specific or unspecific). Hence, axioms and theorems about certain properties of real numbers are possessed by algebraic expressions.

Operations like addition, subtraction, multiplication and division can also be performed using algebraic expressions.

Addition of algebraic expressions involves combining terms. The process of combining similar terms is simplified by using the generalized distributive law which states:

$$ab_1 + ab_2 + ab_3 + \ldots + ab_n = a(b_1 + b_2 + b_3 + \ldots + b_n)$$

The sum of dissimilar terms can only be indicated. This means that there is no single term to represent the sum. The sum is obtained by copying the terms to be added and affixing a plus sign to show that the terms are added together.

In case of addition of algebraic expressions consisting of more than one term, such as a polynomial to another polynomial, the process involves addition of similar terms and dissimilar terms and the application of the commutative and associative laws of real numbers.

Just like when we are adding numbers the sum which is the result in addition can be checked by another operation... subtraction. In case of algebraic expressions we can use numerical substitution or doing the sums again.

Unlike raising a number to a certain power or extracting the root of a number addition is a binary operation, it involves two elements of a set.

The commutative and associative property of real numbers and the use of signs of grouping like parenthesis comes in handy when simplifying and clarifying solutions to addition problems.

Let us give ourselves the chance to enjoy doing sums and knowing that it is the correct sums.

Objectives:

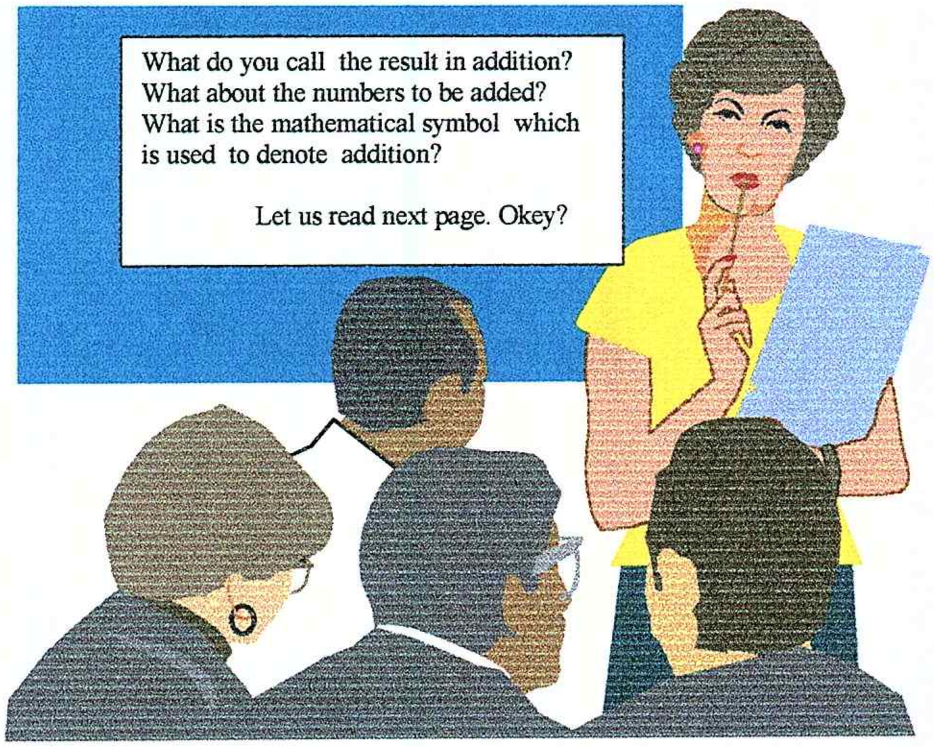
At the end of this lesson the students should be able to:

1. add similar terms using the generalized distributive law.
2. add dissimilar terms by indicating the sum.
3. add polynomials by writing similar terms in the same vertical column.
4. check if the answer in addition is correct by adding the terms again.
5. check if the sum obtained is correct by numerical substitution.
6. apply the commutative, associative and distributive property of real numbers to algebraic expressions.

Input!

Addition of Integers

A very small child learns addition when he started to put together blocks during playtime. It is easy to add and you will agree with me but first let us have a review of some facts involved in the addition process.



The review lessons are presented in question and answer form. Please ask your teacher if some part of it is not very clear to you. It is very important.

Let us have a short review. Okey?

What are integers or signed numbers?

Integers or signed numbers are numbers used to represent opposite ideas. For example going up motion may be represented by positive numbers and going down motion maybe represented by negative numbers. Also a profit of P20.00 maybe represented by +20 and a loss of P100.00 by –100.

The set of integers consist of the negative numbers, zero, and the positive numbers.

In the number line we can show the members of the set of integers by assigning a point corresponding to zero and +1. Once we have this point we can lay off the rest of the members of the set on the line like this below:

<————————————————————————————>

-6 -5 -4 -3 -2 -1 0 1 2 4 5 6

From the correspondence set up between the points on the line and the integers, we can conclude that the set of integers is an infinite set.

What is the absolute value of a number?

The absolute value of a number is the number regardless of the sign. The concept of absolute value of a number is the distance of the number from the origin. Hence it is always positive or zero. It is usually denoted by the symbol | | called the absolute value bars.

The absolute value of a number a is

$|a| = a$ if a is positive
$|a| = -a$ if a is negative
$|a| = 0$ if a is zero.

What do you mean when we say "two numbers have like signs"?

Consider two cases:
Case 1. Two numbers have like signs when both are positive.

Examples: +9 and +6, +3 and +5, +1 and +100, and others.

Case 2. Two numbers have like signs when both are negative.

Examples: –2 and –76, –1 and –4, –21 and –310, –10087 and – 10985, etc.

When are two numbers considered to be having unlike signs?

Two numbers have unlike signs when one of the numbers is a positive number and the other number is negative.

Examples: –2 and +6, +19 and –74, –276 and +313, +1988 and – 13985, etc.

What do you call the terms to be added?

The terms or numbers to be added are called **addends.**

Examples: 2 + 23 = 25, **2** and **23** are the **addends.**
-4 + -47 = -51, **-4** and **–47** are the **addends**.

What is the sum?

The result in addition is called the sum.

Examples: 21 + 33 = 54, **54** is the **sum.**

What do we mean when we say "find the difference"?

To find the difference between two quantities is to subtract the smaller quantity from the bigger quantity.

Examples:
1. Find the difference between 7 and 2.
   7 – 2 = 5 , **5** is the difference between the given numbers.

2. Find the difference between 1 and 9.
   Since 1 is the smaller of the two quantities, so we will subtract 1 from 9. 9 – 1 = 8, Hence the difference is **8.**

What is to prefix the sign?

When we say "prefix the sign" what we mean is we will copy the sign and place or write it before the number.

What do we mean when we say the sign is common to the given numbers?

When we say that the sign is common to the given numbers this means that all the numbers possess the same sign. Either they are all positive or all negative.

What is addition?

Addition is an operation wherein we combine the elements of one set to the elements of another set forming a much bigger set.

What is to "do the sums again"?

"Do the sum again" means the same as to add the elements again may be this time adding the columns starting from the bottom instead of the top.

What is "numerical substitution"?

Numerical substitution is a process by which we substitute a constant to the variable in the expression.

What do we mean when we say "the sum can only be indicated"?

To indicate the sum of an addition process is to show that we have to perform the addition operation and that it is the result of the process.

How do we add numbers having like signs?

Rule:

To add two numbers having like signs, add the absolute value of the numbers and prefix the common sign to the sum.

Consider two cases.

Case 1. Both numbers are positive.

Example 1: Add + 4 and +8.

$$\begin{array}{lllcccll} \text{Solution:} & & +4 & — & |\mathbf{+4}| & = & \mathbf{4} & \\ & + & & & & + & & \\ & & +8 & — & |\mathbf{+8}| & = & \mathbf{8} & \\ & & & & & & \overline{\mathbf{12}} & \text{answer} \end{array}$$

Example 2: Add + 20 and +98.

$$\begin{array}{lllcccll} \text{Solution:} & & +20 & — & |\mathbf{+20}| & = & \mathbf{20} & \\ & + & & & & + & & \\ & & +98 & — & |\mathbf{+98}| & = & \mathbf{98} & \\ & & & & & & \overline{\mathbf{118}} & \text{answer} \end{array}$$

Case 2. Both numbers are negative.

Example 1: Add – 7 and – 3.

$$\begin{array}{lllcccll} \text{Solution:} & & -7 & — & |\mathbf{-7}| & = & \mathbf{7} & \\ & + & & & & + & & \\ & & -3 & — & |\mathbf{-3}| & = & \mathbf{3} & \\ & & & & & & \mathbf{-\overline{10}} & \text{answer} \end{array}$$

Example 2: Add – 12 and – 45.

$$\begin{array}{lllcccll} \text{Solution:} & & -12 & — & |\mathbf{-12}| & = & \mathbf{12} & \\ & + & & & & + & & \\ & & -45 & — & |\mathbf{-45}| & = & \mathbf{45} & \\ & & & & & & \mathbf{-\overline{57}} & \text{answer} \end{array}$$

How do we add numbers having unlike signs?

Rule:

To add two numbers having unlike signs, find the difference of their absolute values and prefix the sign of the addend with the greater absolute value to the sum.

Example 1: Add –4 and +5.

Solution:

$$\begin{array}{lllllll} & -4 & — & |-4| & = & 4 & 5 \\ + & & & & & - & - \\ & +5 & — & |+5| & = & 5 & 4 \\ & & & & & & \overline{\mathbf{1}} \quad \text{answer} \end{array}$$

Example 2: Add –100 and +58.

Solution:

$$\begin{array}{llllll} & -100 & — & |\mathbf{-100}| & = & \mathbf{100} \\ + & & & & & - \\ & +58 & — & |\mathbf{+58}| & = & \mathbf{58} \\ & & & & & \overline{\mathbf{-42}} \quad \text{answer} \end{array}$$

Example 3: Add – 78 and +90.

Solution:

$$\begin{array}{lllllll} & -78 & — & |\mathbf{-78}| & = & \mathbf{78} & \mathbf{90} \\ + & & & & & - & - \\ & +90 & — & |\mathbf{+90}| & = & \mathbf{90} & \mathbf{78} \\ & & & & & & \overline{\mathbf{12}} \quad \text{answer} \end{array}$$

Example 4: Add +45 and – 30.

Solution:

$$\begin{array}{llllll} & +45 & — & |+45| & = & \mathbf{45} \\ + & & & & & - \\ & -30 & — & |\mathbf{-30}| & = & \mathbf{30} \\ & & & & & \overline{\mathbf{+15}} \quad \text{answer} \end{array}$$

Example 5: Add $+23$ and -67.

Solution: $+23$ — $|+23|$ = **23** **67**

$+$ — —

-67 — $|-67|$ = **67** **23**

$-\overline{44}$ answer

How about if there are more than two integers to be added? How do we add these integers?

Rule:

Add algebraically all the positive integers. Separately, add all the negative integers then add the two sums obtained algebraically to obtain the final answer.

Example 1. Add +4, +5, —7 and —3.

Solution: Add all the positive numbers.

Add + 4 and +5.

$+4$ — $|+4|$ = **4**

$+$ $+$

$+5$ — $|+5|$ = **5**

$\overline{9}$ sum of the positive numbers

Add all the negative numbers.

Add -7 and -3.

-7 — $|-7|$ = **7**

$+$ $+$

-3 — $|-3|$ = **3**

$-\overline{10}$ sum of the negative numbers

Add the sum of the positive numbers and the negative numbers.

Add +9 and -10.

$$\begin{array}{llcllcrr}
 & +9 & — & |+9| & = & 9 & & 10 \\
+ & & & & & - & & - \\
 & -10 & — & |-10| & = & 10 & & 9 \\
 & & & & & & - & \overline{1}
\end{array}$$

answer

Example 2. Add +34, +67, +90 and —789.

Solution: Since there is only one negative number so we have to add only the positive numbers.

Add + 34 and +67 and 90.

Applying the associative law we can add +34 and +67 first then their sum is again added to +90.

Add +34 and +67 then

$$\begin{array}{llcllc}
 & +34 & — & |+34| & = & 34 \\
+ & & & & & + \\
 & +67 & — & |+67| & = & 67
\end{array}$$

$$\begin{array}{llcllc}
 & +\overline{101} & — & |+191| & = & 101 \\
+ & & & & & \\
 & +90 & — & |+90| & = & 90 \\
\hline
 & & & & & +191
\end{array}$$

sum of the positive numbers

Add the sum of the positive numbers and the only negative number.

Add +281 and – 789.

$$\begin{array}{llcllcrr}
 & +191 & — & |+191| & = & 191 & & 789 \\
+ & & & & & - & & - \\
 & -789 & — & |-789| & = & 789 & & 191 \\
 & & & & & & - & \overline{598}
\end{array}$$

answer

Example 3. Add +248, —6, 1007, —31, 0.8 and —2.04.

Solution:
1. Add all the positive numbers. No sign means positive.
2. Add all the negative numbers.
3. Find the difference of their sums.
4. Prefix either a + or — sign to the difference depending on the sign of the addend with a greater absolute value.

+ 248.0	-6 .00		1255.80	
1007.0	-31.00	–		
0.8	- 2.04		39.04	
1255.8	-39.04		1216.76	answer

Example 4. Add +628, —3, 2450, 0.9, —6.51 and -3.01.

Solution:
1. Add all the positive numbers. No sign means positive.
2. Add all the negative numbers.
3. Find the difference of their sums.
4. Prefix either a + or — sign to the difference depending on the sign of the addend with a greater absolute value.

+628.0	-3.00		30 278.9	
2450.0	-6.51	–		
0.9	- 3.01		12.52	
30278.9	-12.52		30266.28	answer

Example 5. Add +142, —5, 2113, —43, 0.10 and —1.12.

Solution:
1. Add all the positive numbers. No sign means positive.
2. Add all the negative numbers.
3. Find the difference of their sums.
4. Prefix either a + or — sign to the difference depending on the sign of the addend with a greater absolute value.

$$\begin{array}{r} +\ 142.00 \\ 2113.00 \\ 0.10 \\ \hline 2255.10 \end{array} \qquad \begin{array}{r} -5\ .00 \\ -41.00 \\ -\ 1.12 \\ \hline -47.12 \end{array} \qquad \begin{array}{r} 2255.10 \\ -\quad 47.12 \\ \hline 2302.22 \end{array} \text{ answer}$$

Let us test your computational skills in finding the sum of signed numbers or integers.

TEST YOUR COMPUTATIONAL SKILLS 2.1

Add the given numbers.

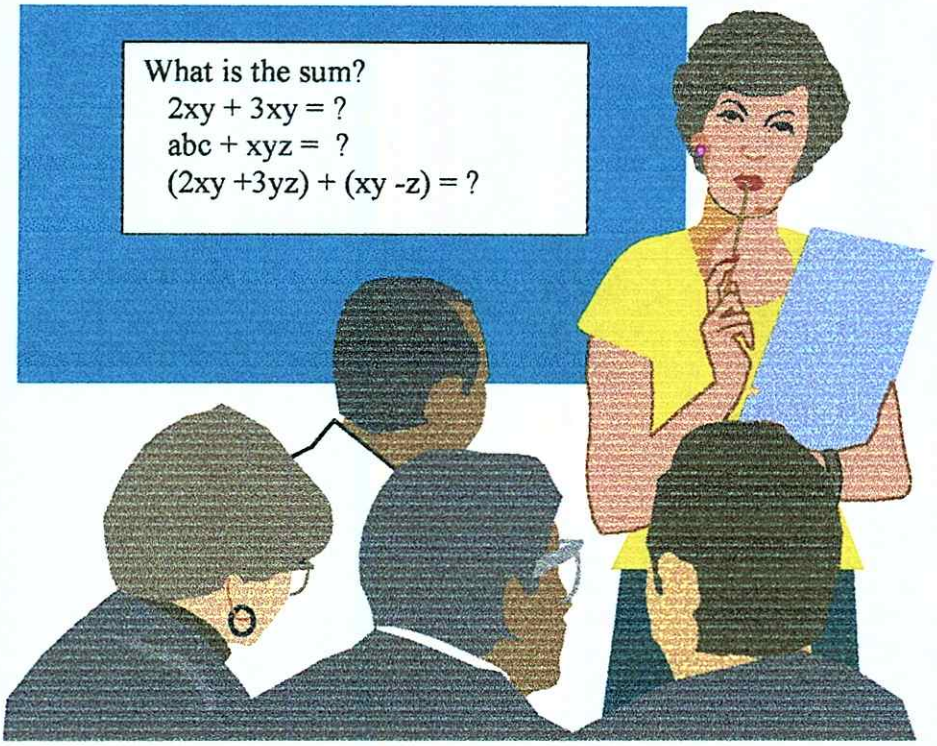
1. +20 and +68
2. 89 and 65
3. –60 and -54
4. –5908 and -34568
5. –345067 and 9876
6. +456 and -67
7. –765 and 676
8. –45 and 654
9. 2345 and -0987
10. +328 and -6754
11. +40, -3, -101, +25, +8
12. +153, -200, -2, +45, 10
13. –100, +300, -7, -3, +40
14. 52, 32, 62, -5, -2, -101
15. 1040, -1040

You are now ready to add algebraic expressions.

Let us start with algebraic expressions which are similar terms. Okey?

Module 2
Lesson 2

Addition of Polynomials



What is the rule for addition of similar terms? dissimilar terms?
How do we add polynomials?

Input!

Addition of Polynomials

(Similar Terms)

Suppose 2x + 3x + 5x –8x –x are to be added , how are we going to do about it?

Since the terms to be added are like terms, so we are going to add only their numerical coefficients, like this

$$2 + 3 + 5 + (-8) + -1 = ?$$

$+ 3 + 5 = (2 + 3) + 5 = 5 + 5 = 10$
$(-8) + (-1) = -9$

Notice:

The coefficients are integers having unlike signs hence we will have to combine all the positive numbers and all negative numbers.

Adding the positive numbers result in +10 and adding all the negative numbers results in –9.

Notice:

Our work is now simpler since we are to add only two numbers +10 and –9. But +10 is +10x and –9 is –9x

$$+10x + (-9x) = +1x = x$$

Suppose, we want to check if our answer is correct, what are we going to do about it?

There are two ways of doing it.

1) by adding the terms again, and
2) by numerical substitution.

Let us try the first one . . . adding the terms again.

Look:

We are to add 2x + 3x + 5x + (-8x) + (-x). Suppose we rewrite the terms to be added like this

$$(-x) + (-8x) + 5x + 3x + 2x.$$

Will the answer obtained be different? What do you think? Now back to our problem at hand. Combine –x and –8x like this

$$-x -8x = (-1-8) = -9 = -9x.$$

Combine +5x; +3x and +2x like this

$$+5x +3x +2x = +5 +3 +2 = +10 = +10x.$$

Now combine the result of the positive numbers and the result obtained by adding all the negative numbers. We have –9x and 10x = -9x +10x = +1x or x.

Notice that we obtain the same answer by adding again.

What does it mean when by adding again we obtain the same answer? Are we sure that our answer is correct? What do you think?

I hope that you know the answer to the three questions asked and your answer is I'm sure that I got the correct answer (Bulls eye!) Okey?

Let us try the second method of checking the sum . . . by numerical substitution.

Add: $2x + 3x + 5x + (-8x) + (-x) = x$

Suppose x is substituted by the number 2.

$2x$ will become $2(2) = 4$
$3x$ will become $3(2) = 6$
$5x$ will become $5(2) = 10$
$-8x$ will become $-8(2) = -16$
$-x$ will become $-1(2) = -2$

Adding $4 + 6 + 10 + (-16) + (-2) = +2$

$4 + 6 + 10 = 20 \qquad -16 - 2 = -18 \qquad +20 - 18 = +2$

But what is +2? +2 is x, since we let $x = +2$. So by numerical substitution we know that our answer obtained is correct. Okey?

Let us look at another example.

Add: $2x^2y +4x^2y +5x^2y -2x +3y^2 -4 +8x -5 +3y^2$

We can group together like or similar terms as $(2x^2y +4x^2y +5x^2y) + (-2x +8x) + (3y^2 +3y^2) + [-4 +(-5)]$. Finding the sum of similar terms we have

$$\begin{aligned} (2x^2y +4x^2y +5x^2y) &= (2 +4 +5)x^2y &&= 11x^2y \\ (-2x +8x) &= (-2 +8)x &&= 6x \\ (3y^2 +3y^2) &= (3 + 3)y^2 &&= 6y^2 \\ [-4 +(-5)] &= (-4 -5) &&= -9 \end{aligned}$$

After combining like terms, we have $11x^2y +6x +6y^2 -9$ is the sum or answer.

Let us check if our answer is correct by adding again $2x^2y +4x^2y +5x^2y -2x +3y^2 -4 +8x -5 +3y^2$.

Suppose we start adding from the last term to the first term as in this example $3y^2 - 5 +8x -4 +3y^2 -2x +5x^2y +4x^2y + 2x^2y$

Grouping together similar terms we have ...

$(3y^2 +3y^2) + [-4 +(-5)] + (-2x +8x) + (2x^2y +4x^2y +5x^2y)$.

We can write similar terms in the same vertical column like this.

$$\begin{array}{r} 3y^2 \\ + \; 3y^2 \\ \hline 6y^2 \end{array} \qquad \begin{array}{r} -5 \\ + \; -4 \\ \hline -9 \end{array} \qquad \begin{array}{r} 8x \\ + \; -2x \\ \hline 6x \end{array} \qquad \begin{array}{r} 5x^2y \\ + \; 4x^2y \\ 2x^2y \\ \hline 11x^2y \end{array}$$

$$(5x^2y + 4x^2y) + 2x^2y =$$

$$9x^2y + 2x^2y = 11x^2y.$$

Note:
Addition is a binary opera tion , so we can operate in only two terms at a time like this

Is this result equal to the first result?

Notice that what we obtained was $11x^2y + 6y^2 - 6x - 9$. Are these two polynomials equal? What do you think?

Recall:
Algebraic expressions are specific or unspecific real numbers hence they possess the commutative and associative property of real numbers. Applying these properties we can show that the first result obtained is equal to the second result.

Look!

$11x^2y + 6y^2 + 6x - 9$

applying grouping $11x^2y + 6y^2 + 6x - 9$ will become

$(11x^2y + 6y^2) + (6x - 9)$

$(6y^2 + 11x^2y) + (-9 + 6x)$ by the commutative property of real nos.

$6y^2 + [11x^2y + (-9 + 6x)]$ regrouping

$6y^2 + [(11x^2y - 9) + 6x]$ associative property

$6y^2 + [(-9 + 11x^2y) + 6x]$ apply commutative property

$6y^2 - 9 + 11x^2y + 6x$ removal of grouping symbols

Another example.

Add $+5(x + 2y) - 2(x + 2y) + 2z - 6x - 8$

Combining and putting similar terms in the same column, we have

$$\begin{array}{l} +5(x+2y) \\ -2(x+2y) \\ \hline \;\;3(x+2y) \quad +2z \; -6x \; +8 = \end{array}$$

applying the distributive property

$3x + 6y + 2z - 6x + 8 =$

$(3x - 6x) + 6y + 2z + 8 =$

$-3x + 6y + 2z + 8$ - answer

Let us find out if you can now add correctly algebraic expressions. Do the Test Your Computational Skills next page.

Test Your Computational Skills 2.2

Add the following algebraic expressions and check using any of the two methods.

1. $(2x + 3x – a) + (6 + c)$
2. $2x^2 y + (4x^2 y - 3yx^2 + 2z)$
3. $(-abc + 8a) + (b + cba)$
4. $4x^2 y + 6x – 8x) + (3x^2y+ 5)$
5. $x^2 y^2 + 2xy) + (4x – 5xy^2) + (4x^2 y^2 - 3x)$
6. $(2 – 4) + (5 – 6)$
7. $(abc – cba) + 2bac$
8. $3m^2 n + (4n^2 m - 6mn)$
9. $(2x^2 y + 4y^2 x) + (8x^2 y – 4y^2 z)$
10. $(ab + 2bc) + (4ac – 3ab + 4)$

Let us recall the processes and skills needed to master addition of polynomials by listing them again. Some tips for you to remember.

When we combine like terms, we use the distributive property. It is usually necessary to change the grouping and the order in which the terms appear. The commutative and associative properties of addition guarantee that when we do this, the sum remains unchanged.

TO COMBINE LIKE TERMS:

1. Identify the like terms by their identical literal parts.

2. Find the sum of each group of like terms by adding their numerical coefficients (then multiply that sum by the literal part of those like terms). When no coefficient is written, it is understood to be one.

Example 1. Combining Like Terms

(a) $2x + 4x = (2 + 4)x = 6x$

(b) $\underset{\wedge}{e} - 9e = (\underset{\wedge}{1} - 9)e = -8e$

when no coefficient is written it is understood to be 1

> **TO ADD POLYNOMIALS**
>
> 1. Remove grouping symbols.
> 2. Combine like terms.
>
> It is helpful to underline like terms with the same kind of line before adding.

Example 2. Adding polynomials

(a) $(3x^2 + 5x - 4) + (2x + 5) + (x^3 - 4x^2 + x)$

$= \mathbf{3x^2} + \underline{5x} - \mathit{4} \ \underline{+2x} \ \mathit{+5} + x^3 \ \mathbf{- 4x^2} \ \underline{+x}$

$= +x^3 \ \mathbf{+3x^2 - 4x^2} \ \underline{+x} + \underline{5x} \ \underline{+2x} \ \mathit{+5} \ \mathit{-4}$

$= +x^3 - x^2 + 8x + 1$

(b) $(5x^3y^2 - 3x^2y^2 + 4xy^3) + (4x^2y^2 - 2xy^2) + (-7x^3y^2 - 6xy^2 - 3xy^3)$

$\mathbf{5x^3y^2} \ \underline{-3x^2y^2} \ \mathit{+4xy^3} \ \underline{+4x^2y^2} \ -2xy^2 \ \mathbf{-7x^3y^2} \ -6xy^2 \ \mathit{-3xy^3}$

$\mathbf{5x^3y^2} \ \mathbf{-7x^3y^2} \ \underline{-3x^2y^2} \ \underline{+4x^2y^2} \ 2xy^2 - 6xy^2 \ \mathit{+4xy^3} \ \mathit{-3xy^3}$

Most addition of polynomials will be done horizontally as already shown. However, in a few cases it is convenient to use vertical addition.

TO ADD POLYNOMIALS VERTICALLY

1. Arrange them under one another so that like terms are in the same vertical line.
2. Find the sum of the terms in each vertical line by adding their numerical coefficients.

Example: Add $(3x^2 +2x -1)$, $(4x + 5)$, and $(4x^3 +7x^2 -6)$ vertically.

$$\begin{array}{rrrr} & 3x^2 & +2x & -1 \\ & & 4x & +5 \\ 4x^3 & +7x^2 & & -6 \\ \hline 4x^3 & +10x^2 & +6x & -2 \end{array}$$

What we have added so far are algebraic expressions consisting of one term only or monomials. Let us look at some examples.

Example 1. Find the sum of

(a) $3x^2y$ and $+18x^2y$
(b) $-4xy^3$ and $-18xy^3$
(c) 2abc, 8abc and abc
(d) $-2Z$, $-8z$, $-7z$ and $-5z$

Solution: The terms to be added are like terms so we can use the distributive law In combining the terms.

$$\text{(a)}\quad 3x^2y \text{ and } +18x^2y \begin{aligned}[t] &= 3x^2y +18x^2y \\ &= (3 +18)\, x^2y \\ &= 21x^2y \end{aligned}$$

$$\text{(b)}\quad -4xy^3 \text{ and } -18xy^3 \begin{aligned}[t] &= -4xy^3 \ -18xy^3 \\ &= (-4 \ -18)\, xy^3 \\ &= -22xy^3 \end{aligned}$$

(c) $2abc$, $8abc$ and abc =
$[2abc + 8abc] + abc$ = apply associative law
$[(2 + 8)abc] + abc$ =
$10abc + abc$ =
$(10+1)abc$ =
$11abc$

(d) $-2Z$, $-8z$, $-7z$ and $-5z$ =
$(-2Z - 8z) + (-7z - 5z)$ = apply associative law
$(-2 - 8)z + (-7 - 5)z$ =
$-10z - 12z$ =
$(-10 - 12)z$ =
$-22z$

We can state our observations about addition of similar terms monomial into a rule.

Rule1. **Addition of Similar Terms (Like Terms)**

To add similar terms, find the algebraic sum of the numerical coefficients and prefix it to their common literal factor.

More about addition next page.

Addition of Polynomials
(Dissimilar Terms)

Let us look at these examples.

Example 2. Find the sum of
(a) $2ab$ and $4c$
(b) x^2y and xy^2
(c) ax^2 and by^2

Solution:

The terms to be added are dissimilar so the sum cannot be expressed as a single sum. This does not mean that we cannot add dissimilar terms, the sum is obtained easily by copying the terms to be added and affixing a plus sign to indicate that the terms are added. Very simple, isn't it?

The answers to the questions in example 2 are:

(a) $2ab + 4c$
(b) $x^2y + xy^2 = xy(x + y)$
(b) $ax^2 + by^2$

Let us state our observations into a rule.

Rule 2. **Addition of Dissimilar Terms**
(Unlike Terms)

To add dissimilar terms, copy the terms to be added and place the plus sign (+) to indicate that the terms were added.

More about addition next page.

Addition of Polynomials

Let us add algebraic expressions called polynomials.

Polynomials are added by combining any similar terms that are contained within those polynomials.

Here are some examples to illustrate the tips given.

Example 1. Add the following polynomials:

1. $(3x^2 + y^2 + 4)$ and $(-3x^2 + 4y^2 - 6)$
2. $(abc + 4cd)$ and $(-abc - 4cd)$
3. $(x^2 + 4x - 4y^2)$ and $(y^2 - 3x^2 + 3x)$

Solutions:

1. $(3x^2 + y^2 + 4) + (-3x^2 + 4y^2 - 6) =$

$$\begin{array}{lccc} & 3x^2 & + y^2 & + 4 \\ + & & & \\ & -3x^2 & + 4y^2 & - 6 \\ \hline & & 5y^2 & -2 \end{array}$$

2. $(abc + 4cd) + (-abc - 4cd) =$

$$\begin{array}{lcc} & abc & + 4cd \\ + & & \\ & -abc & - 4cd \\ \hline & & 0 \end{array}$$

3. $(x^2 + 4x - 4y^2) + (y^2 - 3x^2 + 3x) =$

$$\begin{array}{lccc} & x^2 & + 4x & - 4y^2 \\ + & & & \\ & -3x^2 & + 3x & y^2 \\ \hline & -2x^2 & + 7x & -3y^2 \end{array}$$

rearranged second addend.

Reminders:

1. If the sum of the vertical column is zero but the sum of one or more of the other columns is not zero, indicate the sum of the vertical column which are not zero omit the column with zero sum.
2. If the sum of the vertical column are all zero only one zero is written in the result.

Add $(3x^3y +5x^2 +3x) + (2x^3y -5x^2 +3x)$

Solution: Because the terms in this expressions represents real numbers, they can be rearranged, regrouped and combine.

$(3x^3y +5x^2 +3x) + (2x^3y -5x^2 +3x)$

$$\begin{aligned}
&= (3x^3y + 2x^3y) + (-5x^2 +5x^2) + (3x + 3x)\\
&= (3 + 2)\, x^3y + (-5 + 5)x^2 + (3 + 3)x\\
&= 5x^3y + 0x^2 +6x\\
&= 5x^3y +6x
\end{aligned}$$

We can also add polynomials by placing like terms in the same column. Let's add using the same example.
$(3x^3y +5x^2 +3x) + (2x^3y -5x^2 +3x) =$

Putting like terms in the same column, we have

$$\begin{array}{rrrr}
 & 3x^3y & +5x^2 & +3x \\
+ & & & \\
 & 2x^3y & -5x^2 & +3x \\
\hline
 & 5x^3y & +0x^2 & +6x
\end{array} = 5x^3y \quad +6x$$

We can summarize the procedure of adding polynomials in the following sequence of steps.

Addition of Polynomials

Procedure:

1. If the polynomials are so arranged that similar terms are in vertical column, add each column.
2. Otherwise, place one polynomial under the other so that similar terms are in vertical column and then add each column.
3. In some cases it will be necessary to rewrite the polynomials in descending power of a letter or variable.
4. Check by adding again or by numerical substitution.

TEST YOUR COMPUTATIONAL SKILLS 2.3

Perform the indicated addition. You can use any of the two methods.

1. $(4x^2 + 3x + 4) + (-x^2 + 5x + 8)$
2. $(y^3 + y^2 + x) + (x^2 + x - y^3)$
3. $(abc^3 + 2a^2 b) + (-3a^3b + c)$
4. $(x^2 + y^2 - xy) + (3x^2 - xy + 4y^2)$
5. $(at^2 + bt + c) + (dt^2 + et + d)$
6. $(2m^2 - m + 4) + (3m^2 + m - 5)$
7. $(3x^2 + 4x - 10) + (5x^2 - 3x + 7)$
8. $(-5b^2 + 4b + 8) + (8b^2 + 2b - 14)$
9. $(5n^2 + 8n - 7) + (6n^2 - 6n + 10)$
10. $(2a^2 - 3a + 9) + (3a^2 + 4a - 5)$

Add the polynomials vertically

11.
$$\begin{array}{rrrr} & 17a^3 & +4a & -9 \\ + & 8a^2 & -6a & +9 \\ \hline \end{array}$$

12.
$$\begin{array}{rrrr} & 14x^2y^2 & -11xy^2 & +8xy \\ + & -7x^2y^2 & +6xy^2 & -3xy \\ & -9x^2y^2 & -4xy^2 & -5xy \end{array}$$

CONGRATULATIONS

You have just finished ALGEBRAIC EXPRESSION II
(Addition of Polynomials).
You are now ready to take the PRACTICE TASK
next page.
Please check your answer at the
FEEDBACK TO THE PRACTICE TASK.
It is important.

Practice Task

Nos. 1-5 Combine like terms.

1. $3cd - 8cd + 3cd$
2.
3. $2xy - 5yx + xy$
4. $-5a + 12a$
5. $2xy^2 - 4x^2y + 6$
6. $3ab - 4ac + 4bc$

Nos. 6 – 10 Add the polynomials vertically.

6.
$$\begin{array}{rrrr} 15x^3 & -4x^2 & & -12 \\ 5x^3 & & +9x & -3xy \\ \hline \end{array}$$

7.
$$\begin{array}{rrrrr} 12x^3 & & -14y^2 & +6y & -24 \\ 7x^3 & +2x^3 & & +2y & \\ \hline \end{array}$$

8.
$$\begin{array}{rrrr} & -\ b^3 & +5b^2 & -8 \\ -20b^4 & +2b^3 & & +7 \\ \hline \end{array}$$

9.
$$\begin{array}{rrr} -3a^2b & -2ab^2 & +4ab \\ 12a^2b^2 & +2ab^2 & +2ab \\ -2a^2b^2 & +4ab^2 & -6ab \\ \hline \end{array}$$

10.
$$\begin{array}{rrrr} a^3 & +2a^2 & -2a & +4 \\ 17a^3 & +4a^2 & -2a & +6 \\ -17a^3 & -4a^2 & +4a & -10 \\ \hline \end{array}$$

In exercises 11 – 20 perform the indicated additions.

11. $(2m^2 - m + 4) + (3m^2 + m - 5)$

12. $(3x^2 + 4x - 10) + (5x^2 - 3x + 7)$

13. $(2x^3 - 4) + (4x^2 + 8x) + (-9x + 7)$

14. $(4a^2 + 6 - 3a) + (5a + 3a^2 - 4)$

15. $[(x^2 + 4) + [(x^2 - 5) + (3x^2 + 1)]$

16. $(5n^2 + 8n - 7) + (6n^2 - 6n + 10)$

17. $(2a^2 - 3a + 9) + (3a^2 + 4a - 5)$

18. $(5 + 8z^2) + (4 - 7z) + (z^2 + 7z)$

19. $(8 + 3b^2 - 7b) + (2b + b^2 - 9)$

20. $(3x^2 - 2) + [(4 - x^2) + 2x^2 - 1)]$

You must score 16
or higher of the
PRACTICE TASK.
If you score 10 or less,
please go over
MODULE 2 again.
It is important.

Answers to **Test Your Understanding** and **Test Your Computational Skills** are also provided on separate sheets placed after the Feedback to the Practice Task.

Please check your answers. Okey?

Feedback to the Practice Task

1. $-2cd$
2. $-2xy$
3. 79
4. $4xy^2 - 4x^2 y + 6$
5. $3ab - 4ac + 4bc$
6. $20x^3 - 4x^2 + 9x + 7$
7. $19x^3 + 2x^2 - 14y^2 + 8y - 24$
8. $-20b^4 + b^3 + 5b^2 - 1$
9. $2a^2 b$
10. $a^3 + 2a^2$
11. $5m^2 - 1$
12. $8x^2 + x - 3$
13. $2x^3 + 4x^2 - x + 3$
14. $a^2 + 2a + 2$
15. $5x^2$
16. $11n^2 + 2n + 3$
17. $5a^2 + a + 4$
18. $9z^2 + 9$
19. $4b^2 - 5b - 1$
20. $4x^2 + 1$

TEST YOUR COMPUTATIONAL SKILLS 2.1

ANSWERS

1. 88
2. 154
3. –144
4. –40476
5. –335191
6. 389
7. –89
8. 609
9. 1358
10. –6426
11. –31
12. 6
13. 230
14. 38
15. 0

TEST YOUR COMPUTATIONAL SKILLS 2.2

ANSWERS

1. $-a + c + 5x + 6$
2. $3x^2y + 2z$
3. $8a + b$
4. $7x^2y - 2x + 5$
5. $5x^2y^2 + 2xy + x - 5xy^2$
6. -3
7. $2abc$
8. $3m^2n + 4n^2m - 6mn$
9. $10x^2y + 4y^2x - 4y^2z$
10. $-2ab + 4ac + 2bc + 4$

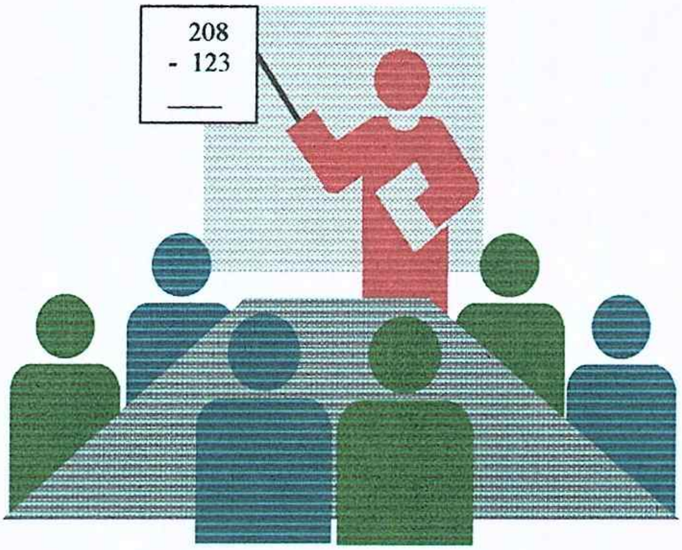
TEST YOUR COMPUTATIONAL SKILLS 2.3

ANSWERS

1. $3X^2 + 9X + 12$
2. $X^2 + 2x + y^2$
3. $-a^2b + abc^2 + c$
4. $4x^2 - 2xy + 5y^2$
5. $(a + d)\,t^2 + (b + e)\,t + (c + d)$
6. $5m^2 - 1$
7. $8x^2 + x - 3$
8. $3b^2 + 6b - 6$
9. $11n^2 + 2n + 3$
10. $5a^2 + a + 4$
11. $25a^2 - 2a$
12. $-2x^2y^2 - 9xy^2$

Module 3
Lesson 1

Subtraction of Integers



How do we subtract integers?

OVERVIEW

Subtraction involves "taking away" or "removing" something from a collection or from a set. It is one of the four fundamental operations. It is a binary operation so it involves two elements of a set ... the **minuend** and the **subtrahend.**

The operation is the inverse of addition. This means that subtraction can be expressed as an addition operation ... the addition of the inverse of the subtrahend to the minuend.

The result of the operation is called the **difference.** The difference can be checked by means of another operation - addition or in case of algebraic expressions by addition or by numerical substitution.

Our concept of subtraction places always the minuend as greater in value than the subtrahend. So, if we have a mathematics sentence about the operation and in the given statement the value of the minuend is less than the subtrahend, we sometime disregard it or become careless. We always assume that the smaller number is subtracted from the bigger number. As a result the answer we get is not the correct answer.

One of the common mistakes regarding this operation comes in when the number to be subtracted is greater in numerical value. If no doubts can be ascertained as to the minuend and the subtrahend the unfortunate answer is "cannot be" meaning that it is impossible to perform the operation in the given set of numbers. With the introduction of the negative numbers our problem was solved.

Translating mathematical statements regarding the operation into symbols is another problem. There is a lot of difference with this two phrases used in subtraction, **"a is subtracted from b"** and **"the difference of a and b"**.

In the first phrase there is no doubt which is our minuend and the subtrahend. But in the second phrase you have two options you can interpret it as **"a - b"** or **"b - a"**. In both cases you will find the difference of a and b.

To understand subtraction let us know the meaning of the following terms ... minuend, subtrahend, difference, minus, subtract and a lot of others.

Subtraction is not the domain of the mathematicians. It is ours too.

OBJECTIVES

At the end of this lesson the students should be able to:

1. write subtraction sentences as addition sentences.
2. find the additive inverse of the given numbers.
3. subtract integers.
4. subtract monomials and polynomials.
5. check if the result obtained in subtraction is correct by addition or numerical substitution.

INPUT

Subtraction of Integers
(Signed Numbers)

The mathematical symbol "**a - b**" is read as "a minus b", "b is subtracted from a", and "the difference of a and b". The minuend is **a** and the subtrahend is **b**. If "a - b = c", then **c** is called the difference. The difference is the name given to the result in subtraction.

Let us examine the definition of subtraction.

Definition of Subtraction

$$a - b = a + (-b)$$

In words: To subtract b from a, add the negative of b to a.

To subtract b from a, we are going to add the negative of b to a. If we try to analyze the definition of subtraction, we are not learning a different operation. Since, this operation involves addition so learning subtraction means mastering addition. What is important is that we know how to find the negative of **b**. It is the negative of b that is to be added to a.

TO find the negative of a number

Change the sign of the number.

The negative of b = -b.
The negative of -b = -(-b) = b.
The negative of 0 = 0.

Let us examine examples of finding the negative of a given number.

Example 1. The negative of a positive number.

(a) The negative of 5 is -5.
(b) The negative of 12 is -12.

Example 2. The negative of a negative number.

(a) The negative of -10 is +10, written -(-10) = 10.
(b) The negative of -14 is +14, written -(-14) = 14.

Let us have a review of addition of signed numbers.

To add two signed numbers

Case 1.	**When the numbers have the same sign**	**1st**	**Add their absolute values.**
		2nd	**The sum has the same sign as both numbers.**
Case 2.	**When the numbers have different signs**	**1st**	**Subtract the smaller absolute value from the larger.**
		2nd	**The sum has the sign of the number that has the larger absolute value.**

Subtraction is the inverse of addition. What do we mean when we say an operation is the inverse of another operation?

Subtraction is called the inverse of addition, because it "undoes" addition. Let us look at the example presented next page.

$$5 \underbrace{+ \ 4}_{\text{addition of 4}} = 9 \qquad\qquad 9 \underbrace{- \ 4}_{\text{subtraction of 4}} = 5$$

inverse operation

Since 5 is where we begun and 5 is where we ended. The operation of addition was "undone" by the operation of subtraction.

Let us subtract signed numbers.

TO subtract one signed number from another

1. **Change the subtraction symbol to an addition symbol, and change the sign of the number being subtracted.**

2. **Add the resulting signed numbers.**

For example: (8) - (+5)

= (8) - (-5) <——Change the sign of the number being subtracted.

= 3.

Change the subtraction symbol to an addition symbol.

Example 1. Subtract (-15) from (+42).

Solution: (+42) - (-15)

= (+42) + (+15)

= +57

Example 2. Subtract (327) from (-1037).

Solution: (-1037) - (+327)

= (-1037) + (-327)

= -1364

Example 3. Find (-15) from (-45).
Solution: (-45) - (-15)
= (-45) + (+15)
= -30

Example 4. Find (+320) from (+1030).
Solution: (+1030) - (+320)
= (+1030) + (-320)
= +710

Example 5. Subtract (-11) from (-7).
Solution: (-7) - (-11) or
= (-7) + (+11)
= +4

$$\begin{array}{r} -\ 7 \\ -\ -11 \\ \hline \end{array} \qquad \begin{array}{r} -\ 7 \\ +\ +11 \\ \hline +\ 4 \end{array}$$

Example 6. Find: (+13) - (-14)
Solution: (+13) - (-14)
= (+13) + (+14)
= +27

$$\begin{array}{r} +13 \\ -\ -14 \\ \hline \end{array} \qquad \begin{array}{r} +13 \\ +\ +14 \\ \hline +27 \end{array}$$

Example 7. Find: -147 -159
Solution: -147 - 159
= -147 - (+159)
= -147 + (-159)
= -206

Example 8. Find: (-4.56) - (-7.48)
Solution: (-4.56) - (-7.48)
= (-4.56) + (+7.48)
= + 2.92

Example 9. Find: (-3 1/2) - (+2 1/4)
Solution: (-3 1/2) - (+2 1/4)
= (-3 1/2) + (-2 1/4)
= (-3 2/4) + (-2 1/4)
= (-5 3/4)

Let us test your computational skills in subtraction of signed numbers next page.

Test Your Computational Skills 3.1

Find the difference.

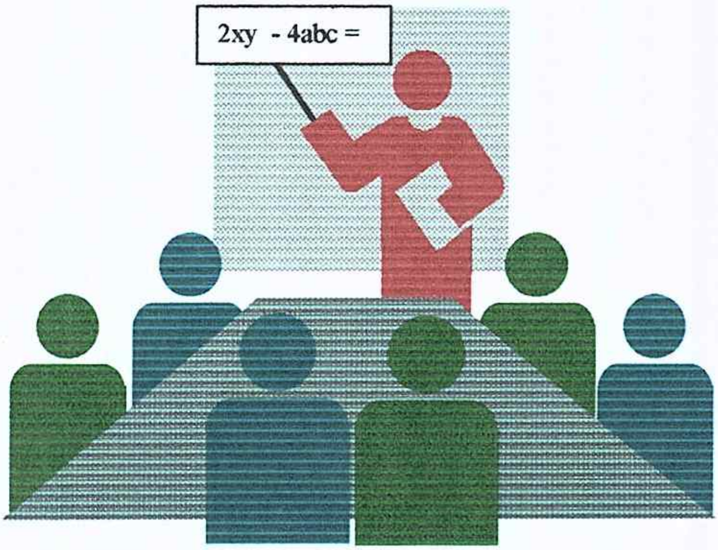
1. (10) - (23)
2. (-23) - (-456)
3. (+234) - (-123)
4. (-134) - (+1256)
5. (+2/3) - (-2 1/3)
6. (-12) - (+21)
7. (-190) - (-190)
8. (-2 4/7) - (-5 3/14)
9. (-7.003) - (+2.078)

2. (-23) - (-456)

10. (+15.61) - (+12.6)

Module 3
Lesson 2

Subtraction of Polynomials



How do we subtract polynomials?

Input!

Subtraction of Polynomials

To subtract b from a, we write a – b, which is the same as a + (–b).

To *subtract b from a* we add the additive inverse (the negative) of b to a.

Now the operation subtraction is reduced to an addition operation.

Recall: The additive inverse of a given number is the negative of the number.

Example 1.

(a) The additive inverse of 4x is –4x.
(b) The additive inverse of –7y is 7y.
(c) The additive inverse of $3x^2 y^3$ is $-3x^2 y^3$
(d) The additive inverse of –1/2 abc is +1/2 abc

Number	Additive Inverse
4x	–4x
–7y	7y
$3x^2 y^3$	$- 3x^2 y^3$
–1/2 abc	1/2abc

Let us look at the numbers and their additive inverses. May I remind you that the additive inverse of a number is **unique**. This means that if a is the additive inverse of –a, there is no other number that can be additive inverse of –a other than a.

Notice that the additive inverse of a number is the negative of a number.

We do not change either the constant factor or the variable in this sample expressions which represent specified or unspecified numbers.

Let us learn subtraction of algebraic expressions and understand some facts related to the operation. Also knowledge of addition is required or needed to master subtraction.

Note:

To get the additive inverse of a given number simply change the sign of the given number. (Do not change the magnitude of the number by changing the constant and variable in the expression).

Example 2. Find the additive inverse of y, $\frac{3}{4}xy^2$, $-1.2xyz$

Solution:

$$
\begin{aligned}
y &= -(y) = -y \\
\tfrac{3}{4}xy^2 &= -(\tfrac{3}{4}xy^2) = -\tfrac{3}{4}xy^2 \\
-1.2xyz &= -(-1.2xyz) = 1.2xyz
\end{aligned}
$$

Subtraction of Monomials
(Similar Terms)

Let us subtract algebraic expressions consisting of one term only or monomials. In subtraction of monomials it is important that we have to read the mathematical statement pertaining to the operation and translate it into mathematical symbols. Be sure that you have correctly identified the minuend and the subtrahend. It is important. The minuend is usually the expression that follows after the word “from” and “is subtracted from”.

Example 3. Subtract 2a from –7a

Solution: $(-7a) - (2a) =$ minuend –7a, subtrahend 2a

$(-7a) + (-2a) =$	Change the operation from subtraction to addition and add the inverse of the subtrahend $-2a$.
$-7a - 2a =$ $(-7 - 2)a =$	Apply the distributive law of multiplication since the terms to be added are similar terms.

$-9a =$ answer (difference)

Example 4. Subtract $-6x$ from $-4x$

Solution: $(-4x) - (-6x) =$	minuend $-4x$, subtrahend $-6x$
$(-4x) + (+6x) =$	Change the operation from subtraction to addition and add the inverse of the subtrahend $-6x$ which is $+6x$.
$-4x + 6x =$ $(-4 + 6)x =$	Apply the distributive law of multiplication since the terms to be added are similar terms.

$+2x =$ answer (difference)

Example 5. Subtract $-9xy$ from $10xy$

Solution: $(10xy) - (-9xy) =$	minuend $10xy$, subtrahend $-9xy$
$(10xy) + (+9xy) =$	Change the operation from subtraction to addition and add the inverse of the subtrahend $-9xy$ which is $+9xy$.
$10xy + 9xy =$ $(10 + 9)xy =$	Apply the distributive law of multiplication since the terms to be added are similar terms.

$19xy =$ answer (difference)

Examples 3 to 5 illustrate subtraction of similar terms. Now let us study subtraction of dissimilar terms in the succeeding examples.

Subtraction of Monomials (Dissimilar Terms)

Let us subtract dissimilar terms.

Example 6. Subtract –4abc from 8xyz

Solution: (8xyz) – (–4abc) =	minuend 8xyz, subtrahend -4abc
(8xyz) + (+4abc) =	Change the operation from subtraction to addition and add the inverse of the subtrahend, –4abc which is +4abc.
8xyz + 4abc =	The terms to be added are dissimilar so we can only indicate their sum. We cannot express the answer as a single sum.

8xyz + 4abc = answer (difference)

Notice:

We cannot combine +8xyz and +4abc since the pair of terms are unlike or dissimilar. So we can only indicate their sum by placing the + sign to show that the terms are added.

Example 7. Subtract $-6x^2y^2$ from $-8x^3y^2$.

Solution: $(-8x^3y^2) - (-6x^2y^3) =$

$-8x^3y^2) + (6x^2y^3) =$	Operation changes from subtraction to addition adding the inverse of the subtrahend $+6x^2y^3$.
$-8x^3y^2 + 6x^2y^3 =$	Terms to be added are dissimilar hence the sum cannot be express as a single term, however we can still apply the distributive law.
$2x^2y^2(4x + 3y) =$	Written in this form the sum can be expressed as a single term but application of the distributive law will give back the answer of two terms.

From the given examples we can formulate a rule for subtraction of polynomials containing only one term or monomials.

Rule 1

To subtract similar terms change the sign of the subtrahend mentally and proceed as in addition of similar terms.

Recall:

The sum of 2 similar terms can be simplified by applying the distributive law of multiplication.

Recall:

The sum of two terms that are dissimilar can only be indicated.

Rule 2

To subtract terms which are dissimilar or unlike change the sign of the subtrahend (mentally) and proceed as in addition of dissimilar terms.

How do we check if our answer in subtraction is correct?

Recall:

Subtraction operation:

$$\begin{array}{rl} & 8 \quad \text{minuend} \\ - & 2 \quad \text{subtrahend} \\ \hline & 6 \quad \text{difference} \end{array}$$

Subtraction check:

$$\begin{array}{rl} & 6 \quad \text{difference} \\ + & 2 \quad \text{subtrahend} \\ \hline & 8 \quad \text{minuend} \end{array}$$

There are two ways in which we can check if our answer is correct in subtraction.

1. checking by addition
2. checking by substitution

Let us try "checking the answer using addition".

Example 8: Subtract $9xy^2$ from $-4xy^2$

Solution:
$$\begin{aligned} (-4xy^2) - (9xy^2) &= \\ (-4xy^2) + (-9xy^2) &= \\ (-4-9)xy^2 &= \\ -13xy^2 &= \text{ Answer (Difference)} \end{aligned}$$

Checking by addition:

We add the difference and subtrahend, this must be equal to the minuend. In this example the difference is $-13xy^2$ and the subtrahend is $9xy^2$. The sum must be equal to the minuend $-4xy^2$.

$$\begin{aligned} (-13xy^2) + (9xy^2) &= -4xy^2 \\ (-13+9)\,xy^2 &= -4xy \\ -4xy^2 &= -4xy^2 \end{aligned}$$

Example 9. Verify if the answer in example 4 is correct

$$(-4x) - (-6x) = 2x$$

Solution:

In this example the difference is 2x the subtrahend is –6x and the minuend is –4x.

$$\begin{aligned}(2x) + (-6x) &= (-4x)\\ (2 - 6)\,x &= -4x\\ -4x &= -4x\end{aligned}$$

Example 10. Verify if the answer in example 6 is correct

$$(8xyz) - (-4abc) = 8xyz + 4abc$$

Let us check if the answer is correct by addition

Solution

In this example the difference is 8xyz + 4abc, then the subtrahend is –4abc and the minuend is 8xyz

$$\begin{aligned}(8xyz + 4abc) + (-4abc) &= 8xyz &&\\ 8xyz + 4abc - 4abc &= 8xyz && \text{Remove the parenthesis}\\ 8xyz + (4abc - 4abc) &= 8xyz && \text{Apply associative law}\\ 8xyz + (4 - 4)\,abc &= 8xyz && \text{Apply the distributive law}\\ 8xyz + 0 &= 8xyz &&\\ 8xyz &= 8xyz &&\end{aligned}$$

Example 11. Check if the answer to example 8 is correct by substitution.

$$(-4xy) - (9xy) = -13xy$$

Solution:

Let $xy = 1$

$$\begin{aligned}-4\,(1) - 9(1) &= -13(1)\\ -4 - 9 &= -13\\ -13 &= -13\end{aligned}$$

Note:

In checking by substitution you can let xy be any number.

Example 12. Verify by numerical substitution if the answer in example 6 is correct.

$$(8xyz) - (-4abc) = 8xyz + 4abc$$

Let $xyz = 1$, $abc = 3$

$8xyz - (-4abc) = 8xyz + 4abc$

$8(1) - [-4(3)] = 8(1) + 4(3)$ Substitute 1 for xyz and 3 for abc

$8 - (-12) = 8+12$ Simplify

$8 + 12 = 20$

$20 = 20$

Let us test your computational skills regarding subtraction of monomials.

Test Your Computational Skills 3.2

1. Subtract $3y^2$ from $-8y^2$
2. Subtract $-4xyz$ from $2xz$
3. Subtract $-abc$ from $4abc$
4. Subtract x^2 from y^2
5. Subtract $3a$ from $3x$

Subtract the lower monomials from the upper monomials.

6. $15x^3$
$8x^3$

7. $-14y^2$
$7y^2$

8. $-(9x^2y^3 + 6xy^2)$
$(-7x^2y^3 \quad + 9)$

9. $12a^2b$
$+8ab^2$

10. $-5x^2y^2$
$+2x^2y^2$

Perform the indicated subtraction.

11. $3x^2 - 5x^2$
12. $(2m^2n) - (-3m^2n)$
13. $(15n^2m) - (-6n^2m)$
14. $-2a^2b - 3ab^2$
15. $17a^3b^2 - (-8a^3b^2)$

Subtraction of Polynomials

For subtraction of algebraic expressions called polynomials, let us study the example presented.

Example 13. Subtract $2x^2 -5xy + y^2$ from $-x^2 +7xy -3y^2$

Solution: Take a look at the solution. You'll notice that the process of subtraction involving more than one terms is the same as subtraction with only one term. We change the operation from subtraction to addition, and we have to add the additive inverse of the subtrahend. We apply the distributive law of multiplication to get the sum.

	$-x^2 + 7xy - 3y^2$	minuend
$-$	$2x^2 - 5xy + y^2$	subtrahend
	$-x^2 + 7xy - 3y^2$	minuend
$+$	$-2x^2 + 5xy - y^2$	inverse of subtrahend
	$-3x^2 + 12xy - 4y^2$	answer

What is important is that we must see to it that similar terms must be in the same vertical column and we add the columns.

Example 14. Subtract $-2a^2 - 3a + 7$ from $-3a +4a^2 +6$.

Notice:

The subtrahend is arranged in descending power of the variable a but not the minuend. To make our work easier we have to arrange the minuend in descending power of the variable a so that similar terms will be in the same vertical column.

$$\begin{array}{r} -3a + 4a^2 + 6 \\ - \qquad\qquad \\ -2a^2 - 3a + 7 \\ \hline \end{array} \qquad \begin{array}{r} 4a^2 - 3a + 6 \\ - \qquad\qquad \\ -2a^2 - 3a + 7 \\ \hline \end{array}$$

Arranging the minuend in descending power of the variable a, $-3a + 4a^2 + 6$ is rewritten as $4a^2 - 3a + 6$.

$$\begin{array}{r} -4a^2 - 3a + 6 \\ + \qquad\qquad \\ 2a^2 + 3a - 7 \\ \hline -2a^2 + 0 - 1 \end{array} = 2a^2 - 1$$

Changing the operation sign from subtraction to addition and adding the additive inverse of the subtrahend $-2a^2 - 3a + 7$ which is $2a^2 + 3a - 7$.

Example 15. Subtract $c^2 - 2cd$ from $4cd - d^2$

$(4cd - d^2) - (c^2 - 2cd)$

$(4cd - d^2) + (2cd - c^2) =$	Operation changes from subtraction to addition.
$(-d^2 + 4cd) + (2cd - c^2) =$	Apply commutative property to the subtrahend.
$-d^2 + (4cd + 2cd) - c^2 =$	Apply commutative property to the minuend.
$-d^2 + (4 + 2)cd - c^2 =$	Group similar or like terms .
$-d^2 + 6cd - c^2 =$	Answer (Difference).

We can state what we have observed as a rule.

Rule 3. To subtract one polynomial called the subtrahend from another polynomial called the minuend, add the minuend to the additive inverse of the subtrahend and combine like terms.

Example 16. Subtract $4a - 3b + c$ from $3x + 2y - 3z$

Solution:

$(3x + 2y - 3z) - (4a - 3b + c) = ?$

$$\begin{aligned}
(3x + 2y - 3z) + (-4a + 3b - c) &= ?\\
3x + 2y - 3z - 4a + 3b - c &= \text{Answer (Difference)}
\end{aligned}$$

Let us check if your answer is correct. Okey?

Now, how do we check if the answer we get in subtraction is correct?

We check by addition or by numerical substitution.

Example 17. $(5a + 2b - 3c) - (4a - 3b + c) = a + 5b - 4c.$

(1) Checking the answer by addition:

Solution: difference plus subtrahend = minuend

$$\begin{aligned}
(a + 5b - 4c) + (4a - 3b + c) &= 5a + 2b - 3c\\
(a + 4a) + (5b - 3b) + (-4c + c) &= 5a + 2b - 3c\\
(1 + 4)a + (5 - 3)b + (-4 + 1)c &= 5a + 2b - 3c\\
5a \quad + \quad 2b \quad - \quad 3c &= 5a + 2b - 3c
\end{aligned}$$

(2) Checking answer by numerical substitution:

$$(5a + 2b - 3c) - (4a - 3b + c) = a + 5b - 4c$$

Solution: Let $a = 1, \quad b = 2, \quad c = 3$

$$\begin{aligned}
(5a + 2b - 3c) - (4a - 3b + c) &= a + 5b - 4c\\
[5(1) + 2(2) - 3(3)] - [4(1) - 3(2) + 3] &= 1 + 5(2) - 4\,(3)\\
(5 + 4 - 9) - (4 - 6 + 3) &= 1 + 10 - 12\\
0 \quad - 1 &= -1\\
-1 &= -1
\end{aligned}$$

After looking at several examples, you are now ready to answer the Test Your Computational Skills.

Test Your Computational Skills 3.3

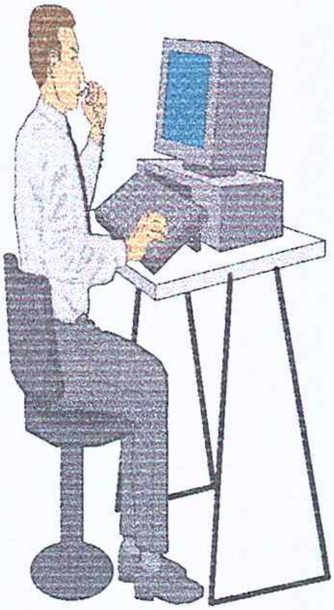
Subtract the first polynomial from the second polynomial and check your answer. For numbers 1-3 by addition and for numbers 4 – 5 by numerical substitution.

1. $x^2 + y^2$, $y^2 - 4$
2. $a^2 b + bc + c^2$, $ab^2 - bc + 4c^2$
3. $at^2 + bt + c$, $et^2 - ft + d$
4. $ax^2 + bx + d$, $-a^2 x^2 + bx + f$
5. $x^2 + y^2 + z$, $3a - 4f - c$

Perform the indicated subtraction.

6. $(2x^2 + 4x - 4) - (2x^2 - 3x + 5)$
7. $(2m^3 - m + 5) - (2m^2 + m - 6)$
8. $(5n^3 + 3n - 4) - (6n^3 - 6n + 1)$
9. $(3a^2 + 4a + 9) - (3a^2 + 4a - 3)$
10. $(17a^2 + 2a + 1) - (8a^2 - 6a + 2)$

CONGRATULATIONS



You have just finished ALGEBRAIC EXPRESSION III
(Subtraction of Polynomials).
You are now ready to take the PRACTICE TASK
next page.
Please check your answer at the
FEEDBACK TO THE PRACTICE TASK.
It is important.

Practice Task:

Perform the indicated subtraction.

1. $(3x^2 \ +4x \ -10) \ - \ (5x^2 - 3x +7)$
2. $(2m^2 \ -m \ +4) \ - \ (3m^2 + m -5)$
3. $(5n^2 \ +8n \ -7) \ - \ (6n^2 - 6n +10)$
4. $(2a^2 \ -3a \ +9) \ - \ (3a^2 + 4a -5)$
5. $(17c^2 +4c \ +9) \ - \ (8c^2 - 6c +9)$

Subtract the lower polynomials from the upper polynomials.

6.
$$\begin{array}{rrrr} 15x^3 & -4x^2 & & +12 \\ 8x^3 & & +19x & -5 \\ \hline \end{array}$$

7.
$$\begin{array}{rrrr} & -14y^2 & +6y & -24 \\ 7y^3 & +14y^2 & -13y & \\ \hline \end{array}$$

8.
$$\begin{array}{rrr} 9x^2y^3 & +6xy^2 & \\ 7x^2y^3 & & +9 \\ \hline \end{array}$$

9.
$$\begin{array}{rrrr} 12a^2b & -8ab^2 & & +4 \\ & +8ab^2 & -3ab & +6 \\ \hline \end{array}$$

10.
$$\begin{array}{rrr} x^3y^2 & -5x^2y^2 & \\ & +2x^2y^2 & +6 \\ \hline \end{array}$$

11. Subtract $(2x^2 -4x +3)$ from the sum of $(5x^2 - 2x +1)$ and $(-4x^2 +6x -8)$.

12. Subtract $(6y^2 -4y -4)$ from the sum of $(-2y^2 \ +y +9)$ and $(8y^2 -2y +5)$.

13. Subtract $(10a^3 -8a +12)$ from the sum of $(11a^3 +9a -14)$ and $(-6a^3 +17a)$.

14. Subtract $(15b^3 -17b^2 +6)$ from the sum of $(14b^2 - 8b-11)$ and $(-9b^3 +12b)$.

15. Subtract $(6r^3t^2 + 6r^2t)$ from the sum of $(16r^3t^2 \ {}^{+}2r^2t)$ and $(r^3t^2 +r^2t)$.

16. Subtract the sum of $(2m^2n–4mn^2 +6)$ and $(-3m^2n + 5mn^2)$ from the sum of $(5 +m^2n -mn^2)$ and $(3 +4m^2n +2mn^2)$.

17. Subtract the sum of $(x^2+2x +5)$ and $(3x +x^2 +6)$ from the sum of $(2x^2 +6)$ and $(5x +5)$.

18. Subtract the sum of $(y^2 +6)$ and $(y^3 –4)$ from the sum of $(2y^2 +6)$ and $(2y +5)$.

19. Subtract the sum of $(8y^2 –5y +7)$ and $(13y^2 –11y +6)$ from the sum of $(-2y^2 –16y +4)$ and $(13y^2 –11y +6)$.

20. Subtract the sum of $(at^2 +bt +c)$ and $(dt^2 +e)$ from the sum of $(ft^2 +gt)$ and $(ht + it^2)$.

You must score 16
or higher of the
PRACTICE TASK.
If you score 10 or less,
please go over
MODULE 3 again.
It is important.

Answers to **Test Your Understanding** and **Test Your Computational Skills** are also provided on separate sheets placed after the Feedback to the Practice Task.

Please check your answers. Okey?

Feedback to the Practice Task:

1. $-2x^2 +7x -17$

2. $-m^2 -2m +9$

3. $-n^2 +14n -17$

4. $-a^2 -7a +14$

5. $7c^2 +10c$

6. $7x^3 -4x^2 -19x +17$

7. $-7y^3 -28y^2 +19y -24$

8. $2x^2y^3 +6xy^2 -9$

9. $12a^2b -16ab^2 +3ab -2$

10. $x^3y^2 -7x^2y^2 -6$

11. $-3x^2 +8x -10$

12. $3y +18$

13. $-5a^3 +4a -26$

14. $-24b^3 +31b^2 +4b -17$

15. $+11r^3t^2 - 3r^2t$

16. $2 +6m^2n -2mn^2$

17. 0

18. $-y^3 +y^2 +2y +9$

19 $-10y^2 -11y -3$

20. $(-a -d + f + i)t^2 +(-b +g + h)t - (c +e)$

Answers

Test Your Computational Skills 3.1

1. -13
2. 433
3. 357
4. -1390
5. 3
6. -33
7. 0
8. 2 9/14
9. -9.081
10. 3.01

Test Your Computational Skills 3.2

1. $-11y^2$
2. $2xz +4xyz$
3. $5abc$
4. $-x^2+y^2$
5. $3x-3a$
6. $7x^3$
7. $-21y^2$
8. $-2x^2y^3 +6xy^2-9$
9. $+12a^2b -8ab^2$
10. $-7x^2y^2$
11. $-2x^2$
12. $+5m^2n$
13. $+21n^2m$
14. $-2a^2b -3ab^2$
15. $25a^3b^2$

Test Your Computational Skills 6.3

1. $-x^2-4$ Check:

$$\begin{array}{cccc} & -x^2 & & -4 \\ + & x^2 & +y^2 & \\ \hline & & y^2 & -4 \end{array}$$

2. $-a^2b + ab^2 - 2bc + 3c^2$ Check:

$$\begin{array}{cccccc} & -a^2b & +ab^2 & -2bc & +3c^2 \\ + & a^2b & & +bc & +c^2 \\ \hline & & +ab^2 & -bc & +4c^2 \end{array}$$

3. $(-a+e)t^2 - (b+f)t + (d-c)$ Check:

$$\begin{array}{cccc} (-a+e)t^2 & -(b+f)t & +(d-c) \\ at^2 & +bt & +c \\ \hline et^2 & -ft & +d \end{array}$$

4. $-(a^2+a)x^2 - (d-f)$ Check: Let $x = 1$, $a = 2$, $b = 6$, $d = 3$, $f = 4$

$$\begin{array}{rcl} -a^2x^2 + bx + f & = & -4(1) + 6 + 4 = 6 \\ ax^2 + bx + d & = & 2 + 6 + 3 = 11 \\ & & 6 - 11 = -5 \end{array}$$

$$-(a^2+a)x^2 - (d-f) = -6 - (3-4) = -5$$

5. $3a - 4f - c - x^2 - y^2 - z$ Check: Let $a = 2$, $c = 3$, $f = 0$, $x = 1$, $y = -1$, $z = 4$

$$\begin{array}{c} 3(2) - 4(0) - 3 = 3 \\ 1 + 1 + 4 = 6 \\ 3 - 6 = -3 \\ 3(2) - 4(0) - 3 - 1 - 1 - 4 = -3 \end{array}$$

6. $7x - 9$

7. $2m^3 - 2m^2 - 2m + 11$

8. $-n^3 + 9n - 5$

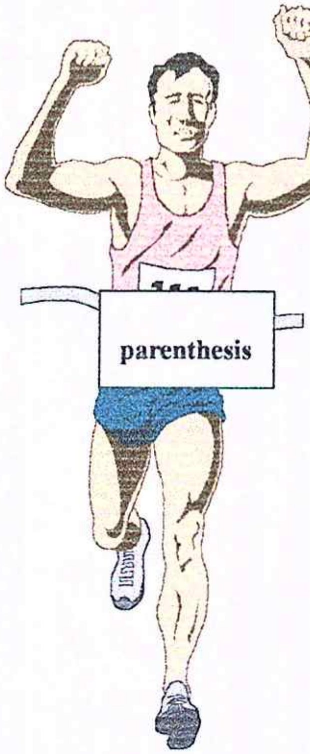
9. 12

10. $9a^2 + 8a - 1$

Module 4
Lesson 1

ALGEBRAIC EXPRESSION IV

(Grouping Symbols)



Grouping Symbols
(Removing & Inserting Grouping Symbols)

OVERVIEW

Grouping symbols are usually used in the solutions of mathematics problems to clarify certain points or facts. The use of grouping symbols simplify computations since we are guided of what operation to perform first.

Grouping symbols when used in mathematical statements indicate more than one operations. The most common grouping symbols used in computations are parenthesis denoted by ( ), bracket denoted by [ ], braces denoted by { }, and the bar denoted by --- used as a fraction line and the bar in square root. Parenthesis, bracket, and braces are generally referred to as parenthesis.

Some mathematical computations require the insertion or removal of all these three grouping symbols. In cases like these the insertion or removal of parentheses from the given expressions must start from the innermost parenthesis.

Knowledge of order of performing operations stated under MDAS Rule, property of real numbers ... associative, commutative, distributive, and the multiplication operation will help in the simplification of problems involving parenthesis.

Let's discover facts about parenthesis. Okey?

Objectives:

At the end of this lesson the students should be able to:

1. remove grouping symbols from algebraic expressions.
2. insert parenthesis or grouping symbols to clarify and simplify solutions of problems.
3. evaluate expressions by removing/inserting grouping symbols.
4. use grouping symbols in evaluating expressions.

INPUT

GROUPING SYMBOLS

Grouping symbols are used to change the normal order of operations. Operations indicated within grouping symbols are carried out before operations outside the grouping symbols.

Example 1. Showing how grouping symbols change the usual order of operations

$5+4 \cdot 6 = 5+24 = 29$	Usual order	- multiplication done before addition
$(5+4) \cdot 6 = 9 \cdot 6 = 54$	Because of parentheses	- addition done before multiplication

All grouping symbols have the same meaning.

$$\begin{aligned}(8 + 4) - (9 - 7) &= [8 + 4] - [9 - 7] \\ &= \{8 + 4\} - \{9 - 7\} \\ &= \overline{8 + 4} - \overline{9 - 7} \\ &= 12 - 2 \\ &= 10\end{aligned}$$

Different grouping symbols can be used in the same expression.

$$\begin{aligned}(8 + 4) - \{9 - 7\} &= [8 + 4] - (9 - 7) \\ &= \{8 + 4\} - [9 - 7] \\ &= \overline{8 + 4} - \overline{9 - 7} \\ &= 12 - 2 \\ &= 10\end{aligned}$$

Example 2. $10 - [3 -(2 - 7)]$

$$\begin{aligned}&= 10 - [3 - (-5)] \\ &= 10 - [3 + 5] \\ &= 10 - [8] \\ &= 2\end{aligned}$$

When grouping symbols appear within other grouping symbols, evaluate the inner grouping first.

$3 - (-5) = 3 + (+5)$ Because of the definition of subtraction of signed numbers.

Example 3. $$\frac{(-4) + (-2)}{8 - 5} \longleftarrow$$

This bar is a grouping symbol for both $\underline{(-4) + (-2)}$ and for $\overline{8 - 5}$. Notice that the bar can be used either above or below the numbers being grouped.

$$\frac{-6}{3} = -2$$

Example 4. $20 - 2\{5 - [3 - 5(6 - 2)]\}$

$= 20 - 2\{5 - [3 - 5(4)]\}$
$= 20 - 2\{5 - [3 - 20]\}$
$= 20 - 2\{5 - [-17]\}$
$= 20 - 2\{5 + 17\}$
$= 20 - 2\{22\}$
$= 20 - 44$
$= -24$

Example 5. $$27 \text{ — } (-3)^2 - 5 \left\langle 6 - \frac{8 - 4}{5} \right\rangle$$

$$= 27 \text{ — } (-3)^2 - 5 \left\langle 6 - \frac{4}{5} \right\rangle$$

$$= 27 \text{ — } (-3)^2 - 5 \left\langle \frac{26}{5} \right\rangle$$

$$= 27 \text{ — } 9 - 5 \left\langle \frac{26}{5} \right\rangle$$

$$= 3 - 26 = -23$$

Example 6. $(2.5)^2 \text{ — } (5.6 - 11.4)$

$= (2.5)^2 \text{ — } (-5.8)$

$= 6.25 \text{ — } (-5.8)$

$= -1.077586207$

$= -1.08$ Rounded to two decimal places

Example 7. $\sqrt{13^2 - 12^2}$ This bar is a grouping symbol. The expression under it is evaluated first, then the square root is taken.

$\sqrt{169 - 144}$

$\sqrt{25} = 5$

In evaluating expressions involving grouping symbols, it is important to apply correctly the rule of order of operations so that we only have one correct answer. Okey?

IMPORTANT

Order of Operations

1. If there are any parentheses in the expression, that part of the expression within a pair of parentheses is evaluated first, then the entire expression.

2. Any evaluation always proceeds in three steps:
   First: Powers and roots are done in any order.
   Second: Multiplication and division are done in order from left to right.
   Third: Addition and subtraction are done in order from left to right.

Let us test your computational skills by evaluating the given expressions next page. Your understanding of grouping symbols will help you in answering the exercises.

Test Your Computational Skills 4.1

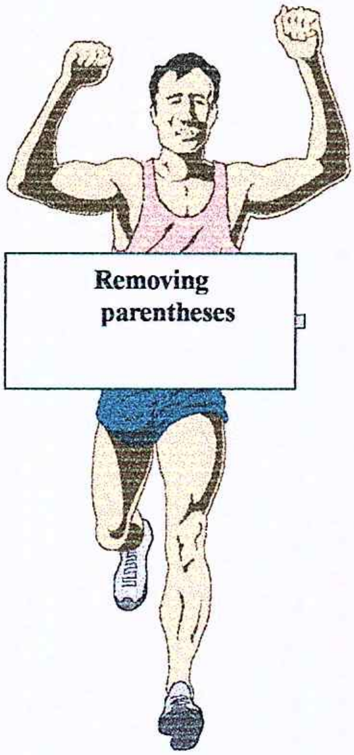
Evaluate each of the following expressions.

1. $2(-6) \div 3(8 - 4)$
2. $5(-4) \div 2(9 - 4)$
3. $24 - [(-6) + 18]$
4. $17 - [(-9) + 15]$
5. $[12 - (-19)] - 16$
6. $[21 - (-14)] - 29$
7. $[11 - (5 + 8)] - 24$
8. $[16 - (7 + 12)] - 22$
9. $20 - [5 - (7 - 10)]$
10. $16 - [8 - (2 - 7)]$
11. $\dfrac{7 + (-12)}{8 - 3}$
12. $\dfrac{(-14) + (-2)}{9 - 5}$
13. $15 - \{4 - [2 - 3(6 - 4)]\}$
14. $17 - \{6 - [9 - 2(2 - 7)]\}$
15. $32 \div (-2)^3 - 5 \left\{ 7 - \dfrac{6 - 2}{5} \right\}$
16. $36 \div (-3)^2 - 6 \left\{ 4 - \dfrac{9 - 7}{3} \right\}$
17. $\sqrt{3^2 + 4^2}$
18. $\sqrt{13^2 - 5^2}$
19. $\sqrt{3^2} + \sqrt{4^2}$
20. $\sqrt{13^2} - \sqrt{5^2}$

Module 4
Lesson 2

ALGEBRAIC EXPRESSION IV

(Grouping Symbols)



Grouping Symbols
(Removing & Inserting Grouping Symbols)

INPUT

REMOVING GROUPING SYMBOLS

The term parenthesis refers to the following grouping symbols: parenthesis ( ), braces { }, and brackets [ ]. These grouping symbols are used to designate in a simple manner more than one operations.

When we write the binomial **(3a + 5b)**, we are considering the sum of 3a and 5b as one quantity. The expression a **- (b + c)** means the sum of b and c is to be subtracted from a.

In performing operations of algebraic expressions, it is sometimes necessary to remove parenthesis or grouping symbols from an expression. Also it is sometimes necessary to insert parenthesis or grouping symbols in our solutions for clarifications.

Removing the grouping symbols means performing the operations that these symbols indicate usually the removal of grouping symbols is done one at a time, starting with the innermost (in cases where there are more than one grouping symbols), following the proper order of operations to be performed.

The removal of parenthesis from expressions is guided by the following rules:

REMOVING GROUPING SYMBOLS

1. **When removing a grouping symbol preceded by a + sign (or no sign) and not followed by a factor:**
   Leave the enclosed terms unchanged. Drop the grouping symbol and the + sign (if there is one) preceding it.

2. **When removing a grouping symbol preceded by a - sign and not followed by a factor:**
   Change the sign of the enclosed terms. Drop the grouping symbol and the - sign preceding it.

3. **When removing a grouping symbol preceded or followed by a factor:**
   Use the distributive property to multiply each enclosed term by the factor, and add these products. Drop the grouping symbol.

4. **When removing a grouping symbol preceded by a - sign and followed by a factor:**
   Use the distributive property to multiply each enclosed term by the factor, and add these products. Change the sign of each enclosed term. Drop the grouping symbol and the - sign preceding it.

5. **When grouping symbols occur within other grouping symbols:**
   It is usually easier to remove the innermost grouping symbols first.

(a) Removing a grouping symbol preceded by a + sign.

Example 1. Removing grouping symbols

$5z + (4y + 7)$

$= 5z + (4y + 7)$

$= 5z + 4y + 7$ Drop the ( ) and the + sign preceding it.

Example 2. Removing grouping symbols and combine like terms.
$3x + (2x - y) + (5x -2y)$

Solution:

$3x + (2x - y) + (5x -2y) =$	
$3x + 2x - y + 5x - 2y =$	Drop the ( ) and the + sign preceding it.
$(3x +2x +5x) + (-y -2y) =$	Combine like terms.
$(3 + 2 + 5)x + (-1 - 2)y =$	Apply distributive property.
$10x - 3y$	Simplify

(b) **Removing a grouping symbol preceded by a - sign.**

Example 3. Removing grouping symbols
$5z - (4y + 7)$

Solution:

$= 5z - (4y + 7)$	Change the sign of each enclosed term and drop the ( ) and the - sign preceding it.
$= 5z - 4y - 7$	

Example 4. Removing grouping symbols and combine like terms.
$3x - (2x - y) - (5x -2y)$

Solution:

$3x - (2x - y) - (5x -2y) =$	Change the sign of each terms enclosed in parenthesis and drop the ( ) and the - sign preceding it.
$3x - 2x + y - 5x + 2y =$	
$(3x - 2x - 5x) + (y + 2y) =$	Combine like terms.
$(3 - 2 - 5)x + (1 + 2)y =$	Apply distributive property.
$-4x + 3y$	Simplify

(c) Removing a grouping symbol preceded by a factor.

Example 5. Removing grouping symbols

$-2x(4 - 5x)$

$= (-2x)4 - (-2x)(5x)$ Multiply each enclosed term by the factor.

$= -8x - (-10x^2)$ Add the products.

$= -8x + 10x^2$

Example 6. Removing grouping symbols and combine like terms.
$3x(2x + y) + 2y(5x + 2)$

Solution:

$3x(2x + y) + 2y(5x + 2) =$ Apply distributive property and multiply.

$(3x)(2x) + (3x)y + 2y(5x) + 2y(2) =$ Drop the ( ) and the + sign preceding it.

$6x^2 + 3xy + 10xy + 4y =$ Combine like terms.

$6x^2 + (3xy + 10xy) + 4y =$ Simplify.

$6x^2 + 13xy + 4y$

(d) Removing a grouping symbol followed by a factor.

Example 7. Removing grouping symbols

$(5 + 3x)(-2x)$

$= 5(-2x) + 3x(-2x)$ Multiply each enclosed term by the factor.

$= -10x + (-6x^2)$ Add the products.

$= -10x - 6x^2$

Example 8. Removing grouping symbols and combine like terms.
$(-2x^2 +xy -5y^2)(-3xy)$

Solution:

$(-2x^2 +xy -5y^2)(-3xy) =$ Apply distributive property when the factors appear at the right and multiply.

$(-2x^2)(-3xy) +(xy)(-3xy) -(5y^2)(-3xy) =$ Simplify.

$6x^3y -3x^2y^2 +15xy^3$

(e) **Removing a grouping symbol preceded by a - sign and followed by a factor.**

Example 9. Removing grouping symbols
$-(6x + 3x^2)(5x)$

$= -[(6x)(5x) +(3x^2)(5x)]$ Multiply each enclosed term by the factor.

$= -[30x^2 + 15x^3]$ Changed the sign of each enclosed term. Drop the grouping symbol and the - sign preceding it.

$= -30x^2 - 15x^3$

Example 10. Removing grouping symbols and combine like terms.
$-(2x + y)3x -(5x +2)(-2y)$

Solution:

$-(2x + y)3x -(5x +2)(-2y) =$ Apply distributive property and multiply.

$-[(2x)(3x) +y(3x)] -[(5x)(-2y) +(2)(-2y)] =$

$-[6x^2 +3xy] -[-10xy +(-4y)] =$ Remove () and + sign.

$-[6x^2 + 3xy] -[-10xy -4y] =$ Changed the sign of each enclosed term. Drop the grouping symbol and the - sign preceding it.

$-6x^2 - 3xy +10xy +4y =$ Combine like terms.
$-6x^2 + 7xy +4y$

A Word of Caution. A common mistake students make is to think that the distributive property applies to expressions like $2(3 \cdot 4)$

$2(3 \cdot 4) = (2 \cdot 3)(2 \cdot 4)$ The distributive property applies only when this is an addition.

$2(12) = 6 \cdot 8$

$24 = 48$

(f) Removing a grouping symbol within other grouping symbol.

Example 11. Removing grouping symbols

(a) $x - [y + (a - b)]$

$= x - [y + (a - b)]$ Remove ( ) preceded by + sign.

$= x - [y + a - b]$ Remove [ ] preceded by - sign.

$= x -y -a +b$

(b) $3 + 2[a -5(x -4y)]$

$= 3 + 2[a -5(x -4y)]$ Remove ( ) using distributive property.

$= 3 + 2[a -5x +20y]$ Remove [ ] using distributive property.

$= 3 + 2a -10x +40y$

(c) $(3a -b) -2\{x -[(y -2) -z]\}$

$= (3a -b) -2\{x -[(y -2) -z]\}$ Remove both pairs of ( ) preceded by no sign.

$= 3a -b -2\{x -[y -2 -z]\}$ Remove [ ] preceded by a - sign.

$= 3a -b -2\{x -y +2 +z\}$ Remove { } using the distributive property.

$= 3a -b -2x +2y -4 -2z$

Example 12. Remove grouping symbols and combine like terms.

(a)	$2x -[3y -2(x +4y)]$	
	$= 2x -[3y -2(x +4y)]$	Remove the innermost sign of grouping (parenthesis). Multiply -2 to all terms inside the parenthesis.
	$= 2x -[3y -2x -8y]$	Remove the outermost sign of grouping (bracket). The bracket is preceded by a minus sign so change the sign of all the terms enclosed by the bracket.
	$= 2x -3y +2x +8y$	Group together like terms.
	$= (2x +2x) + (8y -3y)$	Apply distributive property.
	$= (2 +2)x + (8 -3)y$	Simplify.
	$= 4x + 5y$	
(b)	$x -\{5y +[3x -2(2x- y)]\}$	
	$= x -\{5y +[3x -2(2x- y)]\}$	Multiply -2 to all the terms inside the parenthesis 2x -y to remove parenthesis.
	$= x -\{5y +[3x -4x+ 2y]\}$	Remove bracket without changing the terms enclosed since it is preceded by a positive sign.
	$= x -\{5y +3x -4x + 2y\}$	Remove braces. Change the sign of all terms enclosed since it is preceded by a minus sign.
	$= x -5y -3x +4x - 2y$	Group together like terms.
	$= (x -3x +4x) +(-5y -2y)$	Apply distributive property.
	$= (1 -3 +4)x + (-5 -2)y$	Simplify.
	$= 2x -7y$	

Let us again recall the rules for removing parenthesis from expressions to be sure that we have mastered them.

Rule 1. To remove parenthesis preceded by a plus sign (+), remove parenthesis and rewrite all the terms which are within the parenthesis without changing their signs and combine like terms.

Rule 2. To remove parenthesis preceded by a minus sign (-), remove parenthesis and write all the terms which are within the parenthesis but with the sign changed and combine like terms.

Rule 3. In case of parenthesis within parenthesis, remove one set at a time starting with the innermost parenthesis and combine like terms.

Let us look at some examples which will illustrate clearly the rules on removing parenthesis from expressions.

Rule 1.

To remove parenthesis preceded by a plus sign (+), remove parenthesis and rewrite all the terms which are within the parenthesis without changing their signs and combine like terms.

Example 13. Remove the grouping symbol.

$$\begin{aligned}
\text{(a)}\quad 3a^2 -2a + (2a^2 -5a +4) &= 3a^2 -2a + (2a^2 -5a +4) \\
&= 3a^2 -2a + 2a^2 -5a +4 \\
&= (3a^2 +2a^2) +(-2a -5a) +4 \\
&= 5a^2 -7a +4
\end{aligned}$$

$$\begin{aligned}
\text{(b)}\quad 3x +(4y +5) +6 &= 3x +(4y + 5) + 6 \\
&= 3x + 4y + 5 + 6 \\
&= 3x + 4y +(5 + 6) \\
&= 3x + 4y + 11
\end{aligned}$$

Rule 2.

To remove parenthesis preceded by a minus sign (-), remove parenthesis and write all the terms which are within the parenthesis but with the sign changed and combine like terms.

Example 14. Remove parenthesis and combine like terms.

$$
\begin{aligned}
\text{(a)}\quad 3a^2 -2a -(2a^2 -5a +4) &= 3a^2 -2a -(2a^2 -5a +4) \\
&= 3a^2 -2a -2a^2 +5a -4 \\
&= (3a^2 -2a^2) +(-2a +5a) -4 \\
&= a^2 +3a -4
\end{aligned}
$$

$$
\begin{aligned}
\text{(b)}\quad 3x -(4y +5) +6 &= 3x -(4y + 5) + 6 \\
&= 3x - 4y - 5 + 6 \\
&= 3x - 4y +(-5 + 6) \\
&= 3x - 4y + 1
\end{aligned}
$$

Rule 3.

In case of parenthesis within parenthesis, remove one set at a time starting with the innermost parenthesis and combine like terms.

Example 15. Remove parenthesis and combine like terms.

$$\text{(a)}\quad 3a -\{4a + 5 -[x +2y -\underbrace{(8x +y)}]\} + 4c$$

Remove innermost sign of grouping (parenthesis. The sign of each term enclosed in parenthesis must be changed since the parenthesis is preceded by a - sign.

3a -{4a + 5 -[x +2y -8x -y)]} + 4c	Remove the brackets. Change the sign of each term enclosed by the bracket since the bracket is preceded by a minus (-) sign.
3a -{4a + 5 -x -2y +8x +y} + 4c	Remove the braces. The terms enclosed with braces are preceded by a minus sign so change the sign of the terms within the braces.
3a -4a - 5 +x +2y -8x -y + 4c	Group together like terms.
(3a -4a) + 4c +(x -8x) +(2y -y)	Apply the distributive law.
(3 -4)a +4c +(1 -8)x +(2 -1)y	Simplify.
-a +4c -7x +y	Do not write anymore the numerical coefficient 1 of y and -1 of a.

(b)

4x + {2y -[6x -(2y + 6)] +8} +5	Remove parenthesis.
4x + {2y -[6x -2y - 6] +8} +5	Remove brackets.
4x + {2y -6x +2y +6 +8} +5	Remove braces.
4x + 2y -6x +2y +6 +8 +5	Combine similar terms.
(4x -6x) +(2y +2y) +(6 +8 +5)	Apply distributive law.
(4 -6)x +(2 +2)y + (6 +8 +5)	Simplify.
-2x + 4y +19	

NOTE:

In removing grouping symbols - parenthesis ( ), brackets [ ], and braces { }, see to it that you remove these grouping symbols completely. Remove the whole set not just one of it. The set is consist of a pair always.

Let us look at some examples of removing signs of grouping.

Example 16. Remove parenthesis and combine similar terms.

(a) $8x + 2(x + y) = 8x + 2\underset{\wedge}{(}x + y\underset{\wedge}{)}$

The parenthesis is not preceded by either a plus (+) or a minus (-) sign. It is immediately preceded by a term, in this case it is +2. Apply immediately the distributive property to remove parenthesis.

$= 8x + 2x + 2y$ Combine similar terms.

$= (8x + 2x) + 2y$ Apply distributive property.

$= (8 + 2)x + 2y$ Simplify.

$= 10x + 2y$

(b) $6a + 5(a - 3) = 6a + 5(a - 3)$

^ ^ — The parenthesis is remove by multiplying each term in the parenthesis by the term that precedes it. In this example the quantity (a -3) is thought to be preceded by +5 and not by 6a.

$= 6a + 5a - 15$ Combine similar terms.

$= (6a +5a) -15$ Apply distributive property.

$= (6 +5)a - 15$ Simplify.

$= 11a - 15$

(c) $2b -5(-a - 3b) = 2b -5(-a - 3b)$

^ ^ — The parenthesis is remove by multiplying each term in the parenthesis by the term that precedes it. In this example the quantity (-a-3b) is thought to be preceded by -5 and not by 2b.

$= 2b + 5a +15b$ Combine similar terms.

$= (2b+15b) +5a$ Apply distributive property.

$= (2+15)b + 5a$ Simplify.

$= 5a + 17b$

Let us test your computational skills by answering the exercises next page.

Test Your Computational Skills 4.2

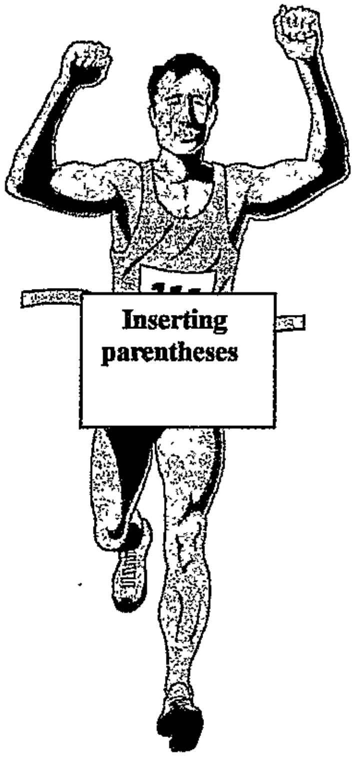
Remove signs of grouping and combine similar terms.

1. $-(2x + 4y)$
2. $+(2a + 3b)$
3. $4x + (2x - 3y)$
4. $a^2 +2b -(a^2 + b)$
5. $3x(4y + 5) + 2y(x + 3)$
6. $-5(x + 2) + 4(x + 2) +2y$
7. $x^3y^2z(x^2 -y^2 -z) + x^3y^2z$
8. $-a^2(b +2a +c) +(2b + 4c^2)$
9. $x^2 +2y -5z(x -y) - 2(2 -y)$
10. $2(a +b) +3(a -2b) -4(b -2a)$
11. $(x^3y^2z^2 - xyz)(x^2yz -x^2y^2z^3)$
12. $x^2 + 2x -[2x^2 +3x^2 -(y +3x)]$
13. $a^2 + b^2 +[2a^2 -4b +(a + b)] + 2$
14. $x^2 +\{4 -[3 + x -(x^2 + 8) +3x] -6\} +9y$
15. $a +2\{b -2[2a +b -3c(a -2b -5) +4a] -2c\} +a$

Module 4
Lesson 2

ALGEBRAIC EXPRESSION IV

(Grouping Symbols)



Grouping Symbols
(Removing & Inserting Grouping Symbols)

INSERTING GROUPING SYMBOLS
(Parentheses)

Let us look at examples of parenthesis or grouping symbol inserted in expressions.

Sometimes we have to insert grouping symbols in our answers for clarity and for making our presentation understandable. If this happens the rules for removal of parenthesis are also applicable to insertion of parenthesis. Just remember that if the terms to be enclosed in parenthesis are to be preceded by a plus (+) sign insert the parenthesis, keep the sign of the terms the same but if it to be preceded by a minus sign (-) sign, insert parenthesis with the sign of the terms to be enclosed changed.

Example11: Insert grouping symbols by writing the last two terms of the given expressions enclosed in parenthesis to be preceded by a positive sign (+).

9. $3x + 2y + 4$
10. $9x^2 + 3x + 11 - y^2 + 3y$

Solution:

9. $3x + 2y + 4$
$3x + (2y + 4)$

The last two terms are +2y and +4. Since it is to be enclosed in parenthesis preceded by a + sign the parenthesis will be inserted without changing the sign of the terms to be enclosed in parenthesis.

10. $9x^2 + 3x + 11 - y^2 + 3y$
$9x^2 + 3x + 11 + (- y^2 + 3y)$

The last two terms are $- y^2 + 3y$. So, the sign of the terms will not be changed when the terms will be placed within parenthesis.

Example 12: Use the same examples as numbers 9 and 10. Enclosed the last two terms within parenthesis to be preceded by a minus (-) sign.

11. $3x + 2y + 4$
12. $9x^2 + 3x + 11 - y^2 + 3y$

Solution:

11. $3x + 2y + 4$
$3x - (-2y - 4)$

The terms to be enclosed in parenthesis is to be preceded by a – sign. The terms within parenthesis take the opposite sign. The term 2y is now –2y, and +4 is now –4.

12. $9x^2 + 3x + 11 - y^2 + 3y$
$9x^2 + 3x + 11 - (y^2 - 3y)$

The terms within parenthesis take the opposite sign of the given term. $-y^2$ becomes $+y^2$ and $+3y$ becomes $-3y$.

Note:

When we put the terms in parenthesis preceded by a minus sign we are taking the additive inverse of the term or the negative of the term. This explains the change in sign.

Let us test your knowledge of inserting parenthesis in algebraic expressions by answering the Test Your Understanding next page.

Test Your Understanding 4.1

Write the given algebraic expressions with the last two terms enclosed in parenthesis preceded by (a) + sign and (b) - sign.

1. $x + 4$
2. $x^2 + y^2 + 6$
3. $3x^2 + 4 - y^2$
4. $a^2 - b^2 + c^2$
5. $x^2 + y^2 + 6$
6. $at^2 + bt + c$
7. $4x + 3y - 8$
8. $-3x^2 - y^2/4 + 3xy^2/8$
9. $a^3 + b^3$
10. $x^3 + y^3 + 8$

CONGRATULATIONS

You have just finished ALGEBRAIC EXPRESSION IV
(Grouping Symbols).
You are now ready to take the PRACTICE TASK
next page.
Please check your answer at the
FEEDBACK TO THE PRACTICE TASK.
It is important.

Practice Task

Evaluate each of the following expressions.

1. $-15\ -4(7-12)$
2. $17 - [(-9) + 15]$
3. $-2 + \{2\ 1/3 + (4 + 3)\}$
4. $16 + 3\{6 - [9 + 2(3 - 1)]\}$
5. $\sqrt[n]{8} - [-\sqrt{16} + 3(2/3)]$

Remove grouping symbols and combine like terms.

6. $2x^2 + (x^2 + 2y)$
7. $(ab^2 + 4ab^2) + 4$
8. $(2x + y) + (y - 3x) + y^2$
9. $4x - (2 + 3y)$
10. $8a - (4a + c) + 2c$
11. $-(a + b) - (b - a) + c$
12. $4 + 3(x + y)$
13. $2x^3 + 4x(x - 8)$
14. $2(x + y) - 3x + 8(x + y)$
15. $-x^2\ (x + y) + y^2$
16. $-2(x\,y - xy)$
17. $-4a(b + c) + 3 - ab(1 + c)$
18. $3x + [4 - 2(y + x)]$
19. $2xy - [2x + (y\ -4)]$
20. $2x + 4y\ [x - 2(y - 3)]$

You must score 16
or higher of the
PRACTICE TASK.
If you score 10 or less,
please go over
MODULE 4 again.
It is important.

Answers to **Test Your Understanding** and **Test Your Computational Skills** are also provided on separate sheets placed after the Feedback to the Practice Task.

Please check your answers. Okey?

Feedback to the Practice Task

1.
$$\begin{aligned} -15\ -4(7-12) &= -15\ - 4(-5) \\ &= -15\ + 20 \\ &= 5 \end{aligned}$$

2.
$$\begin{aligned} 17-[\,(-9)+15] &= 17-[\,(-9)+15] \\ &= 17-(-9\ + 15) \\ &= 17 - 6 \\ &= 11 \end{aligned}$$

3.
$$\begin{aligned} -2+\{2\ 1/3\ +\ (4+3)\} &= -2+\{2\ 1/3\ +\ (4+3)\} \\ &= -2+\{2\ 1/3\ +\ 7\} \\ &= -2+\{9\ 1/3\} \\ &= 7\ 1/3 \end{aligned}$$

4.
$$\begin{aligned} 16\ +3\{6-[9+2(3-1)]\} &= 16\ +3\{6-[9+2(3-1)]\} \\ &= 16\ +3\{6-[9+2(2)]\} \\ &= 16\ +3\{6-[9+4]\} \\ &= 16\ +3\{6-\ 13)\ \} \\ &= 16\ +3\{\ -7\ \} \\ &= 16\ - 21 \\ &= -5 \end{aligned}$$

5.
$$\begin{aligned} \sqrt[3]{8}\ -[-\sqrt{16}\ +\ 3(2/3)] &= \sqrt[3]{8}\ -[-\sqrt{16}\ +\ 3(2/3)] \\ &= 2\ -[-4\ \ +\ 2] \\ &= 2\ -(-2) \\ &= 4 \end{aligned}$$

6. $2x^2 + (x^2 + 2y) = 2x^2 + (x^2 + 2y)$
$2x^2 + x^2 + 2y$
$3x^2 + 2y$

7. $(ab^2 + 4ab^2) + 4 = (ab^2 + 4ab^2) + 4$
$5ab^2 + 4$

8. $(2x + y) + (y - 3x) + y^2 = (2x + y) + (y - 3x) + y^2$
$2x + y + y - 3x + y^2$
$-x + 2y + y^2$

9. $4x - (2 + 3y) = 4x - (2 + 3y)$
$= 4x - 3y - 2$

10. $8a - (4a + c) + 2c = 8a - (4a + c) + 2c$
$= 8a - 4a - c + 2c$
$= 4a + c$

11. $- (a + b) - (b - a) + c = - (a + b) - (b - a) + c$
$= -a - b - b + a + c$
$= -2b + c$

12. $4 + 3 (x + y) = 4 + 3 (x + y)$
$4 + 3x + 3y$

13. $2x^3 + 4x (x - 8) = 2x^3 + 4x (x - 8)$
$= 2x^3 + 4x^2 - 32x$

14. $2 (x + y) - 3x + 8 (x + y) = 2 (x + y) - 3x + 8 (x + y)$
$= 10 (x + y) - 3x$
$= 10x + 10y - 3x$
$= 7x + 10y$

15. $-x^2 (x + y) + y^2 = -x^2 (x + y) + y^2$
$-x^3 - x^2 y + y^2$

16. $$\begin{aligned} -2(x\,y - xy) &= -2(x\,y - xy) \\ &= -2\,x\,y + 2xy \\ &= 0 \end{aligned}$$

17. $$\begin{aligned} -4a(b + c) + 3 - ab(1 + c) &= -4a(b + c) + 3 - ab(1 + c) \\ &= -4a\,b - 4ac + 3 - ab - abc \\ &= -5\,ab - 4ac - abc + 3 \end{aligned}$$

18. $$\begin{aligned} 3x + [4 - 2(y + x)] &= 3x + [4 - 2(y + x)] \\ &= 3x + [4 - 2\,y - 2x] \\ &= 3x + 4 - 2\,y - 2x \\ &= x - 2y + 4 \end{aligned}$$

19. $$\begin{aligned} 2xy - [2x + (y - 4)] &= 2xy - [2x + (y - 4)] \\ &= 2xy - 2x - y + 4 \end{aligned}$$

20. $$\begin{aligned} 2x + 4y\,[x - 2(y - 3)] &= 2x + 4y\,[x - 2(y - 3)] \\ &= 2x + 4y\,[x - 2y + 6] \\ &= 2x + 4xy - 2y^2 + 24y \end{aligned}$$

ANSWERS

Test Your Computational Skills 4.1

1. –1
2. –2
3. 12
4. 11
5. 15
6. 6
7. –26
8. –25
9. 12
10. 3
11. –1
12. –4
13. 23
14. 30
15. –35
16. –16
17. 5
18. 12
19. 7
20. 8

Test Your Computational Skills 4.2

1. $-2x - 2y$
2. $2a + 3b$
3. $6x - 3y$
4. b
5. $14xy + 15x + 6y$
6. $-x + 2y - 2$
7. $x^5 y^2 z \quad -x^3 y^4 z \; -x^3 y^2 z^2 \quad +x^3 y^2 z$
8. $-2a^3 \; -a^2 b - a^2 c + 2b + 4c^2$
9. $x^2 \; + 4y - 5xz \; + 5yz - 4$
10 $13a - 8b$
11 $x^5 y^3 z^3 \; - x^5 y^4 z^5 - x^3 y^2 z^2 \; + x^3 y^3 z^4$
12 $-4x^2 \; + 5x + y$
13 $3a^2 \; + a + b^2 - 3b + 2$
14 $2x^2 \; + 2x \; + 3$
15 $-22a - 2b - 64c + 12ac - 24b$

Test Your Understanding 4.1

(a) preceded by a + sign

1. $+(x + 4)$
2. $x^2 + (y^2 + 6)$
3. $3x^2 + (4 - y^2)$
4. $a^2 + (-b^2 + c^2)$
5. $x^2 + (y^2 + 6)$
6. $at^2 + (bt + c)$
7. $4x + (3y - 8)$
8. $-3x^2 + (-y^2/4 + 3xy^2/8)$
9. $+(a^3 + b^3)$
10. $x^3 + (y^3 + 8)$

(b) preceded by a - sign

1. $-(-x - 4)$
2. $x^2 - (-y^2 - 6)$
3. $3x^2 - (-4 + y^2)$
4. $a^2 - (b^2 - c^2)$
5. $x^2 - (- y^2 - 6)$
6. $at^2 - (- bt - c)$
7. $4x - (- 3y + 8)$
8. $-3x^2 - (y^2/4 - 3xy^2/8)$
9. $-(-a^3 - b^3)$
10. $x^3 - (y^3 - 8)$

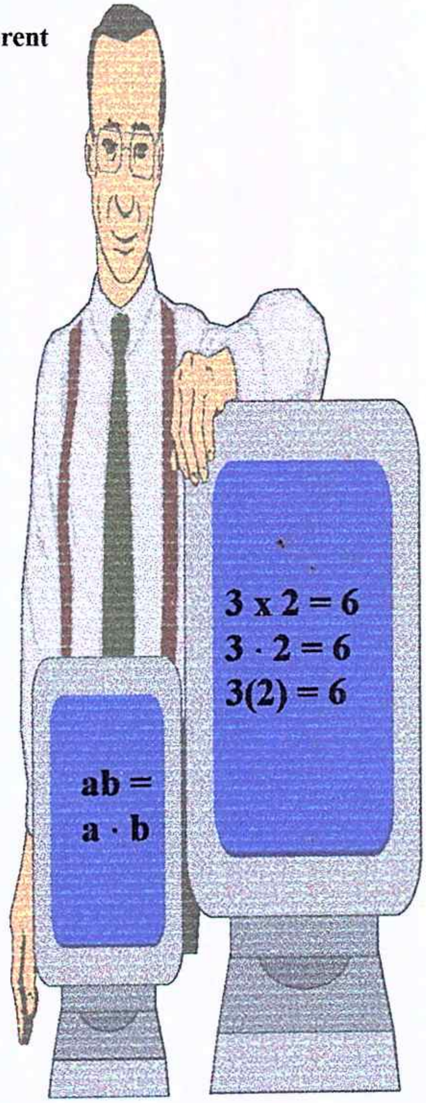
Module 5
Lesson 1

ALGEBRAIC EXPRESSION V

Multiplication of Polynomials

Multiplication may be shown in many different ways.

1. $3 \times 2 = 6$
2. $3 \cdot 2 = 6$ The multiplication dot "$\cdot$ " is written a little higher than the decimal point.
3. $3(2) = 6$ The symbols ( ) are called parentheses.
4. $(3)(2) = 6$ In this kind of multiplication the double parentheses are not necessary.
5. $ab = a \cdot b$ When two letters are written next to each other in this way , it is understood that they are to be multiplied. When two numbers are written next to each other, they are not to be multiplied. For example: 23 does not mean $2 \cdot 3 = 6$.



Multiplication of Signed Numbers

OVERVIEW:

Multiplication is one of the fundamental operations in arithmetic. The operation multiplication can be defined in terms of another operation, i.e., addition.

Multiplication is but repeated addition.

Just like addition, multiplication is a binary operation. We operate on two elements of a set - the multiplicand and the multiplier. The multiplicand and multiplier are also called the factors.

The result of the operation is called the product. To obtain the product as a result of the multiplication process would require knowledge of the following mathematical facts: multiplication of signed numbers, application of the laws of exponents, properties of real numbers, and knowledge on how to combine terms.

The result of multiplication can be checked by another operation --- division or numerical substitution.

The operation multiplication is denoted by the following symbols - dot (.), x, or by symbols of grouping the parentheses; where one of the factor is enclosed within parenthesis and the other factor precedes it.

Our concept of multiplication is that, the product results into a numerical quantity very mush greater in numerical value than any of the two numbers multiplied together, such is not always true in case of products of proper fractions and the products of a negative and a positive number. Also you might think that there is magic in multiplication since the product of two numbers very much less than zero (negative) will yield a number very much greater in value than zero, since negative number times another negative number is equal to a positive number.

Let us explore the magic brought about by this operation. Let's multiply. Okey?

What are signed numbers (integers)?

What kind of set is the set of integers?

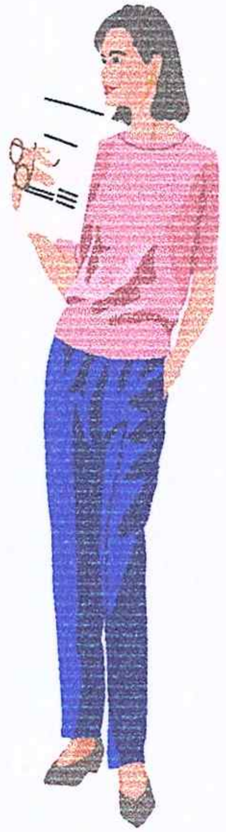
What are the elements of the set of integers?

What is the product of two signed numbers (integers)?

How do you find the product of two signed numbers (integers)?

How do you determine the sign of the product of two integers?

Read Lesson 1 of Module 5 and you will be able to multiply integers.
Okey?



Objective: *At the end of this lesson you should be able to multiply signed numbers (integers).*

Input!

Multiplication of Signed Numbers

Multiplication is a binary operation so we have to operate on two elements of a set. These two elements are called the **multiplicand** and **multiplier** respectively. The multiplicand and multiplier are also called the **factors.**

The result of the operation is called the **product.** In symbols, if ab = c, c is the product and a and b are the factors. Multiplication is commutative so if a is named the multiplicand and b the multiplier we can also named b the multiplicand and a the multiplier.

The symbols used to indicate multiplication are the dot (.) placed in between the multiplicand and multiplier or factors and **x. Parentheses** are also used to indicate multiplication. When parenthesis is used the dot or x is usually omitted. Also, in case of single literal quantities the dot and x are usually omitted. The multiplication of a and b is denoted by a.b, a x b, (a)(b), or ab.

Let us master the multiplication of algebraic expressions.

Mastery of the operation can be achieved if we have knowledge of some facts related to the operation.

For starters let us have a review of multiplication of integers or signed numbers.

The multiplication of integers can be summed up in these two laws below:

Laws for the Multiplication of Integers or Signed Numbers

Law 1. The product of two numbers having the same sign is positive.

Law 2. The product of two numbers having unlike sign is negative.

Remember:

Law 1. The product of two numbers having the same sign is positive.

This means that the product of two negative numbers is positive and the product of two positive numbers is positive.

Example 1. Find the product of

1. (-2) and (-3)
2. (-5) and (-7)
3. (+4) and (+1)
4. (+20) and (+3)
5. (+53) and (+10)

Solution:

→ Factors have like signs

1. (-2)(-3) = 2 x 3 = 6 = + 6 → Multiply the absolute value of the factors

2. (-5)(-7) = 5 x 7 = 35 = +35 → Result of the multiplication of the absolute value of the factors

3. (-4)(-1) = 4 X 1 = 4 = + 4 → Final answer

4. (+20)(+3) =20 x 3 = 60 = +60

5. (+53)(+10) =53 x10 =530 =+530

Recall:

Law 2. The product of two numbers having unlike sign is negative.

This means that the product of a negative and a positive number is negative or the product of a positive and a negative number is negative.

Example 2. Find the product of

1. (+2) and (-3)
2. (+5) and (-7)
3. (+4) and (-1)
4. (-20) and (+3)
5. (+53) and (+10)

Solution:

```
        ┌────────────────────────────> Factors have like
        |        ┌──────────────────┐  signs
        |        |      ┌─────────┐ |
        |        |      |   ┌───┐ | |
1.  (+2)(-3) = 2 x 3 =  6 = - 6 | | └> Multiply the absolute
                                | |    value of the factors

2.  (+5)(-7) = 5 x 7 = 35 = -35 | |
                                | └──> Result of the multi-
3.  (+4)(-1) = 4 x 1 =  4  = -4 |      plication of the
                                |      absolute value of
4. (-20)(+3) =20 x 3 = 60 = -60 |      the factors
                                |
5.(-53)(+10) =53 x10 =530 =-530 └────> Final answer
```

Looking at the examples given, you'll notice that it is easy to master multiplication of signed numbers or integers.

We can summarize the procedure into the following steps:

<u>Steps</u>:

1. Forget about the signs of the factors. If necessary acquire a temporary amnesia. You are actually using the concept of absolute value of a number unknowingly.

   Recall:

   > The absolute value of a number is equal to the number regardless of the sign.

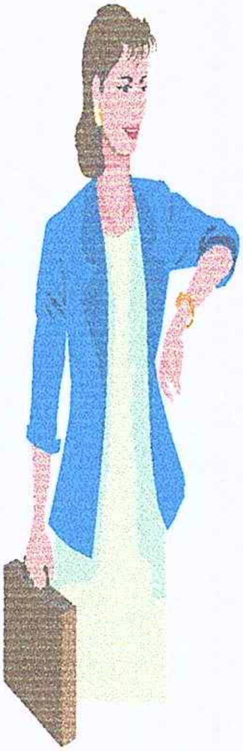
2. Multiply the factors to obtain the product.
3. Open your eyes wide so that you'll see clearly the sign of the factors. You still have to prefix a sign to the final answer.

   Do not forget this

   LIKE SIGN ------ positive (+)
   UNLIKE SIGN ------ negative (-)

4. Depending on your 20-20 vision prefix a (+) or a (-) sign to the product applying Law I and Law II.
5. You are now okey. You have no amnesia, your vision is perfect and you have the photographic memory of Apolinario Mabini. Good luck.

Let us test your computational skills on the multiplication of integers or signed numbers.



Test Your Computational Skills 5.1

Find the product of the given factors.

1. (-10) (+20) =
2. (-8) (-3) =
3. (-80) (-35) =
4. (+60) (+15) =
5. (140) (75) =
6. (-175) (+880) =
7. (-565) (+2) =
8. (+750) (-81) =
9. (280) (-500) =
10. (-480) (-5) =

Ops! Do not hurry. Review your work.

Example 3. Multiply (-3), (+20), and (-5).

Solution: Since multiplication is a binary operation so we have 1) to find the product of any two two of the factors, then 2) the obtained product is to be multiplied again to the remaining factor to get the final answer.

We can multiply (-3) and (+20) first and the resulting product will be multiplied to (-5) or get the product of (+20) and (-5) then multiply it to (-3) or find the product of (-3) and (-5), then multiply it to (+20). In all cases you will obtain exactly the same answers.

Look!!!

1. [(-3)(+20)] (-5) = (-60)(-5) = +300
2. (-3) [(+20)(-5)] = (-3)(-100) = +300
3. (+20) [(-3)(-5)] = (+20)(+15) = +300

Now, suppose you are asked to multiply 3 or more factors with different signs, how will you go about it?

Recall:

Multiplication is both associative and comutative.

What you should do is but apply the laws for multiplication of integers or signed numbers. Okey?

Let us solve some examples.

Example 4. Find the product of the following:

a. (-2)(-4)(+5)(+6)
b. (-1)(+4)(-5)(-1)
c. (+40)(+10)(+5)(-3)(-2)

Solution:

a. (-2)(-4)(+5)(+6) =
[(-2)(-4)][(+5)(+6)] =
(+8)(+30) =
+240

b. (-1)(+4)(-5)(-1) =
[(-1)(+4)][(-5)(-1)] =
(-4)(+5) =
-20

c. $$\begin{aligned} &(+40)(+10)(+5)(-3)(-2) = \\ &(+40)[(+10)(+5)][(-3)(-2)] = \\ &(+40)(+50)(+6) = \\ &(+40)(+300) = \\ &+12{,}000 \end{aligned}$$

Your process in finding the product of several factors may be different from the solution presented but the final answer must be the same.

What is important here is that you should not make mistakes on the sign of the final answer.

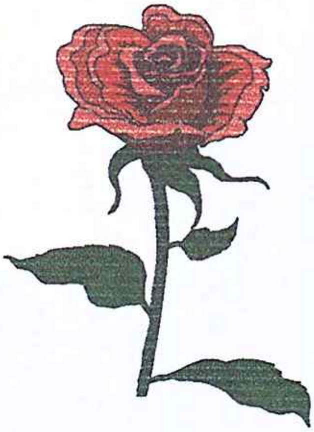
By looking at the examples presented maybe you have already devised a technique for determining the sign of the final answer.

Just count the factors having negative signs. If the number of factors with negative signs is odd the sign of the product is negative, if it is even then the sign of the product is positive.

Why will we not consider the number of positive factors?

Recall:

The product of two positive numbers is **positive.**

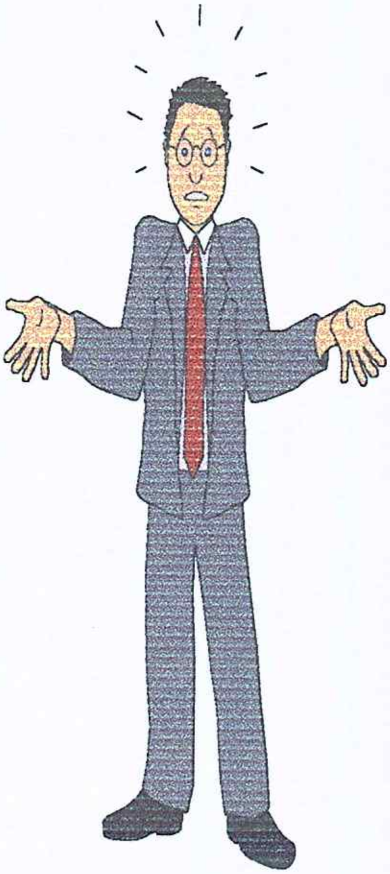


We are sure of the product of the positive factors to be positive, so determining the sign of the final answer will depend on the factors with negative signs.

Recall:

The product of two negative numbers is positive.

Thus, counting only the factors with negative signs we will have an idea of the sign of the final answer.



Odd number of negative sign - - - negative
Even number of negative sign - - positive

If the sign of the product of all the factors with negative signs is negative (odd number), the sign of the final answer is negative. Since, the sign of the product of all the positive factors is positive. So, we are multiplying 2 numbers having unlike signs, the rule states that the resulting product should have a negative sign.

If the sign of the product of all the negative factors is positive, this result when the number of negative factors is even, the sign of the final answer is positive. We are multiplying two factors having like signs and the rule says, the sign of the product of two positive factors is positive. Okey?

Reminder:

To multiply two or more signed numbers, find the product of their absolute values and prefix the negative sign when the number of negative factors is odd and a positive sign when the number of negative factors is even.

Let us test your computational skills in finding the product of the given factors.

Test Your Computational Skills 5.2

Determine the sign of the final answer without doing the actual multiplication.

1. (+20)(-5)(-209)
2. (+6)(+11)(+4)
3. (-5)(-8)(-7)
4. (+20)(+50)(+60)(+9)
5. (-1)(-1)(-5)(-6)(-7)
6. (+5)(+1)(+7)(+5)(-6)
7. (+3)(+7)(+2)(+3)(-4)(-2)
8. (-1)(-1)(-1)(+1)(+1)(+1)(+1)
9. (-7(-8)(-5)(-6)(-3)
10. (1)(2)(3)(4)(5)(-1)(-2)(-3)

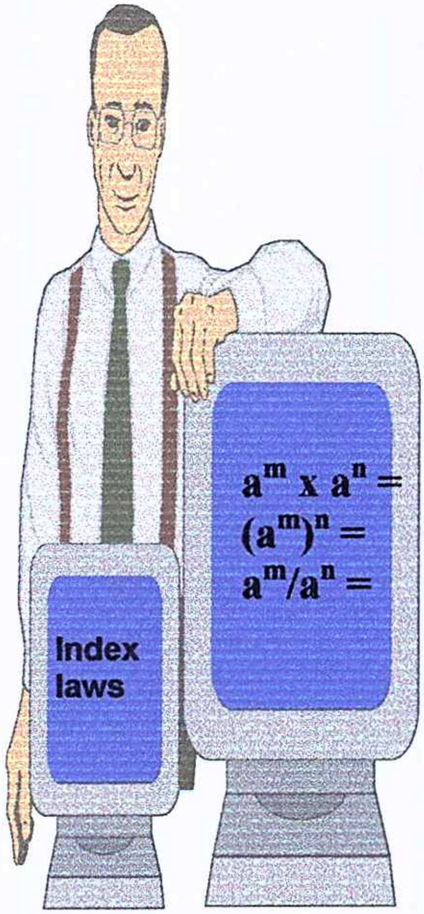
Give the product of the following factors:

11. (-1)(+1)(-2)(+2)(-3)(+3)
12. (10)(9)(8)(7)
13. (-1)(-2)(-3)(-4)(-5)
14. (-1000)(450)(-500)
15. (-5)(-6)(+5)(+6)(-1)(+1)

Module 5
Lesson 2

ALGEBRAIC EXPRESSION V

Multiplication of Polynomials



Index Laws or Laws of Exponents

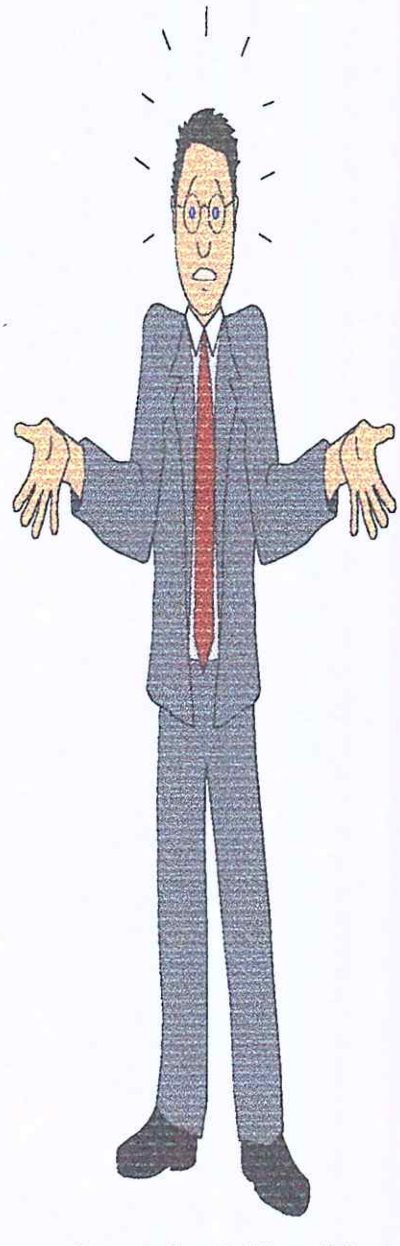
Mathematicians uses some numbers (symbols) to denote or indicate the mathematical operation - **multiplication.**

Exponents are numbers (symbols) written at the upper right of another number or symbols.

Exponents are cute because they represent a more compact way of writing the factors and they are typewritten in smaller type size.

Would you like to know more about exponents?

Read Lesson 2 of Module 5 and know your exponents.
Okey?



Objective: At the end of this lesson the students should be able to:

1. simplify expressions with exponents.
2. apply the laws of exponents in simplifying algebraic expressions.

INPUT!

Let us have a look at exponents for the second time.

Exponents are but symbols that indicate repeated multiplication.

In 2^3 for example, the base is 2 and the exponent is 3. 2^3 means that 2 is used as a factor three times or

$$2^3 = 2 \times 2 \times 2.$$

Notice:

In the expression above, in raising a number to a certain power we involved only one element of a set - the number 2.

$$2^3 = 2 \times 2 \times 2 = 8$$

Raising to a power is a **unary operation**, so it involves only one element of a set.

Mathematical expressions involve quantities raised to a certain specified power and operations like multiplication of these quantities need knowledge of the whereabouts of powers and exponents and the simplification of exponents or indices are governed by the laws of exponents.

Let us have a review of the laws of exponents or the index laws. Okey?

INDEX laws (for m, n positive integers)

a) Numbers with same base

1. $a^m \times a^n = a^{m+n}$
2. $a^m / a^n = a^{m-n}$, $m > n$ and $a = 0$
   $a^m / a^n = a^{n-m}$, $m < n$ and $a = 0$

b) Numbers with different bases

3. $(a^m)^n = a^{mn}$
4. $(ab)^m = a^m \times b^m$
5. $(a/b)^m = (a^m/b^m$, $b = 0$

We can group the laws of exponents (index laws) into 2 categories.

The first group involves quantities or numbers with like bases and the second group involves quantities or numbers with different bases.

Let us look at the first law, the mathematical expression of the first law is

$$a^m \times a^n = a^{m+n}.$$

This means that in multiplying quantities and numbers with like bases, keep the base and add the exponents.

Example 5. Apply the index laws and simplify.

a) $2^3 . 2^4 =$
b) $3^2 . 3 =$
c) $4^2 . 4^2 =$
d) $5 . 5^2 =$

Solutions:

a) $2^3 . 2^4 = 2^{3+4} = 2^7 = 128$
b) $3^2 . 3 = 3^{2+1} = 3^3 = 27$
c) $4^2 . 4^2 = 4^{2+2} = 4^4 = 256$
d) $5 . 5^2 = 5^{1+2} = 5^3 = 125$

Law 2 can be stated as:

> In dividing numbers with like bases keep the base and subtract their exponents.

$$a^m/a^n = a^{m-n},\ m>n$$
$$a^m/a^n = a^{n-m},\ m<n$$

Example:

a. $2^3/2^2 =$
b. $3^2/3 =$
c. $4^2/4^2 =$
d. $6^2/6^4 =$
e. $5/5^2 =$

Solutions:

$m > n$ a. $2^3/2^2 = 2^{3-2} = 2^1 = 2$
$m > n$ b. $3^2/3 = 3^{2-1} = 3^1 = 3$
$m = n$ c. $4^2/4^2 = 4^{2-2} = 4^0 = 1$
$m < n$ d. $6^2/6^4 = 6^{2-4} = 6^{(2)+(-4)}=6^{-2} = 1/6^2 = 1/36$
$m < n$ e. $5/5^2 = 5^{1-2} = 5^{(1)+(-2)}=5^{-1} = 1/5^1 = 1/5$

Note:
Subtraction of exponents is meant to be algebraic subtraction.

In the division process, the divisor must not equal to zero, since division by zero is not defined.

Law 3:

In raising a power to a power, multiply powers.

$$(a^m)^n = a^{mn}.$$

Example 7. Simplify

a. $(2^3)^4 =$
b. $(3^2)^2 =$
c. $(4^3)^2 =$
d. $(5^3)^2 =$

Solution:

a. $(2^3)^4 = 2^{3x4} = 2^{12}$
b. $(3^2)^2 = 3^{2x2} = 3^4$
c. $(4^3)^2 = 4^{3x2} = 4^6$
d. $(5^3)^2 = 5^{3x2} = 5^6$

Law 4:

In raising a product to a power, multiply and raise each factor to the indicated power.

$$(ab)^m = a^m \times b^m.$$

Example B. Simplify.

a. $[(2)(4)]^2 =$
b. $(5 \times 6)^3 =$
c. $(2 \times 3 \times 5)^4 =$
d. $(2 \times 8 \times 9 \times 12)^2 =$

Solutions:

a. $[(2)(4)]^2 = 2^2\ 4^2$
b. $(5 \times 6)]^3 = 5^3\ 6^3$
c. $(2 \times 3 \times 5)^4 = 2^4\ 3^4\ 5^4$
d. $(2 \times 8 \times 9 \times 12)^2 = 2^2\ 8^2\ 9^2\ 12^2$

Law 5:

In raising a quotient to a certain power raised each term of the quotient to the indicated power then carry on the division.

$$(a/b)^m = a^m/b^m, \quad b = 0.$$

Example 9. Apply Law 5 and simplify.

a. $(2/3)^4 =$
b. $(1/3)^2 =$
c. $(5/4)^3 =$
d. $(7/8)^2 =$

Solutions:

a. $(2/3)^4 = 2^4/3^4 = 16/81$
b. $(1/3)^2 = 1^2/3^2 = 1/9$
c. $(5/4)^3 = 5^3/4^3 = 125/64$
d. $(7/8)^2 = 7^2/8^2 = 49/69$

Let us test your computational skills in the application of the index laws. Okey?

Test Your Computational Skills 5.3

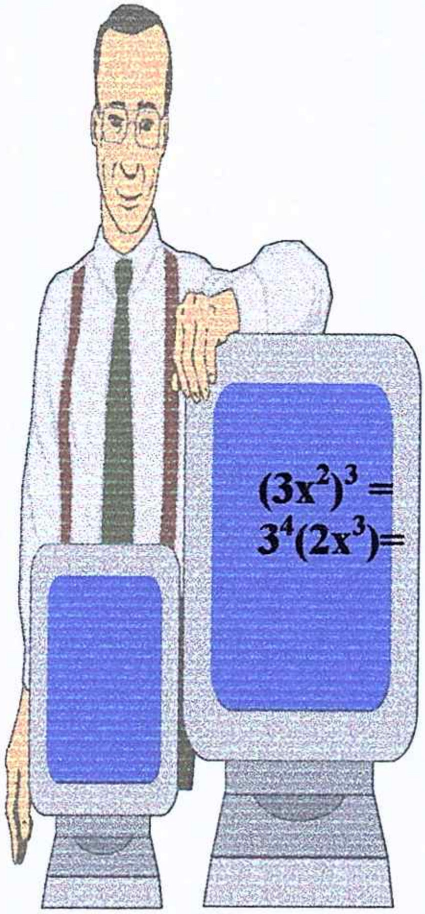
Apply the index laws and simplify.

1. $(ab)^4 (ab)^3 =$
2. $(2xy)^2 (2xy)^3 =$
3. $8^3 . 8$
4. $3^2x^2y^4 . 9x^2y^4 =$
5. $(ab)^4 / (ab)^2$, $a, b = 0$
6. $(2xy)^4 / 2xy$, $x, y = 0$
7. $(2x^2y^3)^4$
8. $(2a^2bc^3)^2$
9. $(z^4)^3$
10. $[2^2xy]^4$
11. $[2a^2b / 3cd]^4$
12. $[x / y]^2$, $y = 0$
13. $[2a / 3b]^2$ $b = 0$
14. $[xy / y^2z^4]^3$, $y, z = 0$
15. $[3^c / 2^d]^2$

Module 5
Lesson 3

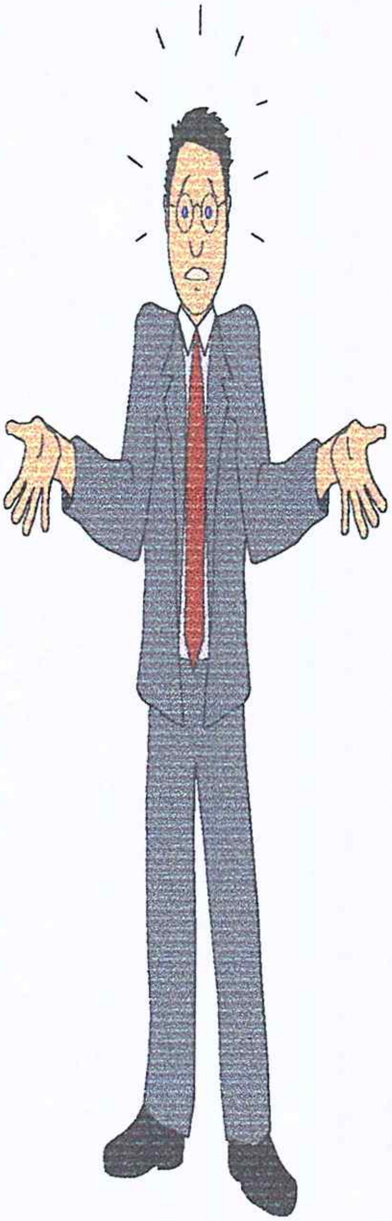
ALGEBRAIC EXPRESSION V

Multiplication of Polynomials



Operations with Integer Exponents

MORE ABOUT EXPONENTS



Let us observe if the index laws still apply to exponents which are not positive integers.

A detailed explanation about exponents is discussed under operations with integer exponents.

Also the examples presented under index laws are all numbers except in the test your computational skills, where you meet variables raised to a power.

If problems occur in answering the test, read Lesson 3, of Module 5. Okey?

You will be exposed once again to exponents.

Objective: At the end of this lesson the students should be able to:

1. multiply expressions with integral powers or exponents.
2. simplify expressions applying the index laws.
3. write expressions as one with positive indices.

INPUT

Operation with Integer Exponents

Multiplication of Powers with the Same Base.

When you multiply monomials with the same base keep the base and add the exponents.

$a^m \cdot a^n = a^{m+n}$

Example 10. Multiply $2x^3y \cdot 4x^2y^3$

Solution:

$$2x^3y \cdot 4x^2y^3 = (2 \times 4)(x^{3+2})(y^{1+3})$$

Multiply the numerical coefficient of the factors to get the numerical coefficient of the resulting product.

Add the exponents of the same literal factors to get the exponent of the literal factors of the product.

The resulting product is a product of the numerical coefficient of the factors and the literal coefficients of the factors with the same letters not repeated . Okey?

Zero as an Exponent

Any real number to the zero power is equal to 1. In symbol, for any real number a,

$$a^0 = 1.$$

Example: Multiply $a^3 \cdot a^0$

Solution; $a^3 \cdot a^0 = a^3 \cdot 1 = a^3$

Recall: The multiplication law of powers states that:

To multiply two powers of the same base keep the base and add the exponents.

$$a^3 \cdot a^0 = a^{3+0} = a^3$$

This implies that $a^0 = 1$, since $1 \cdot a^3 = a^3 \cdot 1 = a^3$.

Negative Power in the Numerator

Any power in the nemerator with a negative exponent can be written in the denominator with a positive exponent.

$a^n = 1/a^n$ for any $a \neq 0$

Example 12. Write the following with positive exponents.

a. p^{-3}
b. $p^{-2} . q^3$
c. $(2x)^{-4}$
d. x^{-5}
e. $x^{-3} y^{-5}$
f. $2x^{-4}$

Recall:

A power with a negative exponent in the numerator can be placed in the denominator with positive exponent.

Also any whole number can be written as a factor with 1 as its denominator.

In the factor a/b, a is the numerator, b is the denominator.

Solutions:

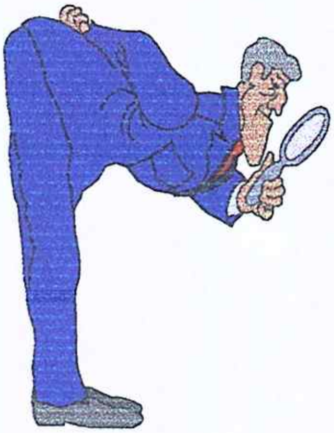
a. $p^{-3} = 1/p^3$
b. $p^{-2} . q^3 = 1/p^2 .q^3 = q^3/p^2$
c. $(2x)^{-4} = 1/(2x)^4 = 1/2^4x^4 = 1/16x^4$
d. $x^{-5} = 1/x^5$
e. $x^{-3}y^{-5} = 1/x^3 . 1/y^5 = 1/x^3y^5$
f. $2x^{-4} = 2 . 1/x^4 = 2/x^4$

Note:

The law for multiplication of powers still holds for negative exponents.

$$\underset{\textbf{powers multiplied}}{a^3 \cdot a^{-2}} = \underset{\textbf{exponents added}}{a^{3+(-2)}} = a^{3-2} = a$$

Let us look at the solution to example **f.** You'll notice that the number 2 remains in the numerator. How come that 2 stayed in the numerator?



Note:

The exponent acts only on the symbol it follows. If you want it to act on more than one symbols, you must enclosed them in parenthesis.

To illustrate this mathematical fact look at example c.

Example 13. Write the following with positive exponents.

a. $2x^{-3}$
b. $(2x)^{-3}$
c. $4x^{-2}/y^3$
d. $(4x)^{-2}/y^3$

Solutions:

a. $2x^{-3} = 2 \cdot 1/x^3 = 2/x^3$
b. $(2x)^{-3} = 1/(2x)^3 = 1/2^3x^3 = 1/8x^3$
c. $4x^{-2}/y^3 = 4 \cdot 1/x^2 \cdot 1/y^3 = 4/x^2y^3$
d. $(4x)^{-2}/y^3 = 1/(4x)^{-2} \cdot 1/y^3 = 1/4^2x^2 \cdot 1/y^3 = 1/16x^2 \cdot 1/y^3 = 1/16x^2y^3$

Negative Exponents in the Denominator

Any power in the denominator wih negative exponents can be written with a positive exponent in the numerator.

$1/x^{-n} = x^n$ where $x \neq 0$

Note:

$$1/x^{-2} = \underset{\substack{\text{definition}\\ \text{of negative}\\ \text{exponent}}}{1/1/x^{2}} = 2 \div 1/x^{2} = \underset{\substack{\text{definition}\\ \text{of division}}}{1 \cdot x^{2}/1} = x^{2}/1 = x^{2}.$$

Try this! Okey ?

Example 14. Write the following without negative exponents.

a. $x^{-3} =$

b. $1/x^{-3} =$

c. $x^{-2}/y^{-3} =$

d. $x^{4}/y^{-3} =$

e. $2x^{-3}/y^{2} =$

f. $5/x^{-2}y^{-3} =$

Solutions:

a. $x^{-3} = 1/x^{3}$

b. $1/x^{-3} = 1 \cdot 1/x^{-3} = 1 \cdot x^{3}/1 = x^{3}$

c. $x^{-2}/y^{-3} = x^{-2} \cdot 1/y^{-3} = 1/x^{2} \cdot y^{3}/1 = y^{3}/x^{2}$

d. $x^{-4}/y^{-3} = x^{-4} \cdot 1/y^{-3} = 1/x^{4} \cdot y^{3}/1 = y^{3}x^{4}$

e. $2x^{-3}/y^{2} = 2x^{-3} \cdot 1/y^{2} = 2/x^{3} \cdot 1/y^{2} = 2/x^{3}y^{2}$

f. $5/x^{-2}y^{-3} = 5 \cdot 1/x^{-2} \cdot 1/x^{-3} = 5 \cdot x^{2}/1 \cdot y^{3}/1 = 5x^{2}y^{3}$

Remember:

If a power with a negative exponent appears as a factor in the denominator, it can be placed in the numerator with positive exponent.

Power Raised to a Power
$(a^x)^y = a^{xy}$

Now look !!!

$(a^2)^3$ means $(a^2)(a^2)(a^2) = a^{2+2+2} = a^6$ also $a^{2\times 3} = a^6$.

This observation leads to the following property:

To raise a power to a power, keep the base, multiply the exponents.

Example 15. Simplify the following:

a. $(x^4)^2 =$
b. $(a^5)^4 =$
c. $(y^3)^5 =$

Multiply exponents

Solutions:

a. $(x^4)^2 = x^{4\cdot 2} = x^8$
b $(a^5)^4 = a^{5\cdot 4} = a^{20}$
c. $(y^3)^5 = y^{3\cdot 5} = y^{15}$

Power of power

Power of a Product
$(a^n . b^m)^x = a^{nx}\cdot b^{mx}$

To see what happens when a product like (ab^2) is raised to a power, use the basic definition of exponent as repeated multiplication.
$(ab2)^3$ means $(ab^2)(ab^2)(ab^2)$.

Therefore:

$(ab^2)^3 = (ab^2)(ab^2)(ab^2)$
Since multiplication is commutative,
$= a \cdot a \cdot a \cdot b^2 \cdot b^2 \cdot b^2$
$= a^{1+1+1}b^{2+2+2} = a^3b^6$

Notice that this is the same result as if we had multiplied each of the exponents.
$(ab^2)^3 = a^{1 x 3}\ b^{2 x 3} = a^3 b^6$.

Let us apply what we have learned to example 16 and 17. Okey?

Example 16. Simplify each of the following:

a. $(x^3y^4)^3 =$
b. $(p^3q)^4 =$

Solutions:

a. $(x^3y^4)^3 = x^{3.x 3}y^{4 x 3} = x^9y^{12}$
b. $(p^3q)^4 = p^{3 x 4}q^{1 x 4} = p^{12}q^4$

Example 17. Simplify $(2a^{-2}b^{-3})^{-2}$
Solution:

$(2a^{-2}b^{-3})^{-2}$	
$= 2^{-2}\ (a^{-2})^{-2}\ (b^{-3})^{-2}$	power of a product
$= 2^{-2}\ a^{-4}\ b^6$.	power of a power
$= 1/2^2 \cdot 1/a^4 \cdot b^6$	negative exponents
$= 1/4 \cdot 1/a^4 \cdot b^6$	evaluate $1/2^2$
$= b^6/4a^4$	multiply

Power of a Quotient
$(a^x/b^y)^n = a^{nx}/b^{ny}$ where $b \neq 0$.

Quotients may also be raised to a power.

$(a/b)^2$ means $a/b \cdot a/b = a^2/b^2$ and
$(x^2/y^3)^2$ means. $x^2/y^3 \cdot x^2/y^3 \cdot x^2/y^3 = x^4y^6$

Let us look at example 18.

Example 18. Simplify the following:

a. $(a^3/b^4)^2 =$
b. $(-3a^5/4b^3)^2 =$
c. $(2a^4/3b^5)^{-3} =$
d. $(2a^3b^{-2}/4c^{-4})^{-3} =$

Solutions:

a. $(a^3/b^4)^2 = a^{3\times 2}/b^{4.\times 2}$
$= a^6/b^8$
b. $(-3a^5/4b^3)^2 = -3^2a^{5\times 2}/4^2b^{3\times 2}$
$= 9a^{10}/16b^6$
c. $(2a^4/3b^5)^{-3} = 2^{-3}a^{4.(-3)}/3^{-3}b^{5(-3)}$
$= 2^{-3}a^{-12}/3^{-3}b^{-15}$
$= 3^3b^{15}/2^3a^{12}$
$= 27b^{15}/ 8a^{12}$
d. $(2a^3b^{-2} /4c^{-4})^{-3} = 2^{-3}a^{3(-3)}b^{-2(-3)}/4^{-3}c^{-4(-3)}$
$= 2^{-3}a^{-9}b^6/4^{-3}c^{12}$
$= 4^3b^6/2^3a^9c^{12}$
$= (2^2)^3b^6/2^3a^9c^{12}$
$= 2^6b^6/2^3a^9c^{12}$
$= 2^{6-3}b^6 /a^9c^{12}$
$= 2^3b^6/a^9c^{12}$
$= 8b^6/a^9c^{12}$

Division of Powers
with like Base

When you divide monomials with the same base, keep the base and subtract the exponents.

$a^m/a^n = a^{m-n}$

Exampe 19. Divide $12x^3/4x^2$

Solution:

$$12x^3/4x^2 \;=\; 12/4\,x^{3-2} \;=\; 3x^1 \;=3x$$

Divide the numerical coefficients to get the coefficient of the quotient.	Subtract the exponents to get the exponent of the literal factor of the quotient

$2x^4/x^7 =?$

What if the exponent of the denominator is greater than the exponent of the numerator?

Does the rule still apply?

Let's try a simple case using the definition of exponents to divide.

$$a^3/a^5 = a \times a \times a / a \times a \times a \times a \times a \;=\; 1/a \times a \;=1/a^2$$

If the law for division of powers is followed, $a^3/a^5 = a^{3-5} = a^{-2}$ therefore, we can define $a^{-2} = 1/a^2$. Suppose the numbers to be multiplied have fractional indices such as, $a^{1/2} \cdot a^{1/2} = ?$, $a^{1/4}.a^{1/2} = ?$ will the index law still apply?

Let us look at this example, Okey?

$$a^{1/2} \cdot a^{1/2} = a^{1/2+1/2} \;= a^{2/2} = a$$

What is $a^{1/2}$? $a^{1/2}$ or $\sqrt{a}$, both represent the positive square root of a only. The square root of a is the number which when multiplied by itself gives a.

$$a^{1/2} \cdot a^{1/2} = a^{1/2+1/2}$$
$$= a^{2/2}$$
$$= a$$

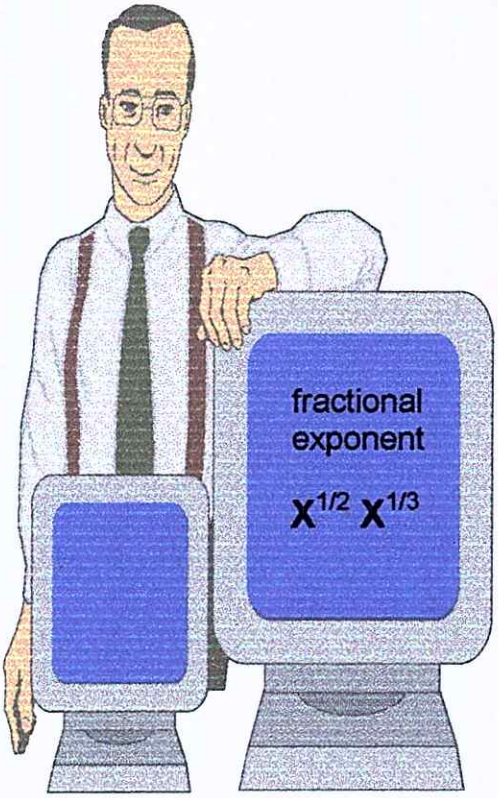
application of the index law

$$a^m \cdot a^n = a^{m+n}$$

Module 5
Lesson 4

ALGEBRAIC EXPRESSION V

Multiplication of Polynomials



Operations with Fractional Exponents

Recall:

There are two square roots of "a" the positive square root and the negative square root.

The negative root of a when multiplied by itself gives "a" also.
$-a^{1/2} \cdot -a^{1/2} = a^{1/2+1/2} = a^{2/2} = a.$

Let us look at another example. $a^{1/3} \cdot a^{1/3} \cdot a^{1/3} = a^{1/3+1/3+1/3} = a^{3/3} = a.$ In this example the index law for multiplying numbers with the same base is again applicable.

Let us define $a^{m/n}$. $a^{m/n} = \sqrt[n]{a^m}$

Restrictions:
The base **a** is a positive real number,
m is an integer and
n is a positive integer.

By restricting the base to a positive number we could avoid expression such as $(-4)^{1/2}$, $(-16)^{1/2}$ which are not defined.

Some examples of problems involving fractional exponents are given below.

Example 20. Evaluate $64^{2/3}$.

Solution: This can be done in either of two ways:

a) factoring the base

$$64^{2/3} = (2^6)^{2/3} = 2^{12/3} = 2^4 = 16$$

b) $64^{2/3} = \sqrt[3]{(64)^2} = 4^2 = 16$

Example 21. Simplify $81^{-0.25}$.
Solution:

$$81^{-0.25} = (3^4)^{-1/4} = 3^{4(-1/4)} = 3^{-4/4} = 3^{-1} = 1/3$$

Expand $(a^{1/2} - q^{1/2})^2$.
Solution:

$$\begin{aligned}(a^{1/2} - a^{1/2})^2 &= (a^{1/2})^2 - 2a^{1/2}a^{1/2} + (a^{1/2})^2\\ &= a^{2/2} - 2a^{1/2+1/2} + a^{2/2}\\ &= a^1 - 2a^{2/2} + a^1\\ &= a - 2a + a\\ &= 2a - 2a\\ &= 0\end{aligned}$$

Let us test your computational skills about fractional exponents. Okey?

Test Your Computational Skills 5.4

Simplify and express with positive indices:

1. $a^{1/2} \cdot a^{1/4}$
2. $25^{1/2}$
3. $27^{2/3}$
4. $b^{5/6} \cdot b^{1/2}$
5. $32^{1/5}$
6. $(5)^3 \cdot 5$
7. $16^6 \cdot 16$
8. $8^3 \cdot 8$
9. $q^{1.2} \cdot q^{0.2}$
10. $(2^{2.5})2$

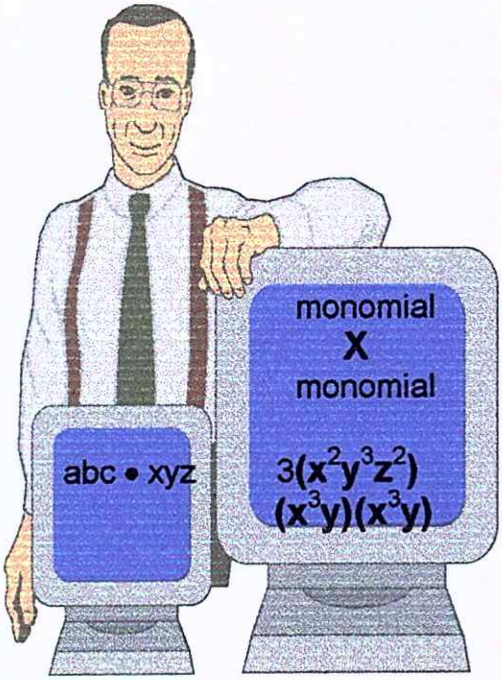
You are now ready to multiply algebraic expressions.

Let us start with multiplication of monomials. Okey?

Module 5
Lesson 5

ALGEBRAIC EXPRESSION V

Multiplication of Polynomials



Multiplication of Monomials

Multiplication of Monomials

Procedure:

1. Multiply their numerical coefficients using the law of signs for multiplication.

2. Add the exponents of the same letter.

Example:

To multiply a^4 by a^3 means to multiply $a \cdot a \cdot a \cdot a$ by $a \cdot a \cdot a$. Here the literal number a is used 7 times as a factor in multiplication. So $a^4 \times a^3 = a^7$.

3. If the letters are unlike, rewrite the letters in alphabetical order without changing exponents.

Example:

$x^4 \times y^2 = x^4y^2$

4. Multiply the product of the numerical coefficients by the product of the literal factors.

Example:

$4x^2 \times 2y^3 = 8x^2y^3$

This is done by writing the numerical product infront of the literal product.

Numerical coefficients should always precede the literal factors.

5. Check by numerical substitution or division.

Example 22.

1.
$$\begin{array}{r} -5a^3b^4 \\ \times\ 2ab^2 \\ \hline \end{array} \qquad \begin{array}{rl} -5 & a^3b^4 \\ \times\ 2 & ab^2 \\ \hline -10 & a^4b^6 \end{array} \quad = \quad -10a^4b^6$$

2. $$\begin{array}{rr} & -bx \\ x & -3 \\ \hline \end{array} \qquad \begin{array}{rrr} & -1 & bx \\ x & -3 & \\ \hline & 3 & bx \end{array} \quad = \quad 3bx$$

3. $$\begin{array}{rr} & 4a^2 \\ x & -c \\ \hline \end{array} \qquad \begin{array}{rrr} & 4 & a^2 \\ x & -1 & c \\ \hline & -4 & a^2c \end{array} \quad = \quad -4a^2c$$

4. $$\begin{array}{rr} & -5bx^2 \\ x & -3xy^2 \\ \hline \end{array} \qquad \begin{array}{rrr} & -5 & bx^2 \\ x & -3 & xy^2 \\ \hline & 15 & bx^3y^2 \end{array} \quad = \quad 15bx^3y^2$$

5. $(-2dx)(dx^2) = (-2)(1)(d \cdot d)(x \cdot x^2) = -2d^{1+1}x^{1+2} = -2d^2x^3$

6. $$\begin{aligned}(3xy^2z)(-2x^2y^4) &= (3)(-2)(x \cdot x^2)(y^2 \cdot y^4)(z) \\ &= -6x^{1+2}y^{2+4}z \\ &= -6x^3y^6z\end{aligned}$$

7. $(2a^3b^2)^3 = 2^3(a^3)^3(b^2)^3 = 2^3a^9b^6 = 8a^9b^6$

8. $$\begin{aligned}(2x^3y)^3\,(3xy^{2)2} &= [2^3(x^3)^3y^3][3^2x^2(y^2)^2] \\ &= [8x^9y^3]\,[9x^2y^4] \\ &= (8)(9)(x^9x^2)(y^3y^4) \\ &= 72\,x^{9+2}y^{3+4} \\ &= 72x^{11}y^7\end{aligned}$$

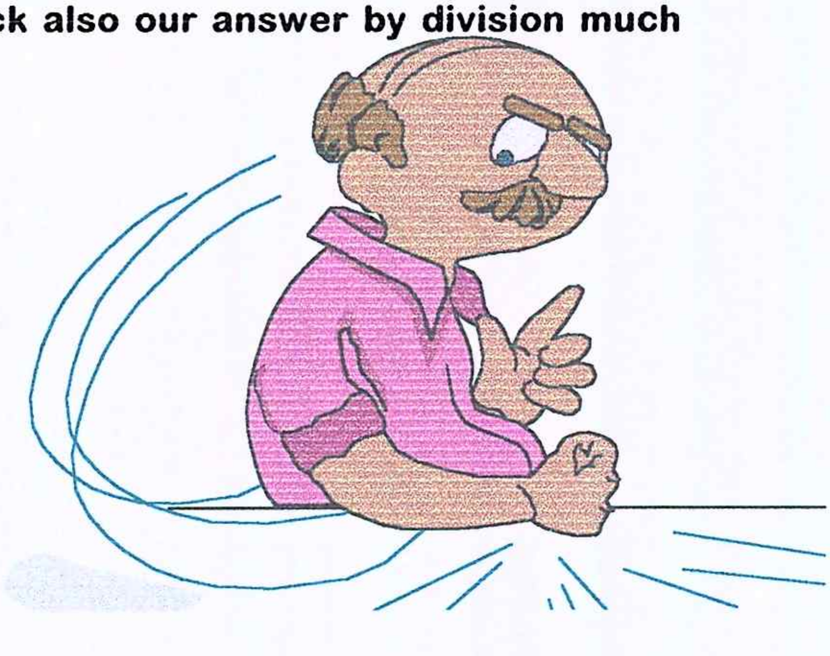
9. $(-3a^2b^2)^2\ (-2a^3b^3)^3\ (-bc^2)^4$ =
Solution: Apply associative property.

$$
\begin{aligned}
&(-3a^2b^2)^2(-2a^3b)^3(-bc^2)^4\\
&= [(-3a^2b^2)^2\ (-2a^3b)^3](-bc^2)^4\\
&= \{[(-3^2(a^2)^2(b^2)^2][(-2)^3(a^3)^3b^3]\}[-1^4b^4(c^2)^4]\\
&= \{[9a^4b^4][-8a^9b^3]\}[b^4c^8]\\
&= \{[(9)(-8)(a^4 \cdot a^9)(b^4b^3)]\}[b^4c^8]\\
&= (-72a^{13}b^7)(b^4c^8)\\
&= (-72)(1)(a^{13})(b^7 \cdot b^4)(c^8)\\
&= -72a^{13}b^{11}c^8
\end{aligned}
$$

10. $(-a^2b^4)^3\ \ (2ab^3c)^2\ \ (a^3c^2)^3$ =
Solution: Apply associative property.

$$
\begin{aligned}
&(-a^2b^4)^3(2ab^3c)^2(a^3c^2)^3\\
&= [(-a^2b^4)^3(2ab^3c)^2](a^3c^2)^3\\
&= \{[-1^3(a^2)^3(b^4)^3][2^2a^2(b^3)^2c^2]\}[(a^3)^3(c^2)^3]\\
&= \{[-1a^6b^{12}][4a^2b^6c^2]\}(a^9c^6)\\
&= (-4a^8b^{18}c^2)(a^9c^6)\\
&= (-4)(1)(a^8 \cdot a^9)(b^{18})(c^2c^6)\\
&= -4a^{17}b^{18}c^8
\end{aligned}
$$

Let us check if our answer is correct by numerical substitution. We an check also our answer by division much later.



Note:
You can choose any number to sustitute for the variable. It depends on what number is most convenient to you.
Okey?

Example 23. $-5a^3b^4 \text{ x } 2ab^2 = -10a^4b^6$
Solution: Let $a = 1$ and $b = 2$

$$\begin{aligned}
-5\,(1)^3(2)^4 \text{ x } 2(1)(2)^2 &= -10(1)^4(2)^6 \\
-5(1)(16) \text{ x } 2(1)(4) &= -10(1)(64) \\
-80 \text{ x } 8 &= -640 \\
-640 &= -640
\end{aligned}$$

Example 24. $(-bx)\,(-3) = 3bx$
Solution: Let $b = 1$ and $x = -1$

$$\begin{aligned}
[-(1)(-1)](-3) &= 3(1)(-1) \\
(1)(-3) &= -3 \\
-3 &= -3
\end{aligned}$$

Example 25. $(4a^2)(-c) = -4a^2c$
Solution: Let $a = 1$ and $c = 3$

$$\begin{aligned}
[4(1)^2](-3) &= -4(1)^2(3) \\
(4)(-3) &= -4\,(3) \\
-12 &= -12
\end{aligned}$$

Example 26. $(-5bx^2)(-3xy^2) = 15bx^2y^2$
Solution: Let $b = 1$, $x = -1$, and $y = 2$

$$\begin{aligned}
[-5(1)(-1)^2][-3(-1)(2)^2] &= 15(1)(-1)^3(2)^2 \\
(-5)(12) &= -60 \\
-60 &= -60
\end{aligned}$$

Example 27. $(-2dx)(dx^2) = -2d^2x^3$
Solutions: Let $d = 1$ and $x = -1$

$$\begin{aligned}
[-2(1)(-1)]\,[(1)(-1)^2] &= -2(1)^2(-1)^3 \\
(2)\,(1) &= 2 \\
2 &= 2
\end{aligned}$$

The check to numbers 6 to 10 is left to you. Let us test your computational skills in multiplying monomials.

Test Your Computational Skills 5.5

I. Give the product.

1. $x^2y \cdot xy^2z$
2. $ab^3 \cdot ab^2c$
3. $2a(4ac)$
4. $a^{1/2}b^{1/2}(ab)^2$
5. $4/b \cdot 12abc/d$
6. $xyz/2 \cdot 2xz/y$
7. $(3a)^3 \cdot (bcd)^2$
8. $xy^2z^4(9x^2yz^3)^4$
9. $(2a^3)^3 \cdot (4a^2c)^2$
10. $[2abc/3](3)^2/(2abc)(2abc)^3$

* * *

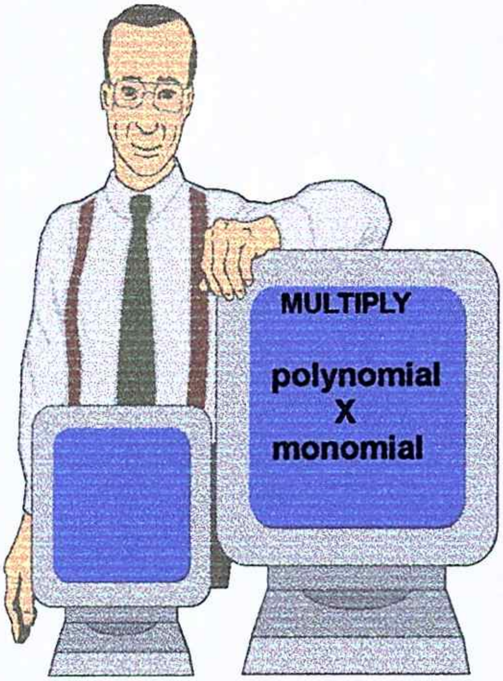
Suppose one of the factor is a polynomial the other factor is a monomial, will the process be different from obtaining the product when both factors are monomials ?

Let us look at the procedure in the next lesson. Okey?

Module 5
Lesson 6

ALGEBRAIC EXPRESSION V

Multiplication of Polynomials



Multiplication of a Polynomial by a Monomial

INPUT

Multiplication of a Polynomial by a Monomial

Procedure:

1. **Multiply each term of the polynomial by the monomial and combine the results.**
2. **Check by numerical substitution or by division.**

Let us look at examples. Okey?

Example 28. Find the product.

1. $-5\,(2a^2-3a+1)$
2. $b\,(5a-b+4)$
3. $3x^2y\,(x^2-2xy-3y^2)$
4. $4a\,(2c^2+8cd-3d^3)$
5. $-3cd\,(4c^2-5cd+d^2)$
6. $(-3a^3b)^2\,(2ab^2-5a^2b+7)$

Solution:

1. $-5(2a^2-3a+1) = (-5)(2a^2) + (-5)(-3a) + (-5)(1)$
   $= -10a^2 + 15a - 5$

2. $b(5a-b+4) = (b)(5a) + (b)(-b) + (b)(4)$
   $= 5ab - b^2 + 4b$

3. $3x^2y(x^2-2xy-3y^2) = (3x^2y)(x^2) + (3x^2y)(-2xy) + (3x^2y)(-3y^2)$
   $= 3x^4y - 6x^3y^2 - 3x^2y^3$

4. $4a(3c^2+8cd-3d^3) = (4a)(3c^2) + (4a)(8cd) + (4a)(-3d^3)$
   $= 12ac^2 + 32acd - 12ad^3$

5. $-3cd\,(4c^2-5cd+d^2) = (-3cd)(4c^2) + (-3cd)(-5cd) + (-3cd)(d^2)$
   $= -12c^3d + 15c^2d^2 - 3cd^3$

6. $(-3a^3b)^2(2ab^2-5a^2b+7) =$

Solution:

$= (-3a^3b)^2(2ab^2) \quad +(-3a^3b)^2(-5a^2b) \quad +(-3a^3b)^2(7)$

$= [-3^2(a^3)^2b^2](2ab^2) \quad +[-3^2(a^3)^2b^2](-5a^2b) \quad +[-3^2(a^3)^2b^2](7)$

$= (9a^6b^2)(2ab^2) \quad +(9a^6b^2)(-5a^2b) \quad +(9a^6b^2)(7)$

$= [(9)(2)(a^6a)(b^2b^2)] \quad + [(9)(-5)(a^6a^2)(b^2b)] \quad + [(9)(7)a^6)(b^2)]$

$= 18a^7b^4 \quad -45a^8b^3 \quad + 63a^6b^2$

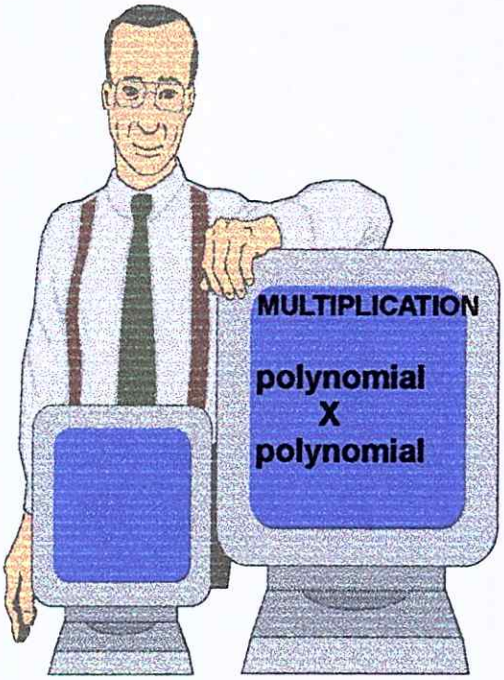
You have just performed multiplication of polynomials by monomials. You' ve noticed that we apply the distributive property of real numbers, the laws of exponents for like bases, and multiplication of signed numbers.

You are now ready for more complicated tasks. Okey? Let us now multiply a polynomial by another polynomial.

Module 5
Lesson 7

ALGEBRAIC EXPRESSION V

Multiplication of Polynomials



Multiplication of a Polynomial by a Polynomial

INPUT

Multiplication of a Polynomial by a Polynomial

Procedure:

1. Arrange the terms of both multiplicand and multiplier in descending or ascending power of one single letter or variable.
2. Starting at the left, multiply all the terms in the multiplicand by each term of the multiplier.
3. Add the partial products. This is done by writing similar terms in column.
4. Check by numerical substitution or by division.

Examples:

1.

$$\begin{array}{rrr} & x & +\ 6 \\ \times & x & -\ 2 \\ \hline x^2 & +\ 6x & \\ & -\ 2x & -12 \\ \hline x^2 & +4x & -12 \end{array}$$

NOTE:
We can write the product of two polynomials as products of a monomial and a polynomial.

$$x(x+6) + (-2)(x+6) =$$
$$x(x+6) = x^2 + 6x$$
$$-2(x+6) = -2x-12$$
$$(x^2 + 6x) + (-2x-12) = x^2 + (6x-2x) - 12$$
$$= x^2 +4x -12$$

2.

$$\begin{array}{rrr} & 3x & -\ 4y \\ \times & 5x & -\ 2y \\ \hline 15x^2 & -\ 20xy & \\ & -\ 6xy & +\ 8y^2 \\ \hline 15x^2 & -\ 26xy & +\ 8y^2 \end{array}$$

$$5x\ (3x-4y) + (-2y)(3x-4y) = \ ?$$

$$5x\ (3x-4y) = 15x^2 - 20xy$$
$$-2y\ (3x-4y) = -6xy + 8y^2$$

$$(15x^2-20xy) + (-6xy+8y^2) =$$

$$15x^2+(-20xy - 6xy) + 8y^2 =$$

$$15x^2 - 26xy + 8y^2$$

- 3. $4a - d$
- $\times \quad 4a + d$
- ---------------
- $16a^2 - 4ad$
- $+4ad - d^2$
- _______________
- $16a^2 \qquad -d^2$

$4a\,(4a - d) + d(4a - d) = ?$

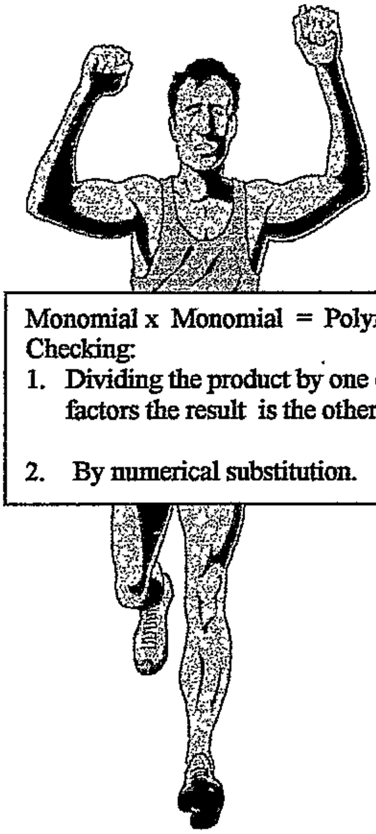
$4a\,(4a - d) = 16a^2 - 4ad$

$d\,(4a - d) = 4ad - d^2$

$(16a^2 - 4ad) + (4ad - d^2) =$

$16a^2 + (4ad - 4ad) - d^2 = 16a^2 - d^2$

- If you have doubts regarding
- your answers you can check
- it by numerical substitution
- or by division.
- Division check is done by
- dividing the obtained product by any of the factors.
- The result is the other
- factor. Okey?
- More about division in the
- next lesson.



Monomial x Monomial = Polynomial
Checking:

1. Dividing the product by one of the factors the result is the other factor.
2. By numerical substitution.

- Let us test your computational skills in multiplying polynomials by
- polynomials next page. Okey?

Test Your Computational Skills 5.6

Give the product.

1. $(b^3 + b^6)(c+3)$

2. $(-3r^4 + 8r^3)\ (n-4)$

3. $(-x^5y + x^6y)\ \ (b+5)$

4. $(-2ab^2 + c^2)\ (3a^2b + c^3)$

5. $-(3a + 4d)\ (d-1)$

6. $(-9c-d-5)\ (2a + 4)$

7. $(a+b^2)\ (3b^4 - b^3 - b^2 + 2b)$

8. $$\begin{array}{r} x + 4 \\ {}^{\times}\ x + 3 \\ \hline \end{array}$$

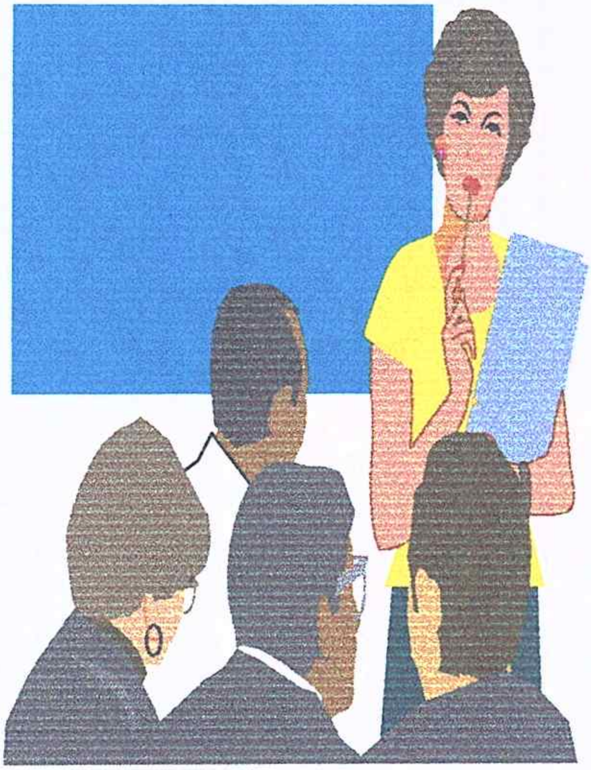
9. $(5b + 4)(ax - 3)$

10. $$\begin{array}{r} 2x - 7 \\ {}^{\times}\ 4x + 3 \\ \hline \end{array}$$

Reread the review lessons on multiplication of signed numbers, index laws, operations on integral exponents and fractional indices when necessary.

Check your answers to Test Your Computational Skills, it is on separate sheets placed after the Feedback to the Practice Task. There is no substitute for mastery. Okey?

CONGRATULATIONS



You have just finished ALGEBRAIC EXPRESSION V
(Multiplication of Polynomials).
You are now ready to take the PRACTICE TASK
next page.
Please check your answer at the
FEEDBACK TO THE PRACTICE TASK.
It is important.

Practice Task

Find the product.

1. $(x^3y^3)^3 (xy)^2$
2. $(a^2b^3)^2 (a^4b^4)^2$
3. $(2cd^2-4c) (4c^2d^4)$
4. $(m^2n^2 + 5m)^2 (mn)^3$
5. $(5x^2y^2 -2)(x-2)$
6. $(4x^3y^2-4x^2+ 3)(x^2)$
7. $-2 (5x^4y) (3x^2y^3)$
8. $(6x^2y)(2xy^2)^2(-3xy)$
9. $8x^2y (2x^2z)^2 (-2xy^2)^2$
10. $2x^2y[(x^2-y +z) (x-1)]$
11. $(x+1)^2 (2x-1)^2$
12. $(b-2)^2(b+3)^2$
13. $[(c^2-4) + (c^2+1)]^2$
14. $[(b+3) - (b+4)^2$
15. $4c^2d^3[(2cd + c^2) -(c^2+ 2)]^2$
16. $[(a-3)^2 + (b-2)^2]^2$
17. $[(x+y)^2 - (x-y)^2]^2$
18. $[(x+y)(x^2 -xy+y^2)] [(x-y)(x^2+ xy+ y^2)]$
19. $[(x+2)^2 (x+2)^3]^2$
20. $[(a-1)(a^2+a+1)] [(a+1) (a^2 -a + 1)]$

You must score 16
or higher of the
PRACTICE TASK.
If you score 10 or less,
please go over
MODULE 5 again.
It is important.

Answers to **Test Your Understanding** and **Test Your Computational Skills** are also provided on separate sheets placed after the Feedback to the Practice Task.

Please check your answers. Okey?

Feedback to the Practice Task

1. $x^{11}y^{11}$
2. $a^{12}b^{14}$
3. $8c^3d^6-16c^3d^4$
4. $m^7n^7+10m^6n^5+25m^5n^3$
5. $5x^3y^2-2x-10x^2y^2+4$
6. $4x^5y^2-4x^4+3x^2-12x^3y^3+12x^2y-9y$
7. $-30x^6y^4$
8. $-72x^5y^6$
9. $128x^8y^5z^2$
10. $2x^5y-2x^4y-2x^3y^2+2x^2y^2+2x^3yz-2x^2yz$
11. $4x^4+4x^3-3x^2-2x+1$
12. $b^4+2b^3-11b^2-12b+36$
13. c^4+3c^2-3
14. 1
15. $16c^4d^5-32c^3d^4+16c^2d^3$
16. $(a^2-6a+b^2-4b+13)^2$
17. $4x^4+4x^2y^2+4y^4$
18. $(x+2)^{10}$
19. $(x^2-y^2)(x^4+x^2y^2+y^4)$
20. $(a^4+a^2+1)(a^2-1)$

ANSWERS

Test Your Computational Skills 5.1

1. –200
2. 24
3. 2800
4. 900
5. 10500
6. –154000
7. –1130
8. –60750
9. –140000
10. 2400

Test Your Computational Skills 5.2

1. 20900
2. 264
3. –280
4. 540000
5. –210
6. –1050
7. 1008
8. –1
9. –5040
10. – 720
11. –36
12. 5040
13. –120
14. 225000000
15. –900

Test Your Computational Skills 5.3

1. $a^7 b^7$
2. $32x^5y^5$
3. 4096
4. $81x^4y^8$
5. a^2b^2
6. $8x^3y^3z^3$

7. $16x^8y^{12}$
8. $4a^4b^2c^6$
9. z^{12}
10. $256x^4y^4$
11. $16a^8b^4/81c^4b^4$
12. x^2/b^2
13. $4a^2/9b^2$
14. x^3 / y^3z^{12}
15. $3^{2c} / 2^{2d}$

Test Your Computational Skills 5.4

1. $a^{3/4}$
2. 5
3. 9
4. $b^{4/3}$
5. 2
6. 5^4
7. 6^7
8. 8^4
9. $q^{1.4}$
10. $2^{3.5}$

Test Your Computational Skills 5.5

1. $x^3y^3 z$
2. a^2b^5c
3. $8a^2c$
4. $a^{5/2}b^{5/2}$
5. $48ac/d$
6. $x^2 z^2$
7. $27a^3b^2c^2d^2$
8. $6561x^9y^6z^{16}$
9. $128a^{13}c^2$
10. $3/8a^3b^3c^3$

Test Your Computational Skills 5.6

1. $b^3c + b^6c + 3b^3 + 3b^6$
2. $-3nr^4 + 12r^4 + 8nr^3 - 32r^3$
3. $-bx^5y - 5x^5y + bx^6y + 5x^6y$

4. $-6a^{3}b^{3} - 2ab^{2}c^{3} + 3a^{2}bc^{2} + c^{5}$
5. $-3ad + 4d - 4d^{2+}4d$
6. $18ac - 2ad - 10a - 36c - 4d - 20$
7. $3ab^{4} - ab^{3} - ab^{2} + 2ab + 3b^{6} - b^{5} - b^{4} + 2b^{3}$
8. $x^{2} + 7x + 12$
9. $5abx - 15b + 4ax - 12$
10. $8x^{2} - 22x - 21$

Module 6
Lesson 1

ALGEBRAIC EXPRESSION VI

(Division of Polynomials)

Division of Integers

(Signed Numbers)

Overview:

Division as an operation is taught at home by our parents. A very small child is made to understand the concept of division by his parents when they try to divide foods, toys, and goodies equally among the siblings.

As the child grows older his concept of division is reinforced with his contact with other kids during playtime. He tries to share what he possesses with his friends.

The Bible even provided for the concept of division when Jesus Christ broke the bread to feed thousands of people.

Division is a binary operation. It involves two elements of a set, the dividend and the divisor. The result of the operation is called the quotient.

Division is the inverse operation of multiplication, hence division can be stated as a multiplication operation the product of the dividend and the reciprocal of the divisor.

The fraction, a/b indicates division, i.e., a is divided by b. There are other symbols that represent or indicate division aside from the horizontal bar in fractions. a/b can be represented in symbols by a ÷ b, b)a ‾ , and b|a.

We can check if the quotient we obtain in division is correct by (1) numerical substitution, and (2) by multiplying the quotient and the divisor and comparing the product obtained with the dividend. The product obtained must be equal to the dividend.

Mistakes are often made because of carelessness and lack of understanding when it comes to the use of mathematical phrases in division problems. There is a lot of difference between "a is divided by b" and "a divides b". When these phrases are translated into symbols, "a is divided by b" is denoted by "a/b" and "a divides b" is symbolized by "b/a" or "a|b.

Another common error is the number which is bigger in value is considered as the dividend and the smaller number the divisor without the benefit of reading and understanding the accompanying phrase or instruction and if by chance the instruction is very clear that the bigger number is the divisor and the smaller number is the dividend the unfortunate answer is " Ma'am cannot be" meaning that the division process is not possible.

Also they don't know the correct answers to the following division problems, n/0, 0/n, 0/0. Another problem is in understanding of mathematical phrases like "division by zero is not defined", while they know how to read the mathematical meaning of the statement is lost to them.

Acquiring the necessary mathematical skills to master division of algebraic expressions would require knowledge of multiplication of algebraic expressions, laws of exponents, and division of signed numbers.

So, let's do it. United we stand and divide we will not fall or fail. Divide and conquer mathematics.

Objectives:

At the end of this lesson the students should be able to:

1. find the quotient of two integers.
2. determine the sign of the quotient of two or more integers.
3. apply the laws of exponents in simplifying algebraic expressions which are quotients.
4. divide a monomial by a monomial, a polynomial by a monomial, and a polynomial by a polynomial.
5. check the quotient obtained by numerical substitution and by multiplication.

Input!

Division is the inverse operation of multiplication. If $ab = c$, $c/a = b$ and $c/b = a$. Division can be expressed as a multiplication operation, i.e. the product of the dividend and the reciprocal of the divisor. a/b can be stated as **a x 1/b**. The quantity/number **1/b** is the reciprocal of the divisor or the multiplicative inverse of the divisor. The result in division is called the quotient.

Find the ratio, quotient, and divide are but some of the terms used to command division operation.

Division is a binary operation, so we have to operate on two elements of a set, the dividend and the divisor.

The division of "a by b" is denoted by the following symbols: bla, a/b, a) b and $a \div b$.

The mathematical symbol that is used to indicate division is the horizontal bar used in fractions, ÷, and the $)\overline{}$, which is used in long division algorithm.

In some cases a/b will give a quotient c which is not an integer. If there is a remainder, we say that the division is not exact. If $c = q + r$, we can express the result as $q + r/b$ or q r/b.

In dividing polynomials if the divisor contains variables we often see restriction, "provided the divisor is not zero". Division of any number by zero is not defined or it has no meaning since we don't know its answer. The number a divided by zero is not equal to c. In symbol, $a/0 \neq c$. But $a/0 \neq a$ and $a/0 \neq 0$. To check if our answer is correct in division we multiply the quotient and the divisor.

The obtained product must be equal to the dividend. But 0 x 0 ≠ a, a x 0 ≠ a, c x 0 ≠ a. So we do not allow division by zero because the answer is not defined. The answer is not 0, a, or c.

0 x a = 0, 0/a = 0, but 0/0 ≠ a? The number zero is the only number which has no multiplicative inverse. The multiplicative inverse of a number **a** answers to the question, what number when multiplied to "a" gives 1, the multiplicative identity? All other numbers except zero has an inverse so we say that division of zero is not defined.

Mastering division of algebraic expressions, needs other basic skills like division of signed numbers, laws of exponent for like bases, the cancellation law and others.

For our starting venture into the land of division let's view division of signed numbers next page. Okay?

Division of Integers
(Signed Numbers)

Let us study the laws for dividing signed numbers.

Law 1. The quotient of two numbers having like sign is positive.

Law 2. The quotient of two numbers having unlike sign is negative.

Law 1 can be restated like this: the quotient of two positive numbers is positive and the quotient of two negative numbers is positive.

Examples:

1. + 12 ÷ +4 = +3
2. + 18 ÷ +3 = +6
3. –20 ÷ –2 = +10
4. – 15 ÷ –3 = +5

Law 2 states that the quotient of two numbers having unlike sign is negative. This means that the sign of the quotient of a positive and a negative number is negative. Also, the sign of the quotient of a negative number and a positive number is negative.

Examples:

1. + 12 ÷ –4 = –3
2. + 18 ÷ –3 = –6
3. –20 ÷ +2 = –10

4. − 15 ÷ −3 = −5

Let us test you computational skills in dividing signed numbers.

Test Your Computational Skills 6.1

Find the quotient.

1. − 10 ÷ − 2 =
2. − 27 ÷ − 9 =
3. − 8 ÷ +4 =
4. − 175 ÷ +25 =
5. + 100 ÷ +10 =
6. 180 ÷ +20 =
7. 69 ÷ 3 =
8. 500 ÷ −725 =
9. − 45 ÷ 90 =
10. + 40 ÷ −240 =
11. + 17 ÷ − 2 =
12. 181 ÷ 5 =
13. 68 ÷ 3 =
14. 503 ÷ 4 =
15. − 42 ÷ − 8 =

Let us study numbers 8, 9 and 10 of test your computational skills. It is easy to determine the sign of the quotient using laws 1 and 2. But to obtain the quotient is another problem since the divisor is a bigger number compared to the dividend.

Is the division possible or not?

$$\frac{\text{Smaller number}}{\text{Bigger number}} = ?$$

Let us study the following examples:

$$\begin{array}{lccccccccc}
1. & \dfrac{4}{18} & = & \dfrac{2 \times 2}{2 \times 9} & = & \dfrac{2}{2} \times \dfrac{2}{9} & = & 1 \times \dfrac{2}{9} & = & \dfrac{2}{9} \\[2em]
2. & \dfrac{50}{750} & = & \dfrac{50 \times 1}{50 \times 15} & = & \dfrac{50}{50} \times \dfrac{1}{15} & = & 1 \times \dfrac{1}{15} & = & \dfrac{1}{15} \\[2em]
3. & \dfrac{250}{750} & = & \dfrac{250 \times 1}{250 \times 3} & = & \dfrac{250}{250} \times \dfrac{1}{3} & = & 1 \times \dfrac{1}{3} & = & \dfrac{1}{3} \\[2em]
4. & \dfrac{300}{400} & = & \dfrac{100 \times 3}{100 \times 4} & = & \dfrac{100}{100} \times \dfrac{3}{4} & = & 1 \times \dfrac{3}{4} & = & \dfrac{3}{4} \\[1em]
 & & & (1) & & (2) & & (3) & & (4)
\end{array}$$

Observe that in (1) we find the greatest common factor of the dividend and the divisor; (2) we express the fraction (dividend divided by the divisor) as a product of factors. One of the factors is a quotient of the greatest common factor of the dividend and the divisor; (3) we replace the factor which is equal to 1 by the number 1. Since any number multiplied by 1 is the number itself so in (4) we obtain the final answer which is the other factor.

Take note that what we have done so far is to reduce the fraction (dividend divided by divisor) into its lowest term. So if the divisor is a bigger number than the dividend the quotient is found by reducing the fraction formed into its lowest term.

Our answer to item numbers 8, 9 and 10 is not Ma'am cannot be or that the division is not possible. Okay?

Numbers 11 to 15 is another problem. Dividing the dividend by the divisor gives a quotient which is not an integer. The division is not exact. The quotient c is of the form q + r. In this case how are we going to express our answer? Let us study the following example.

$$\frac{15}{4} = 3 + \frac{3}{4}$$

$$= 3\tfrac{3}{4}$$

$$= 3.75$$

	3	Quotient
Divisor	4) 15 -12	Dividend
	3	Remainder

3.75 is also a correct answer, since if we continue the division process by adding 2 more zeros after the last digit of the dividend the division algorithm will terminate. Let us study the example presented.

$$\begin{array}{rl}
3.75 & \\
4\,)\,\overline{15.00} & \\
-12 & 3 \times 4 = 12 \\
\cline{1-1}
30 & \\
-28 & 7 \times 4 = 28 \\
\cline{1-1}
20 & \\
-20 & 5 \times 4 = 20 \\
\cline{1-1}
0 &
\end{array}$$

In case the division is not exact, it is in the form of quotient plus remainder, we can state the remainder in fraction form. The divisor is the denominator of the fraction and the remainder the numerator or we add zeros after the last digit and continue the division algorithm until it terminates or obtain a zero remainder.

In the division algorithm, how can we tell if the new dividend is already the remainder?

Note: If the new dividend is less than the divisor then it is a remainder.

We can check if the result we obtain in division is correct by taking the product of the quotient and the divisor and comparing it with the dividend. If the two figures are equal then we are sure that our quotient is correct. We can also check our answer by numerical substitution. Let us take a look at the example presented.

$\frac{15}{4} = 3.75$ Is 3.75 x 4 = 15? The answer is **yes.**

Hence, $15 \div 4 = 3.75$. Okay.

How about the other answers 3 + ¾ and 3 ¾ ?

Let us check if **3 + ¾** is one of the correct answers.

If $\frac{15}{4} = 3 + ¾$, then 3 + ¾ x 4 = 15/4 x 4 = 15. The product of the divisor and the quotient is equal to the dividend. The answer is correct.

How about the other answer, 3 ¾ ? The mixed number 3 ¾ is equal to 15/4 and 15/4 x 4 = 15, hence 3 ¾ is correct.

Suppose our result in division is not an integer, as in the example below, it is of the form q + r , how do we check our answer ?

17 − 2 = 8 remainder 1.

To check our answer let us multiply the quotient, 8 by the divisor, 2 and add the remainder of 1. The result 17 must equal to 17, the dividend. Since 17 = 17, so our answer is correct.

If we write the final answer in this form 8 ½ will the product of the quotient and the divisor be equal to the dividend?

17/2 = 8 ½. Is 8 ½ x 2 = 17?

8 ½ = 17/2 x 2 = 17, but 17 = 17, hence our answer is also correct if it is written as 8 ½ .

Let us test your computational skills in dividing numbers which are exact, with remainders, with divisors which are bigger numbers compared to the dividend and check the obtained results.

Test Your Computational Skills 6.2

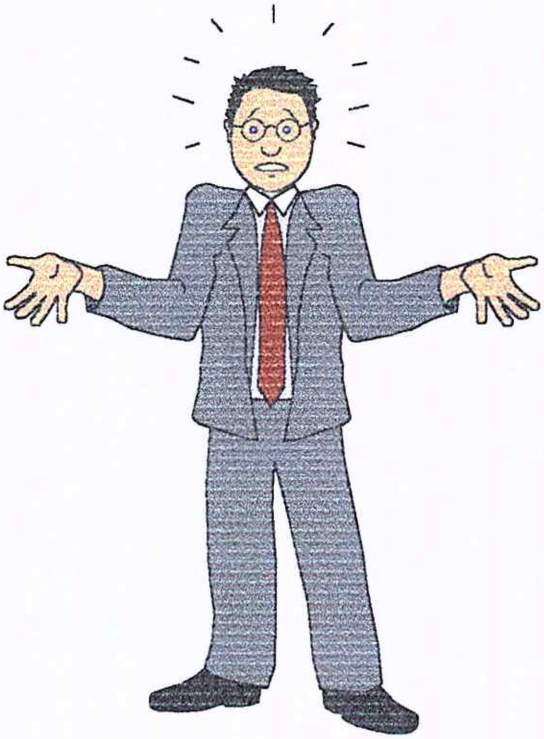
Give the quotient.

1. –80 ÷ 100 =
2. +250 ÷ 11000 =
3. –48 ÷ +192 =
4. –10 ÷ -160 =
5. 50 ÷ -250 =
6. –110 ÷ +25 =
7. –88 ÷ 44 =
8. 69 ÷ 3 =
9. 488 ÷ 488 =
10. –150 ÷ -50 =
11. –49 ÷ +21 =
12. –84 ÷ 42 =
13. 100 ÷ 3 =
14. –150 ÷ +750 =
15. +2/3 ÷ -2/3 =

Module 6
Lesson 2

ALGEBRAIC EXPRESSION VI

(Division of Polynomials)



Division of Polynomials

Input!

Division of Polynomials

We will now consider two types of division – division of a polynomial **by a monomial** and division of a polynomial **by a polynomial of more than one term.**

Division of a Polynomial by a Monomial

TO DIVIDE A POLYNOMIAL BY A MONOMIAL

Divide *each* term in the polynomial by the monomial, then add the results.

Consider the example:

$$\frac{4x^3 -6x^2}{2x} = \frac{1}{2x} \cdot \frac{4x^3 - 6x^2}{1} = \frac{1}{2x}(4x^3 - 6x^2)$$

$$= \left(\frac{1}{2x}\right)(4x^3) + \left(\frac{1}{2x}\right)(-6x^2)$$ By the distributive property

$$= \frac{4x^3}{2x} + \frac{-6x^2}{2x}$$ Dividing each term of the polynomial by the monomial

$$= 2x^2 - 3x$$

In the example presented, the division of a polynomial by a monomial is expressed as a multiplication operation, the product is the result of multiplying the dividend and the reciprocal of the divisor.

Example 1. Dividing a polynomial by a monomial

(a) $$\frac{4x + 2}{2} = \frac{4x}{2} + \frac{2}{2} = 2x + 1$$

(b) $$\frac{9x^3 -6x^2 +12x}{3x} = \frac{9x^3}{3x} + \frac{-6x^2}{3x} + \frac{12x}{3x} = 3x^2 -2x +4$$

(c) $$\frac{4x^2 -8x +16}{-4x} = \frac{4x^2}{-4x} + \frac{-8x}{-4x} + \frac{16}{-4x} = -x +2 - \frac{4}{x}$$

(d) $$\frac{15x^2y +20y^2z -10xz^2}{5xyz} = \frac{15x^2y}{5xyz} + \frac{20y^2z}{5xyz} + \frac{-10xz^2}{5xyz}$$

$$= \frac{3x}{z} + \frac{4y}{x} - \frac{2z}{y}$$

How can we know if our answer in dividing a polynomial by a monomial is correct?

Let us check if our answers are correct using (1) numerical substitution; (2) multiplication.

Let us check the quotient obtained in example 1.

1. By numerical substitution

Let $x = 1,\ y = 2,\ z = 3$

(a) $$\frac{4x + 2}{2} = 2x + 1$$

$$\frac{4(1) + 2}{2} = 2(1) + 1$$

$$\frac{6}{2} = 2 + 1$$

$$3 = 3$$

(b) $$\frac{9x^3 - 6x^2 + 12x}{3x} = 3x^2 - 2x + 4$$

$$\frac{9(1)^3 - 6(1)^2 + 12(1)}{3(1)} = 3(1)^2 - 2(1) + 4$$

$$\frac{9 - 6 + 12}{3} = 3 - 2 + 4$$

$$5 = 5$$

(c) $$\frac{4x^2\ -8x\ +16}{-4x} = -x\ +2 - \frac{4}{x}$$

$$\frac{4(1)^2\ -8(1)\ +16}{-4(1)} = -(1)\ +2\ - \frac{4}{(1)}$$

$$\frac{4\ -8\ +16}{-4} = -1\ +2\ -4$$

$$-3 = -3$$

(d) $$\frac{15x^2y\ +20y^2z\ \ -10xz^2}{5xyz} = \frac{3x}{z} + \frac{4y}{x} - \frac{2z}{y}$$

$$\frac{15(1)^2(2)\ +20(2)^2(3)\ -10(1)(3)^2}{5(1)(2)(3)} = \frac{3(1)}{3} + \frac{4(2)}{1} - \frac{2(3)}{2}$$

$$\frac{30\ +240\ -90}{30} = \frac{3}{3} + \frac{8}{1} - \frac{6}{2}$$

$$1\ +\ 8\ -\ 3 = 1\ +\ 8\ -\ 3$$

$$6 = 6$$

2. By multiplying the quotient and the divisor to yield the dividend.

(a) $$\frac{4x + 2}{2} = 2x + 1$$

Check:

$$2(2x + 1) = 4x + 2$$

(b) $$\frac{9x^3 -6x^2 +12x}{3x} = 3x^2 -2x +4$$

Check:

$$3x (3x^2 -2x +4) = 9x^3 -6x^2 +12x$$

(c) $$\frac{4x^2 -8x +16}{-4x} = -x +2 - \frac{4}{x}$$

Check:

$$-4x (-x +2 - \frac{4}{x}) = 4x^2 -8x +16$$

(d) $$\frac{15x^2y +20y^2z -10xz^2}{5xyz} = \frac{3x}{z} + \frac{4y}{x} - \frac{2z}{y}$$

Check:

$$5xyz (\frac{3x}{z} + \frac{4y}{x} - \frac{2z}{y}) = 15x^2y +20y^2z -10xz^2$$

Let us test your computational skills in dividing a polynomial by a monomial.

Test Your Computational Skills 6.3

1. $\dfrac{3x + 6}{3}$

2. $\dfrac{10x + 15}{5}$

3. $\dfrac{6x + 8y}{2}$

4. $\dfrac{5x - 10y}{5}$

5. $\dfrac{5x^2 + 3x^2}{x}$

6. $\dfrac{4y^2 - 3y}{y}$

7. $\dfrac{3a^2 b - ab}{ab}$

8. $\dfrac{5mn^2 - mn}{mn}$

9. $\dfrac{12z^3 - 16z^2 + 8z}{-4z}$

10. $\dfrac{-15^4 + 12^3 - 18a^2}{-3a^2}$

Division of a Polynomial by a Polynomial

The method used to divide a polynomial by a polynomial is like long division of whole numbers in arithmetic. It is based on the fact that division is repeated subtraction.

Example 2. $966 \div 23$

$$\text{First term in quotient} = \frac{\text{First term of dividend}}{\text{First term of divisor}} = \frac{9}{2} = 4$$

$$\begin{array}{rl} & \underline{\;\;42} \\ 23\,) & 966 \\ & -92 \\ \hline & \;\;46 \\ & -46 \\ \hline & \;\;\;0 \end{array} \qquad \begin{array}{l} \\ \\ \text{Subtracting } 4 \bullet 23 = 92 \\ \\ \text{Subtracting } 2 \bullet 23 = 46 \\ \\ \\ \end{array}$$

$$\text{Second term in quotient} = \frac{4}{2} = 2$$

Example 3. $(x^2 -3x -10) \div (x+2)$

$$\text{First term in quotient} = \frac{\text{First term in dividend}}{\text{First term in divisor}} = \frac{x^2}{x} = x$$

$$\text{Second term in quotient} = \frac{5x}{x} = 5$$

$$\begin{array}{rl} & \underline{\;\;\;\;\;\;x - 5} \\ x + 2\,) & x^2 -3x -10 \\ & x^2 +2x \\ \hline & \;\;\;\;-5x - 10 \\ & \;\;\;\;-5x - 10 \\ \hline & \;\;\;\;\;\;\;\;\;\;\;\;0 \end{array} \qquad \begin{array}{l} \\ \\ \text{Subtracting } x(x + 2) = x^2 + 2x \\ \\ \text{Subtracting } (-5)(x + 2) = -5x -10 \\ \\ \\ \end{array}$$

Therefore, $(x^2 -3x - 10) \div (x + 2) = x - 5$

Example 4. $(6x^2 + x - 10) \div (2x + 10)$

$$\begin{array}{r l}
 & \quad 3x - 4 \qquad R\ 2 \\
2x + 3 & \overline{)\, 6x^2 + x - 10} \\
 & \underline{\ 6x^2 + 9x\ } \\
 & \qquad -8x - 10 \\
 & \qquad \underline{-8x - 12} \\
 & \qquad\qquad 2 \quad \text{Remainder}
\end{array}$$

Therefore, $(6x^2 + x - 10) \div (2x + 3) = 3x - 4 + \dfrac{2}{2x + 3}$

Example 5. $(27x - 19x^2 + 6x^3 + 10) \div (5 - 10x)$

The terms of the dividend and the divisor should be arranged in *descending powers* of the variable *before* beginning the division.

$$\begin{array}{r l}
 & \quad -2x^2 + 3x - 4 \qquad R\ 30 \\
-3x + 5 & \overline{)\, 6x^3 - 19x^2 + 27x + 10} \\
 & \underline{\ 6x^3 - 10x^2 \qquad\qquad} \\
 & \qquad -9x^2 + 27x \\
 & \qquad \underline{-9x^2 + 15x \qquad\quad} \\
 & \qquad\qquad\quad 12x + 10 \\
 & \qquad\qquad\quad \underline{12x - 20} \\
 & \qquad\qquad\qquad\quad 30 \quad \text{Remainder}
\end{array}$$

Therefore:

$$(6x^3 - 19x^2 + 27x + 10) \div (-3x + 5) = -2x^2 + 3x - 4 + \frac{30}{-3x + 5}$$

Example 6. $(x^3 - 1) \div (x + 1) =$

$$\begin{array}{r l}
 & x^2 + x + 1 \\
x - 1) & \overline{x^3 - 0x^2 + 0x - 1} \\
 & x^3 - x^2 \\
 & \overline{ x^2 - 0x} \\
 & x^2 - x \\
 & \overline{ x - 1} \\
 & x - 1 \\
 & \overline{ 0}
\end{array}$$

It is helpful to leave space for missing powers by using zeros in this way

Therefore, $(x^3 - 1)) \div (x - 1) = x^2 + x - 1$

Example 7. $(2x^4 + x^3 - 8x^2 - 5x - 2) \div (x^2 - x - 2)$

When the divisor is a polynomial of more than two terms, exactly the same procedure is used.

$$\begin{array}{r l l}
 & 2x^2 + 3x - 1 & \text{R } -4 \\
x^2 - x - 1) & \overline{2x^4 + x^3 - 8x^2 - 5x - 2} & \\
 & 2x^4 - 2x^3 - 4x^2 & \\
 & \overline{ 3x^3 - 4x^2 - 5x} & \\
 & 3x^3 - 3x^2 - 6x & \\
 & \overline{ -x^2 + x - 2} & \\
 & -x^2 + x + 2 & \\
 & \overline{ -4} & \text{Remainder}
\end{array}$$

Therefore:

$$(2x^4 + x^3 - 8x^2 - 5x - 2) \div (x^2 - x - 2) = = 2x^2 + 3x - 1 + \frac{-4}{x^2 - x - 2}$$

Let us check if our answers in the examples presented are correct.

Example 2

$$\begin{aligned} 966 \div 33 &= 42 \\ 33 \times 42 &= 966 \\ 966 &= 966 \end{aligned}$$

Example 3

$$\begin{aligned} (x^2 - 3x - 10) \div (x + 2) &= x - 5 \\ (x + 2)(x - 5) &= x^2 - 3x - 10 \\ x^2 - 3x - 10 &= x^2 - 3x - 10 \end{aligned}$$

Example 4

$$(6x^2 + x - 10) \div (2x + 3) = 3x - 4 + \frac{2}{2x + 3}$$

$$(2x + 3)\ (3x - 4 + \frac{2}{2x + 4}) = 6x^2 + x - 10$$

$$6x^2 + x - 10 = 6x^2 + x - 10$$

Example 5

$$(27x - 19x^2 + 6x^3 + 10) \div (5 - 3x) = -2x^2 + 3x - 4 + \frac{30}{-3x + 5}$$

$$6x^3 - 19x^2 + 27x + 10 = (5 - 3x)\,[-2x^2 + 3x - 4 + \frac{30}{-3x + 5}]$$

$$6x^3 - 19x^2 + 27x + 10 = 6x^3 - 19x^2 + 27x + 10$$

Example 6

$$(x^3 - 1)) \div (x - 1) = x^2 + x + 1$$

$$(x - 1)\ (x^2 + x + 1) = x^3 - 1$$

$$x^3 - 1 = x^3 - 1$$

Example 7

$$(2x^4 + x^3 - 8x^2 - 5x - 2) \div (x^2 - x - 4) = 2x^3 + 3x - 1 - \frac{4}{2x^2 - x - 4}$$

$$(x^2 - x - 4)\ (2x^2 + 3x - 1 - \frac{4}{2x^2 - x - 4}) = 2x^4 + x^3 - 8x^2 - 5x - 2$$

$$2x^4 + x^3 - 8x^2 - 5x - 2 = 2x^4 + x^3 - 8x^2 - 5x - 2$$

Let us test your computational skills in dividing a polynomial by another polynomial and your skills in checking your answer by using either numerical substitution or multiplication.

Test Your Computational Skills 6.4

Perform each division with a polynomial divisor. Check your answer by multiplication or by numerical substitution.

1. $\dfrac{x^2 + 5x + 6}{x + 2}$

2. $\dfrac{x^2 - 9x + 20}{x - 4}$

3. $\dfrac{6x^2 + 5x - 6}{3x - 2}$

4. $\dfrac{8x - 4x^3 + 10}{2 - x}$

5. $\dfrac{a^3 - 8}{a - 2}$

6. $\dfrac{20x^2 + 13x - 15}{5x - 3}$

7. $\dfrac{5x^3 + 1}{x + 1}$

8. $\dfrac{4x^4 - 1}{2x^2 - 1}$

9. $\dfrac{12x - 15 - x^3}{3 - x}$

10. $\dfrac{c^3 - 27}{c - 3}$

CONGRATULATIONS

You have just finished ALGEBRAIC EXPRESSION VI
(Division of Polynomials).
You are now ready to take the PRACTICE TASK
next page.
Please check your answer at the
FEEDBACK TO THE PRACTICE TASK.
It is important.

Practice Task

Perform each indicated division.

1. $\dfrac{(x^3\, y^3)^3}{(xy)^2}$

2. $\dfrac{(a^2\, b^3)^2}{(a^4\, b^4)^2}$

3. $\dfrac{(2cd^2 - 4c)}{(4c^2 d^4)}$

4. $\dfrac{(m^2\, n^2 + 5m)^2}{(mn)^3}$

5. $\dfrac{(5x^2\, y - 20xy^2)}{5xy}$

6. $\dfrac{(4x^3\, y^2 - 4x^2\, + 3)}{(x^2 - 3y)}$

7. $\dfrac{-2\,(5x^4\, y)(3x^2 y^3)}{xy^3}$

8. $\dfrac{(6x^2 y)\,(2xy^2)^2}{(-3xy)}$

9. $$\frac{8x^2 y(2x^2 z)^2}{(-2xy^2)^2}$$

10. $$\frac{2x^2 y \ [(x^2 \ -y + z)(x-1)]}{x^2 \ -y + z}$$

11. $$\frac{(x + 1)^2 (2x - 1)^2}{(x + 1)^2 (2x - 1)^2}$$

12. $$\frac{(b-2)^2 (b+3)^2}{4b -8}$$

13. $$\frac{(c^2-4) + (c^2 \ +1)^2}{(c^2-4) + (c^2 \ +1)^2}$$

14. $$\frac{(b + 3) \ (b+4)^2}{(b + 3)^2}$$

15. $$\frac{(2cd +c^2) - (c^2 \ +2)^2}{4c^2 d^3}$$

16. $$\frac{(a-3)^2 (b-2)^2}{(a-3)^3 (b-2)^3}$$

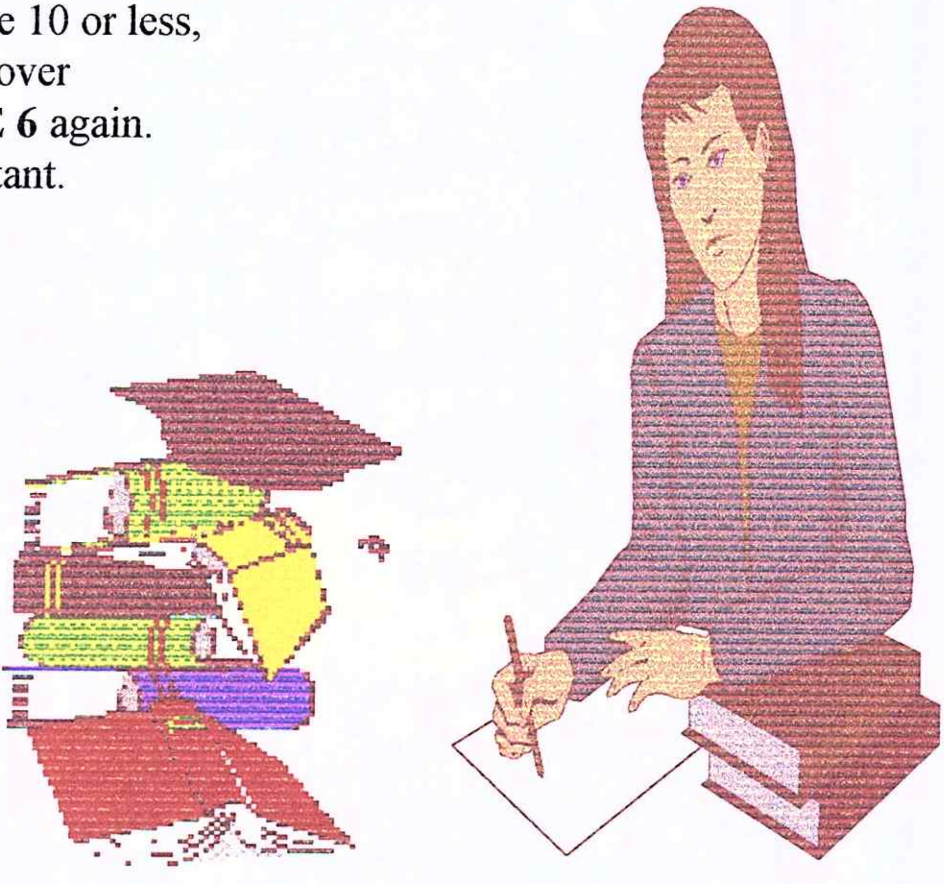
17. $$\frac{(x - y)^2 - (x + y)^2}{(x + y)\ (x - y)}$$

18. $$\frac{-(x + 2)^2 (x + 2)^3}{(x + 2)^5}$$

19. $$\frac{[(x + y)(x^2 - xy + y^2)]\ [(x - y)(x^2 - xy + y^2)]}{[(x + y)(x^2 + xy + y^2)]}$$

20. $$\frac{(a - 1)(a^2 + a + 1)(a + 1)\ (a^2 - a + 1)}{-(a - 1)(a^2 + a + 1)^2\ (a + 1)\ (a^2 - a + 1)}$$

You must score 16
or higher of the
PRACTICE TASK.
If you score 10 or less,
please go over
MODULE 6 again.
It is important.



Answers to **Test Your Understanding** and **Test Your Computational Skills** are also provided on separate sheets placed after the Feedback to the Practice Task.

Please check your answers. Okey?

Feedback to the Practice Task

1. $$\frac{(x^3 y^3)^3}{(xy)^2} = \frac{x^9 y^9}{x^2 y^2} = x^7 y^7$$

2. $$\frac{(a^2 b^3)^2}{(a^4 b^4)^2} = \frac{a^4 b^6}{a^8 b^8} = \frac{1}{a^4 b^2}$$

3. $$\frac{(2cd^2 - 4c)}{(4c^2 d^4)} = \frac{2c(d^2 - 2)}{2c(cd^4)} = \frac{d^2 - 2}{cd^4}$$

4. $$\frac{(m^2 n^2 + 5m)^2}{(mn)^3} = \frac{m^4 n^4 + 10m^3 n^2 + 25m^2}{m^3 n^3} = mn + \frac{10}{n} + \frac{25}{mn^3}$$

5. $$\frac{(5x^2 y - 20xy^2)}{5xy} = \frac{5xy(x - 4y)}{5xy} = x - 4y$$

6. $$\frac{(4x^3 y^2 - 4x^2 + 3)}{(x^2 - 3y)} = 4xy^2 - 4 + \frac{12xy^3 - 12y + 3}{x^2 - 3y}$$

$$4xy^2 \quad -4$$

$$x^2 - 3y \;/\; \overline{4x^3 y^2 \quad -4x^2 \quad +3)}$$

$$\underline{4x^3 y^2 - 12xy^3}$$

$$-4x^2 \quad +12xy^3$$

$$\underline{-4x^2 \quad {}^{+}12y}$$

$$12xy^3 \quad -12y + 3$$

7. $$\frac{-2(5x^4 y)(3x^2 y^3)}{xy^3} = \frac{(-10x^4 y)(3x^2 y^3)}{xy^3} = \frac{-30x^6 y^4}{xy^3} = -30x^5 y$$

8. $$\frac{(6x^2y)(2xy^2)^2}{(-3xy)} = \frac{(6x^2y)(4x^2y^4)}{-3xy} = \frac{24x^4y^5}{-3xy} = -8x^3y^4$$

9. $$\frac{8x^2y(2x^2z)^2}{(-2xy^2)^2} = \frac{8x^2y(4x^4z^2)}{4x^2y^4} = \frac{32x^6yz^2}{4x^2y^4} = \frac{8x^4z^2}{y^3}$$

10. $$\frac{2x^2y\ [(x^2-y+z)(x-1)]}{x^2-y+z} = 2x^2y\,(x-1)$$

11. $$\frac{(x+1)^2\,(2x-1)^2}{(x+1)^2\,(2x-1)^2} = 1$$

12. $$\frac{(b-2)^2(b+3)^2}{4b-8} = \frac{(b-2)^2(b+3)^2}{4\,(b-2)} = \frac{(b-2)\ (b+3)^2}{4}$$

13. $$\frac{(c^2-4)+(c^2+1)^2}{(c^2-4)+(c^2+1)^2} = 1$$

14. $$\frac{(b+3)\ (b+4)^2}{(b+3)^2} = \frac{(b+4)^2}{(b+3)}$$

15. $$\frac{(2cd+c^2)-(c^2+2)^2}{4c^2d^3} = \frac{(2cd+c^2)-(c^4+4c^2+4)}{4c^2d^3}$$

$$= \frac{2cd-3c^2-c^4-4}{4c^2d^3}$$

$$= \frac{2cd}{4c^2d^3} - \frac{3c^2}{4c^2d^3} - \frac{c^4}{4c^2d^3} - \frac{4}{4c^2d^3}$$

$$= \frac{1}{2cd^2} - \frac{3}{4d^3} - \frac{c^2}{4d^3} - \frac{1}{c^2d^3}$$

16. $$\frac{(a-3)^2(b-2)^2}{(a-3)^3(b-2)^3} = \frac{1}{(a-3)\ (b-2)}$$

17. $$\frac{(x-y)^2 - (x+y)^2}{(x+y) - (x-y)} = \frac{[(x-y)-(x+y)][(x-y)+(x+y)]}{(x-y) - (x+y)} = (x-y)+(x+y)$$

18. $$\frac{-(x+2)^2(x+2)^3}{(x+2)^5} = -1$$

19. $$\frac{[(x+y)(x^2-xy+y^2)]\ [(x-y)(x^2-xy+y^2)]}{[(x+y)(x^2-xy+y^2)]} = (x-y)(x^2-xy+y^2)$$

20. $$\frac{(a-1)(a^2+a+1)(a+1)\ (a^2-a+1)}{-(a-1)(a^2+a+1)^2\ (a+1)\ (a^2-a+1)} = \frac{1}{-a^2+a+1}$$

ANSWERS

Test Your Computational Skills 6.1

1. 5
2. 3
3. -2
4. -7
5. 10
6. 9
7. 23
8. –20/29
9. – ½
10. –1/6
11. –8.5
12. 36.2
13. 22.67
14. 125.75
15. 5.25

Test Your Computational Skills 6.2

1. –4/5
2. 1/44
3. –1/4
4. 1/16
5. –1/5
6. -4
7. -2
8. 23
9. –1
10. +3
11. – 7/3
12. –2
13. 33 1/3
14. –1/5
15. -1

Test Your Computational Skills 6.3

1. $x + 2$
2. $2x + 3$
3. $3x + 4y$
4. $x - 2y$
5. $8x$
6. $4y - 3$
7. $3a - 1$
8. $5n - 1$
9. $-3z^2 + 4z - 2$
10. $-16299/a^2 + 6$

Test Your Computational Skills 6.4

1. $$\frac{x^2 + 5x + 6}{x + 2} = x + 3$$

$$\begin{array}{r l}
 & x \quad +3 \\
x + 2 \; / & \overline{x^2 \quad +5x \quad +6} \\
 & x^2 \quad +2x \\
 & \overline{\qquad\qquad} \\
 & \qquad 3x \quad +6 \\
 & \qquad 3x \quad +6 \\
 & \qquad \overline{\qquad\quad 0}
\end{array}$$

Check: $(x + 2)(x + 3) = x^2 + 5x + 6$
$x^2 + 5x + 6 = x^2 + 5x + 6$

2. $$\frac{x^2 - 9x + 20}{x - 4} = x - 5$$

$$\begin{array}{r l}
 & x \quad -5 \\
x - 4 \; / & \overline{x^2 \quad -9x \quad +20} \\
 & x^2 \quad -4x \\
 & \overline{\qquad\qquad} \\
 & \qquad -5x \quad +20 \\
 & \qquad -5x \quad +20 \\
 & \qquad \overline{\qquad\quad 0}
\end{array}$$

Check: $(x - 4)(x - 5) = x^2 - 9x + 20$
$x^2 - 9x + 20 = x^2 - 9x + 20$

3. $$\frac{6x^2+5x-6}{3x-2} = 2x+3$$

$$\begin{array}{rrrr}
 & & 2x & +3 \\ \cline{2-4}
3x-2\,/ & 6x^2 & +5x & -6 \\
 & 6x^2 & -4x & \\ \cline{2-3}
 & & +9x & -6 \\
 & & +9x & -6 \\ \cline{3-4}
 & & & 0
\end{array}$$

Check: $(3x-2)(2x+3) = 6x^2+5x-6$

$6x^2+5x-6 = 6x^2+5x-6$

4. $$\frac{8x-4x^3+10}{2-x} = \frac{4x^3+8x+10}{-x+2} = 4x^2+8x+8+\frac{6}{x-2}$$

$$\begin{array}{rrrrr}
 & & 4x^2 & +8x & +8 \\ \cline{2-4}
-x+2\,/ & -4x^3 & +0x^2 & +8x & +10 \\
 & -4x^3 & +8x^2 & & \\ \cline{2-3}
 & & -8x^2 & +8x & \\
 & & -8x^2 & +16x & \\ \cline{3-4}
 & & & -8x & +10 \\
 & & & -8x & +16 \\ \cline{4-5}
 & & & & -6
\end{array}$$

Check:

$(-x+2)(4x^2+8x+8+\frac{6}{x-2}) = -4x^3+8x+10$

$-4x^3+8x+10 = -4x^3+8x+10$

5. $$\frac{a^3-8}{a-2} = a^2+2a+4$$

$a^2 \ +2a \ +4$

$a -2 \, / \, \overline{a^3 \ +0a^2 \ +0a \ -8}$ Check:

$a^3 \ -2a^2$

$\overline{\quad 2a^2 \ +0a}$

$2a^2 \ {}^{-}4a$

$(a-2)(a^2 +2a +4) = a^3 -8$

$a^3 -8 = a^3 -8$

$\overline{\quad 4a \ -8}$

$4a \ -8$

$\overline{\quad 0}$

6. $$\frac{20x^2 \ +13x \ -15}{5x \ -3} = 4x + 5$$

$4x \ +3$

$5x - 3 \, / \, \overline{20x^2 \ +13x \ -15}$

$20x^2 \ -12x$

Check: $(5x-3)(4x+3) = 20x^2 \ +13x \ -15$

$20x^2 \ +13x -15 = 20x^2 +13x \ -15$

$\overline{\quad +25x \ -15}$

$+25x \ -15$

$\overline{\quad 0}$

7. $$\frac{5x^3 \ + 1}{x + 1} = 5x^2 - 5x \ +5 \ - \frac{4}{x+1}$$

$\underline{5x^2 \ -5x +5}$

$x +1 \, / \, 5x^3 +0x^2 \ +0x \ +1$

$5x^3 \ {}^{+}5x^2$

Check: $(x+1)(5x^2 -5x +5) = 5x^3 \ +1$

$5x^3 \ +1 = 5x^3 +1$

$\overline{\quad -5x^2 \ +0x}$

$\underline{-5x^2 \ \ -5x}$

$5x \ +1$

$\underline{5x +5}$

-4

8. $$\frac{4x^4 -1}{2x^2 -1} = 2x^2 -1$$

$$\begin{array}{r}
\overline{2x^2 \qquad +1} \\
2x^2 -1 /\ 4x^4 +0x^2 +0x +1 \\
4x^4 \ {}^-2x^2 \qquad\qquad\quad \\
\hline
2x^2 \qquad +1 \\
\underline{2x^2 \qquad +1} \\
0
\end{array}$$

Check: $(2x^2 +1)(2x^2 -1) = 4x^4 -1$

$4x^4 -1 = 4x^4 -1$

9. $$\frac{12x -15 -x^3}{3 -x} = x^2 +3x -3 + \frac{6}{x-3}$$

$$\begin{array}{r}
\overline{x^2 \quad +3x \quad -3} \\
-x +3 /\ -x^3 +0x^2 +12x -15 \\
-x^3 \ {}^+3x^2 \qquad\qquad\quad \\
\hline
-3x^2 \ +12x \quad \\
\underline{-3x^2 \ + 9x} \quad \\
3x \ -15 \\
\underline{3x - 9} \\
-6
\end{array}$$

Check:

$(-x +3)(x^2 +3x -3 + \frac{6}{x-3}) = -x^3+12x -15$

$-x^3+12x -15 = -x^3+12x -15$

10. $$\frac{c^3 -27}{c - 3} = c^2 +3c +9$$

$$\begin{array}{r}
\overline{c^2 \ +3c \quad +9} \\
c -3 /\ c^3 \ +0c^2 \ +0c \ -27 \\
c^3 \ -3c^2 \qquad\qquad\quad \\
\hline
+3c^2 \ +0c \qquad \\
+3c^2 \ {}^-9c \qquad \\
\hline
+9c \ -27 \\
+9c \ -27 \\
\hline
0
\end{array}$$

Check:

$(c -3)(c^2 +3c +9) = c^3 -27$

$c^3 -27 = c^3 -27$

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APPENDICES

APPENDIX A

SAMAR STATE POLYTECHNIC COLLEGE
Catbalogan, Samar

The Dean of Graduate Studies
Samar State Polytechnic College
Catbalogan, Samar

Sir:

In my desire to start writing my thesis proposal, I have the honor to submit for your approval one of the following research problems, preferably No. 1

1. DEVELOPMENT OF SELF INSTRUCTIONAL MATERIALS IN MATHEMATICS 101 (College Algebra).
2. READING COMPREHENSIONS AND COMPUTATIONAL SKILLS OF COLLEGE STUDENTS OF SSPC.
3. DEVELOPMENT OF MODULE ON SOME SELECTED TOPICS IN COLLEGE ALGEBRA, SSPC.

I hope for your early and favorable action on this matter.

Very truly yours,

(SGD.) ANESIA M. JAROMAY

Recommending Approval:

(SGD.) TERSITO A. ALIPOSA, Ed.D./Ph.D.
Head of Research and Publication

APPROVED:

(SGD.) SENECIO D. AYONG, DPA/Ed.D.
College Dean

APPENDIX B

Samar State Polytechnic College
Catbalogan, Samar

The President
Samar State Polytechnic College
Catbalogan, Samar

Sir:

In connection with my thesis entitled **DEVELOPMENT OF SELF INSTRUCTIONAL MATERIALS IN MATHEMATICS 101 (College Algebra)**, I would like to request permission to conduct a diagnostic test to the BSIT, DIT, BSIE and ENG'G students.

The diagnostic test is a vital component of my thesis writing.

Your approval on this request is highly appreciated.

Very truly yours,

(SGD.) ANESIA M. JAROMAY

Noted by:

(SGD.) FLORIDA B. MARCO
Adviser

Approved by:

(SGD.) BASILIO S. FRINCILLO
President

APPENDIX C

Samar State Polytechnic College
Catbalogan, Samar

June 18, 1992

The President
Samar State Polytechnic College
Catbalogan, Samar

Sir:

I have the honor to request your good office to utilize the computer equipment for my desire to input my thesis writing entitled **DEVELOPMENT OF SELF INSTRUCTIONAL MATERIALS IN MATHEMATICS 101 (College Algebra)**, effective this date up to January 1993, scheduled Wednesday morning, Saturday and Sunday.

I do hope that this request will merit your favorable action.

Respectfully yours,

(SGD.) **ANESIA M. JAROMAY**
(Researcher)

Recommending Approval:

(SGD.) **Engr. EDDIE FRINCILLO**
Computer In-charge

(SGD.) **Engr. ARTURO RAMA**
Head of Eng'g Dept.

APPROVED:

(SGD.) **BASILIO S. FRINCILLO**
President

APPENDIX D

SAMAR STATE POLYTECHNIC COLLEGE
Catbalogan, Samar

August 2, 1995

The President
Samar State Polytechnic College
Catbalogan, Samar

Sir:

In connection with my study entitled **DEVELOPMENT OF SELF INSTRUCTIONAL MATERIALS IN MATHEMATICS 101 (College Algebra)**, I have the honor to ask permission to validate my module using the following selected BSIT students effective August 2 to 31, 1995:

	Experimental Group			**Control Group**	
1.	Campos,	Ruel	1.	Dizon,	Alison
2.	Dacles,	Nadia	2.	Cabuello,	Erwin
3.	Quebec,	Pedro	3.	Bernate,	Randy
4.	Pacanza,	Conrado	4.	Caveiro,	Adan
5.	Arao,	Glunilo	5.	Villaflor,	Haidee
6.	Encarnacion,	Nelson	6.	Saoy,	Rolando
7.	Pabunan,	Reynaldo	7.	Dalwatan,	Pompio
8.	Pahayahay,	Arlene	8.	Cemania,	Gemma
9.	Cananua,	Expedito	9.	Tan,	Nelia
10.	Curiano,	Rolando	10.	Cabadsan,	Joel
11.	Orale,	Daniel	11.	Abulog,	Jane
12.	Santiago,	Salome	12.	Abantao,	Alfredo
13.	Salamida,	Orlando	13.	Ybanez,	Merlita
14.	Dacutanan,	Adelino	14.	Ecle,	Arlet
15.	Solayao,	Antonio	15.	Enano,	Maricor
16.	Valles,	Van Jose	16.	Siapo,	Evelyn
17.	Bello,	Marvin	17.	Cuna,	Paulo
18.	Santiago,	Romeo	18.	Dacuma,	Florina
19.	Sobremonte,	Edmond	19.	Dacula,	Rommel
20.	Babatio,	Irenio	20.	Pomida,	Chona
21.	Redaja,	Christina	21.	Rafon,	Maricel
22.	Turla,	Rommel	22.	Doza,	Alfredo
23.	Pelande,	Jances	23.	Dacutan,	Alfredo
24.	Catalan,	Jurez	24.	Catalan,	Pureza
25.	Cordowa,	Sherwin	25.	Velos,	Jesica

These students are under Prof. Elisa Porcil. The approval of my request will mean that we will set a separate schedule.

Hoping for a favorable action on my request for the experiment. I remain.

Very truly yours,

(SGD.) **ANESIA M. JAROMAY**
(Researcher)

Recommending Approval:

(SGD.) **RIZALINA M. URBIZTONDO, Ed.D.**
Dean of Graduate Studies

APPROVED:

(SGD.) **DOMINADOR Q. CABANGANAN, Ed.D.**
College President

APPENDIX E

Republic of the Philippines
SAMAR STATE POLYTECHNIC COLLEGE
Catbalogan, Samar

January 13, 1992

The President
Samar State Polytechnic College
Catbalogan, Samar

Sir:

In my desire to finish writing my thesis proposal leading to the degree of Master of Arts in Teaching Mathematics (MAT), in Samar State Polytechnic College, I would like to request from your good office that I be granted study leave with pay, as stated in Section 18 under Leave Privileges of the College Code, I have applied last year but due to some constraints the office promised me that I be granted this year 1992-1993, starting from the first semester of June 1992.

I hope that this year this request will merit your favorable consideration.

Very truly yours,

(SGD.) **ANESIA M. JAROMAY**

NOTED:

(SGD.) **TERSITO A. ALIPOSA, Ed.D./Ph.D.**
Head of Research and Publication

Recommending Approval:

(SGD.) **SENECIO D. AYONG, DPA/Ed.D.**
Dean of Graduate School

(SGD.) **BERNARDO S. OLIVA, Ph.D**
College Dean

APPROVED:

(SGD.) **BASILIO S. FRINCILLO**
President

APPENDIX F

Samar Sate Polytechnic College
Catbalogan, Samar

The Dean of Graduate Studies
Samar State Polytechnic College
Catbalogan, Samar

Madam/Sir:

I hereby respectfully request that I be scheduled for final defense of my Thesis entitled **DEVELOPMENT OF SELF INSTRUCTIONAL MATERIALS IN MATHEMATICS 101 (College Algebra)** on a date convenient for your office.

Thank you.

Very truly yours,

(SGD.) ANESIA M. JAROMAY
(Researcher)

Noted by:

(SGD.) FLORIDA B. MARCO
Adviser

Approved:

(SGD.) RIZALINA M. URBIZTONDO, Ed.D.
Dean of Graduate Studies

APPENDIX G

Samar State Polytechnic College
Catbalogan, Samar

To The Student

Dear student,

You are lucky to have been chosen as a taker in this diagnostic test in mathematics.

This instrument is not designed to test your ability in College Algebra but rather to help you identify the difficulties you encounter in your study of this course.

Rest assured that in no way the results of this diagnostic test affects your grade in mathematics subject, you are presently enrolled in. The results will solely be used as a bases for upgrading mathematics instruction in our college.

This test is only good for one hour; so do not stay too long in a single item. Answer all the items to the best of your ability and **DO NOT LEAVE ANY ITEM BLANK.**

Thank You!

(SGD.) **ANESIA M. JAROMAY**
Researcher

Directions: Fill in the spaces on the Answer Sheets. This is an 80 item test. Select the best answer and blacken the circle under each letter on the Answer Sheets. If you do not find a correct answer among the first four choices, blacken the circle under N means a correct answer is not among the four choices. Only one answer should be marked for each problem. Do your figuring on scratch papers.

1\. The set of directed numbers is usually called as

A. negative
B. positive
*C. integers
D. fraction
N. none of these

2\. -5(6)(-3) =

A. -90
B. - 2
C. 27
*D. 90
N. none of these

3\. (-9-(-5))/2

A. - 7
B. 2
C. -13/2
*D. - 2
N. none of these

4\. If a negative number is subtracted from itself the result is

*A. 0
B. a negative no.
C. 1
D. a positive no.
N. none of these

5\. What number can replace both question marks?

1/? = ?/64

A. 1
*B. 8
C. 32
D. 64
N. none of these

6\. The sum of

17 4/9
8 7/3
5 5/6

A. 30 5/9
B. 31 1/8
C. 31 7/9
D. 32 1/6
*N. none of these

7. The simplified form of the answer

$$\frac{3\ 1/2 + 4\ 3/4}{7 - 2\ 1/4}$$

A. 1 4/19
B. 32/19
C. 33/19
*D. 1 6/19
N. none of these

8. Which of the numerals below is read as

"Two hundred five and sixty four thousandth"

A. 2005.64000
*B. 205.064
C. 205.640
D. 2005.064
N. none of these

9. From nine hundred eight subtract nine hundred and eight thousands.

*A. - 7.992
B. -8.028
C. 0.7992
D. 1.7992
N. none of these

10. The decimal equivalent of 3/5

A. 0.35
B. 0.25
*C. 0.60
D. 0.75
N. none of these

11. Multiply

$$\begin{array}{r} .014 \\ .014 \\ \hline \end{array}$$

A. 196
B. 19.6
*C. 0.000196
D. 0.00196
N. none of these

12. The decimal number 86.49952 rounded off to the nearest thousandths.

*A. 86.500
B. 86.005
C. 86.050
D. 86.450
N. none of these

13. A combination of numbers and symbols, expression connected by the fundamental operations in algebra.

A. polynomial
*B. algebraic expression
C. equation
D. monomial
N. none of these

14. The expression $3a(2b - a)$ when simplified is

A. $3a-b$
B. $4a-b$
C. $2a-2b$
*D. $6ab-3a^2$
N. none of these

15. The expression $2x-2y^2-5ab$ is

A. binomial
B. monomial
*C. trinomial
D. multinomial
N. none of these

16. Polynomial expression consisting of one term

*A. monomial
B. binomial
C. trinomial
D. polynomial
N. none of these

17. The product of $-4c^2 (cf-1/2c +f)$ is

A. $-32cf +2c$
B. $2c-32c^2$
C. $-2f^3-cf$
*D. $-4c^3f+2c^3-4c^2f$
N. none of these

18. $$\frac{3^5 \text{ x } 4^8}{4^8 \text{ x } 3^5}$$

A. 0
B. 3/10
*C. 1
D. 3
N. none of these

19. The quotient obtain from

$$25\ /\ \overline{625}$$

A. 40
B. 12
C. 32
*D. 25
N. none of these

20. The volume of a cube of side equal to r is

*A. r^3
B. $3/4r^3$
C. $3r^3$
D. $3r$
N. none of these

21. In the expression x^n n is called the ___

A. base
B. power
*C. exponent
D. radical
E. none of these

22. Simplifying $a^{x/y}$ will result to an expression which is called as

A. algebraic
*B. radical
C. logarithmic
D. exponential
N. none of these

23. Law of exponent define

as $x^m \cdot x^b$

*A. $x^{m + b}$
B. $x^{y/b}$
C. x^{m-b}
D. x/y
N. none of these

24. The expression $5y^{-3} - 6c^{-6}$ is equal to ____ when written without negative exponents.

*A. $5/y^3-6/c^6$
B. $5y^3 + 6c^6$
C. $5y^3 - 6c^6$
D. $5y - 6c$
N. none of these

25. Evaluating the expression

$\sqrt{3600}$

*A. 60
B. 10
C. 35
D. 25
N. none of these

26. Simplifying the expression

$(8a^3b^6)/4ab^3$

A. 3ab
B. $2a^2b$
*C. $2a^2b^3$
D. 4ab
N. none of these

27. The diameter of an object near our sun is 0.0000000001 cm. What is the compact form of writing this number?

A. $1x10^{-14}$
B. $1x10^{-15}$
C. $1x10^{-12}$
*D. $1x10^{-10}$
N. none of these

28. The radical form of

$x^{1/2}$ is

*A. $\sqrt{x}$
B. $\sqrt{2x}$
C. $2\sqrt{x}$
D. 4x
N. none of these

29. Changing the expression

$x^{2/5}/y^{-2/5}$

to its radical form.

A. $xy\sqrt{3/5}$
B. $3\sqrt{xy^5}$
C. $5\sqrt{3xy}$
*D. $\sqrt{(x/y)}^5$
N. none of these

30. Multiplying ___
$(5\sqrt{1/8})(5\sqrt{2})$

A. $25\sqrt{1/4}$
B. $25\sqrt{8/2}$
C. $25\sqrt{2/2}$
*D. $25/2$
N. none of these

31. The quotient of the expression

$$\frac{8m^3ny^2 - 4m^8ny^4}{2mny}$$

A. $4mn + 6m^3$
*B. $4m^2y - 2m^7y^3$
C. $9my-3y^2$
D. $5xy + 2xy$
N. none of these

32. In the expression $2x\sqrt{3xy}$ when 2x is incorporated to the radicand is equal to ___.

*A. $\sqrt{12x^3y}$
B. $\sqrt{2x^3xy}$
C. $\sqrt{24x^4y/2}$
D. $\sqrt{32x^4}$
N. none of these

33. Rationalizing the denominator of the radical expression

$1/1 + \sqrt{2}$

*A. $\sqrt{2} - 1$
B. $1 - \sqrt{2}$
C. $1 + \sqrt{2}$
D. 0.1
N. none of these

34. Expressing the sum as a single radical
$3\sqrt{2} - 1\sqrt{5}$

$2\sqrt{2} + 3\sqrt{5}$

A. $5\sqrt{2} - 2\sqrt{5}$
*B. $5\sqrt{2} + 2\sqrt{5}$
C. $7\sqrt{10} - 1\sqrt{3}$
D. $11\sqrt{10} + \sqrt{3}$
N. none of these

35. Expressing in its simplest form the given expression
$4\sqrt{36} - \sqrt{2} - \sqrt{18}$

A. $4\sqrt{6} - 4\sqrt{2}$
B. $2\sqrt{6} - 3\sqrt{2}$
*C. $24 - 4\sqrt{2}$
D. $6\sqrt{56}$
N. none of these

36. The $\sqrt{625}$ is

A. 65
B. 35
*C. 25
D. 50
N. none of these

37. Which of the given choices is not a linear equation?

A. $3x-1 = 7$
B. $3x+y = 0$
C. $x-7y+2 = 0$
*D. $x^2-7y+2 = 0$
N. none of these

38. An equation form $ax+by+c = 0$ is a linear equation in

A. four unknowns
*B. two unknowns
C. five unknowns
D. three unknowns
N. none of these

39. What property of equality is used in simplifying the given equation?

$2x = 10$
$x = 5$

A. addition property
*B. multiplication
C. transitive
D. reflexive
N. none of these

40. The value of x in the equation $3x-t+5 = 0$

*A. $t-5/3$
B. $-t+5/3$
C. $t+5/3$
D. $t-5/3$
N. none of these

41. What is x in the equation $5x-4 = 3x-6$?

A. -3
B. -4
C. -5
*D. -1
N. none of these

42. What is y in the equation $2y + 4y = 36$?

A. 2
B. 3
C. 4
*D. 6
N. none of these

43. When $x = 3$, find the value of the expression

$$x^2+5x+9$$

*A. 33
B. 34
C. 30
D. 31
N. none of these

44. The length of the rectangle exceeds its width by 2 feet. If its width is 6 feet, what is the lenght?

A. 24
B. 12
C. 6
*D. 8
N. 2

45. An expression which has a sign of equality is called ________.

*A. equation
B. algebra
C. multiplication
D. division
N. subtraction

46. What is the value of x and y in the given equations?

$$\begin{array}{l} 4x+2y = 24 \\ \underline{3x-2y = 25} \end{array}$$

A. $x = 7$, $y = 28$
B. $x = 4$, $y = 7$
C. $x = 6$, $y = 2$
*D. $x = 7$, $y = -2$
N. none of these

47. The value of x in the equation $3x-a+5 = 0$

A. $d-5/3$
B. $d+5/3$
*C. $a-5/3$
D. $a+3/2$
N. none of these

48. An equation which is true where no or only some particular value or values for the letters it contains.

A. linear equation
*B. conditional
C. identical
D. function
N. none of these

49. Solve for the value of n in the equation

$$10n+5 = 55$$

A. 18
*B. 5
C. 3
D. 10
N. none of these

50. Solve for x in the expression

$2x-5 = 11$

A. 5
B. 3
C. 6
*D. 8
N. none of these

51. $5x-6 = x+14$.
what is x?

*A. 5
B. 9
C. 6
D. 8
N. none of these

52. The solution of the system of simultaneous equations is

$$\begin{aligned} 2x+ y &= 7 \\ 2x-3y &= 11 \end{aligned}$$

A. $x=4,\ y=-1$
B. $x=4,\ y= 1$
C. $x=7,\ y=18$
*D. $x=4,\ y=-1$
N. none of these

53. What type of equations are the following:

$$\begin{aligned} 2x-3y &= 12 \\ 2x-3y &= 7 \end{aligned}$$

A. consistent
*B. inconsistent
C. conditional
D. identical
N. none of these

54. The given equation $x^2-x+2 = 0$ is

A. linear equation
*B. quadratic
C. independent
D. dependent
N. none of these

55. A quadratic equation in x is an expression in the form

A. ax^2+bx+c
B. ax^2+bx-c
C. $y = ax^2+bx+c$
*D. $ax^2+bx+c = 0$

56. Which of the choices is not a quadratic equation.

A. $x^2-4x-5 = 0$
B. $(ax+b)^2-cx = 0$
C. $3x^2-2x = x^2$
*D. $(a+b)(a+b)$
N. none of these

57. The difference of the two squares

a^2-b^2

*A. (a+b) (a-b)
B. (a-b) (a-b)
C. (a-c) (a+c)
D. (a+b) (a+c)
N. none of these

58. Taking the square roots of both sides of the equation

$(x-3)^2 = (17)^2$
will results in

*A. x = 20, -14
B. x = 17, -17
C. x = +3, -3
D. x = 20, 14
N. none of these

59. The equation $ax^2 -bx +c = 0$ when a = 1, b = -5, c = 6 will give the value of x equal to _______.

*A. {3, 2}
B. {-3, 2}
C. {3, -2}
D. {-3, -2}
N. none of these

60. A positive value of the discriminant $b^2 -4ac$ means that the roots of the given quadratic equation are ___.

A. real and equal
B. real and unequal
C. imaginaries
*D. real numbers
N. none of these

61. Which of the given choices is a sum of two cubes?

A. (a+b)(a-b)
B. $(a-b)^3(a^3+ab+b^2$
C. $(a+b)^3$
D. $(a+b)^2$
*N. none of these

62. Which of the given choices is the given expression

a^6-b^6 equal to?

A. $(a+b)(a^3-2ab+b^2)$
B. $(a-b)(a^2+ab+b^2)$
C. $(a+b)^3\ (a-b)^3$
D. $(a+b)^6\ (a-b)^6$
*N. none of these

63. Which are the factors of the algebraic expression

$27x^2-12y^2$

*A. 3(3x+2y)(3x-2y)
B. 3(5x+y)(4+y)
C. $3(2x+y)(x+y^3)$
D. $2(2x+y)(x+y^2)$
N. none of these

64. The solution set of the equation $4x^2-9 = 0$ is

*A. (3/2, -3/2)
B. (3/2, 3/2)
C. (-3/2, -3/2)
D. (2/3, -2/3)
N. none of these

65. If x+5 = y, which of the following statements is true about x and y?

A. x more than y
*B. x is less than y
C. x is more than 5
D. x is less than 5
N. none of these

66. If x/y = 3/5, which of the following statements is true about x and y?

Statement 1. x is less than y
Statement 2. x = 3 and y = 5
Statement 3. 5x = 3y

A. Statement I
B. Statement I and II
*C. Statement I and III
D. Statement II
N. none of these

67. Let S {0,1,2} by substituting each number in set S, What is the solution of 3x = 15?

A. 0
B. 1
C. 2
D. 3
*N. none of these

68. Beth is 3 yrs. older than Alma. Which table of number pairs below correctly associates the ages of the two?

A.

Alma's age	6	7	8
Beth's age	3	3	3

B.

Alma's age	6	7	8
Beth's age	3	4	5

C.

Alma's age	6	7	8
Beth's age	9	12	13

*D.

Alma's age	6	7	8
Beth's age	9	10	11

N. none of these

69. Susana is taking typing lessons. Her record for the first few weeks is shown below.

No. of weeks	No. of words/minute
1st	32
2nd	38
3rd	44
4th	50

If she improved at the same rate how many more weeks can she expect to attain a typing speed of 71 words per minute?

A. 2
B. 2 1/2
C. 3
*D. 3 1/2
N. none of these

70. Which set contains the ordered pairs pictured in the arrow diagram below?

1 -------> 4
2 -------> 3
3 -------> 2
4 -------> 1
5 -------> 0

A. {(4,1), (3,2), (2,3), (1,4), (0,5)}
*B. {(1,4), (2,3), (3,2), (4,1), (5,0)}
C. {(1,2), (2,3), (3,4), (4,5), (2,1)}
D. {(4,3), (3,2), (2,1), (1,0), (2,3)}
N. none of these

71. Which of the arrow diagram below pictures a function?

```
-4 ———————> 16 ———————> +16 ———————> -4
+4                    > -16 ———————> +4

-5 ———————> 25 ———————> +25 ———————> -5
+5                    > -25 ———————> +5

-6 ———————> 36 ———————> +36 ———————> -6
+6                    > -36 ———————> +6

-7 ———————> 49 ———————> +49 ———————> -7
+7                    > -49 ———————> +7

       I          II            III
```

A. I only
B. II only
*C. I and III only
D. All of them
E. none of these

72. What ordered pair describes the point located on the fourth quadrant and 3 units away from both axis.

*A. (3, -3)
B. (-3, 3)
C. (3, 3)
D. (-3, -3)
N. none of these

73. What are the factors of $16x^2-40x+25$?

A. (4x+5) (4x+5)
*B. (4x-5) (4x-5)
C. (5x+4) (5x+4)
D. (5x-4) (5x-4)
N. none of these

74. Which of the following expressions is equivalent to

$\log_b x = y$?

A. $x = y_b$
B. $y = x_b$
C. $y = b_x$
*D. $x = b^y$
N. none of these

75. A bag contains more than 15 marbles. That is, the contents of the bag is ____ than 15.

A. <
*B. >
C. ≤
D. ≥
N. none of these

76. An envelope is known to contain at least 12 pesos. That is, the contents of the envelope ____ 12 pesos.

A. >
B. <
C. ≤
*D. ≥
E. none of these

77. Mother told Mario "You may spend at most 70 centavos for merienda. That is the amount Mario may spend is _____ 70 centavos.

A. <
B. >
C. ≥
*D. ≤
E. none of these

78. $x > -2$ and $x < 2$
What are the possible values of x?

*A. (-2, 2)
B. [-2, 2]
C. (-2, 2]
D. [-2, 2)
N. none of these

79. What is the solution set?

$x > -2$ or $x < 2$

A. { ... -2, 2, 3, ...}
B. { ... -2, -1, ... 2}
C. { ... -2, 3, 4, ...}
D. { ... -2, -1, ... 2}
*N. none of these

80. $-2 < x < 2$
What are the possible values of x in the set of integers?

*A. { -1, 0, 1}
B. { ... -1, 0, 1, ...}
C. { ... -2, 0, 1}
D. { -2, 0, 1, 2}
N. none of these

APPENDIX H

ANSWER SHEETS

Name: ______________________________ Date: ______________________
Year & Section: __________________ Score: ____________________

	A	B	C	D	N		A	B	C	D	N
1.	0	0	0	0	0	41.	0	0	0	0	0
2.	0	0	0	0	0	42.	0	0	0	0	0
3.	0	0	0	0	0	43.	0	0	0	0	0
4.	0	0	0	0	0	44.	0	0	0	0	0
5.	0	0	0	0	0	45.	0	0	0	0	0
6.	0	0	0	0	0	46.	0	0	0	0	0
7.	0	0	0	0	0	47.	0	0	0	0	0
8.	0	0	0	0	0	48.	0	0	0	0	0
9.	0	0	0	0	0	49.	0	0	0	0	0
10.	0	0	0	0	0	50.	0	0	0	0	0
11.	0	0	0	0	0	51.	0	0	0	0	0
12.	0	0	0	0	0	52.	0	0	0	0	0
13.	0	0	0	0	0	53.	0	0	0	0	0
14.	0	0	0	0	0	54.	0	0	0	0	0
15.	0	0	0	0	0	55.	0	0	0	0	0
16.	0	0	0	0	0	56.	0	0	0	0	0
17.	0	0	0	0	0	57.	0	0	0	0	0
18.	0	0	0	0	0	58.	0	0	0	0	0
19.	0	0	0	0	0	59.	0	0	0	0	0
20.	0	0	0	0	0	60.	0	0	0	0	0

APPENDIX H (Cont'd)

	A	**B**	**C**	**D**	**N**		**A**	**B**	**C**	**D**	**N**
21.	0	0	0	0	0	61.	0	0	0	0	0
22.	0	0	0	0	0	62.	0	0	0	0	0
23.	0	0	0	0	0	63.	0	0	0	0	0
24.	0	0	0	0	0	64.	0	0	0	0	0
25.	0	0	0	0	0	65.	0	0	0	0	0
26.	0	0	0	0	0	66.	0	0	0	0	0
27.	0	0	0	0	0	67.	0	0	0	0	0
28.	0	0	0	0	0	68.	0	0	0	0	0
29.	0	0	0	0	0	69.	0	0	0	0	0
30.	0	0	0	0	0	70.	0	0	0	0	0
31.	0	0	0	0	0	71.	0	0	0	0	0
32.	0	0	0	0	0	72.	0	0	0	0	0
33.	0	0	0	0	0	73.	0	0	0	0	0
34.	0	0	0	0	0	74.	0	0	0	0	0
35.	0	0	0	0	0	75.	0	0	0	0	0
36.	0	0	0	0	0	76.	0	0	0	0	0
37.	0	0	0	0	0	77.	0	0	0	0	0
38.	0	0	0	0	0	78.	0	0	0	0	0
39.	0	0	0	0	0	79.	0	0	0	0	0
40.	0	0	0	0	0	80.	0	0	0	0	0

APPENDIX I

Key to Correction of Diagnostic Test

1. C
2. D
3. D
4. A
5. B
6. N
7. N
8. B
9. A
10 C
11 C
12. A
13. B
14. N
15. D
16. A
17. N
18. C
19. D
20. D
21. C
22. B
23. D
24. A
25. C
26. D
27. D
28. A
29. B
30. C
31. B
32. A
33. A
34. B
35. D
36. C
38. B
39. B
40. D
41. D
41. D
42. D
43. A
44. B
45. A
46. N
47. C
48. A
49. C
50. B
51. D
52. N
53. B
54. C
55. B
56 D
57. A
58. N
59. A
60. C
61. C
62. N
63. A
64. N
65. B
66. C
67. N
68. D
69. D
70. B
71. A
72. A
73. B
74. D
75. C
76. D
77. B
78. A
79. B
80. A

APPENDIX J

Item Analysis of Diagnostic Test

Item	a		b		c		d		e		f		key			Diff.	Inter	Dis	Re	p	pq
No.	H	L	H	L	H	L	H	L	H	L	H	L		PH	PL	index	pret	cri	mark		
1.	19	8	8	8	8	15	8	6	15	4	5	5	a	.70	.30	.50	MD	.40	Ret.	.50	.25
2.	11	9	10	9	6	2	12	9	3	3	3	2	d	.44	.33	.38	D	.11	Imp.	.62	.24
3.	6	3	17	6	6	3	7	7	13	1	1	0	b	.63	.22	.42	MD	.41	Ret.	.58	.24
4.	8	7	6	5	20	3	6	6	1	1	5	1	c	.74	.11	.42	MD	.63	Ret	.58	.24
5.	16	2	5	4	5	5	11	1	4	1	1	0	a	.59	.07	.33	D	.52	Ret	.67	.22
6.	6	3	17	13	5	3	5	2	13	3	1	1	b	.63	.11	.37	D	.52	Ret.	.63	.19
7.	18	4	4	4	14	3	12	1	2	2	2	0	a	.67	.15	.41	MD	.52	Ret.	.59	.21
8.	17	4	7	4	4	4	4	4	2	3	3	1	a	.63	.15	.39	D	.48	Ret.	.61	.18
9.	2	1	12	1	2	1	24	3	3	21	2	0	d	.69	.48	.68	MD	.41	Ret	.32	.22
10.	2	1	2	1	2	1	3	2	0	0	3	1	f	.11	.04	.07	VD	.07	Rej.		
11.	5	2	5	2	6	2	18	5	1	0	3	1	d	.67	.18	.42	MD	.49	Ret.	.58	.24
12	9	8	9	8	17	7	19	6	3	0	6	13	d	.70	.22	.46	MD	.48	Ret.	.54	.25
13.	22	11	12	12	8	17	8	5	6	4	15	3	a	.81	.41	.61	MD	.40	Ret.	.39	.24
14.	6	5	6	5	5	2	2	1	3	2	1	4	e	.11	.07	.09	VD	.04	Rej.		
15.	15	2	5	2	16	3	4	3	14	3	3	3	c	.59	.11	.35	D	.24	Imp.	.65	.23
16.	12	12	12	11	6	6	9	8	18	5	5	3	e	.67	.18	.42	MD	.49	Ret.	.58	.24
17.	13	11	3	4	6	5	3	13	3	12	2	0	a	.48	.41	.44	MD	.07	Rej.		
18.	16	4	6	8	6	12	4	3	11	0	1	0	a	.59	.15	.37	D	.44	Ret	.63	.23
19.	10	8	10	3	19	7	20	8	3	3	4	4	d	.74	.30	.52	MD	.44	Ret	.48	.25
20.	12	2	12	10	22	11	2	10	1	1	2	0	c	.81	.41	.61	MD	.40	Ret.	.39	.24
21.	10	10	20	11	7	4	6	4	6	4	4	3	b	.74	.41	.58	MD	.33	Imp.	.42	.24
22.	3	1	3	11	3	0	12	0	13	1	3	2	e	.48	.04	.26	D	.44	Ret.	.74	.19
23.	14	2	4	2	23	2	3	11	26	12	3	1	e	.96	.44	.07	E	.52	Ret.	.30	.21
24.	1	4	17	4	5	1	16	5	14	2	0	1	b	.63	.15	.39	D	.48	Ret	.61	.24
25.	2	2	2	12	1	0	10	0	2	1	2	2	e	.07	.04	.06	VD	.03	Rej.		
26.	13	0	3	20	11	1	1	0	22	10	0	0	e	.81	.37	.57	MD	.44	Ret	.41	.24
27.	13	2	3	2	14	1	2	0	10	0	2	1	c	.52	.04	.28	D	.48	Ret	.71	.20
28.	16	3	15	3	7	6	5	4	14	1	2	0	a	.59	.11	.35	D	.48	Ret.	.65	.23
29.	6	2	16	2	5	4	15	3	13	2	3	1	b	.59	.07	.33	D	.52	Ret	.67	.22
30.	19	7	9	7	16	1	13	1	2	1	13	3	a	.70	.26	.48	MD	.44	Ret	.52	.21
31.	16	4	6	4	14	1	11	1	2	2	2	14	a	.59	.07	.33	D	.44	Ret	.63	.23
32.	6	5	2	5	2	26	10	4	20	3	1	2	c	.96	.37	.67	MD	.59	Ret.	.33	.22
33.	5	0	25	10	24	6	12	4	13	11	20	1	c	.93	.37	.65	MD	.56	Ret.	.35	.23
34.	3	1	13	1	3	2	4	4	12	2	11	0	b	.48	.04	.26	D	.44	Ret.	.74	.19
35.	16	2	6	2	14	2	12	2	1	0	12	2	a	.22	.07	.15	D	.15	Imp.	.85	.13
36.	8	3	8	3	5	4	11	3	3	2	14	0	d	.41	.11	.26	D	.30	Imp.	.74	.19
37.	4	1	4	4	6	0	15	3	2	11	3	12	d	.56	.11	.34	D	.45	Ret.	.66	.22
38.	15	1	5	3	2	1	1	3	2	12	2	1	a	.56	.04	.30	D	.52	Ret.	.70	.21
39.	13	3	2	6	8	5	3	11	3	0	3	0	a	.70	.11	.41	MD	.59	Ret.	.59	.24

APPENDIX J (Cont'd.)

40.	5	3	7	8	5	4	2	2	4	1	4	1	f	.15	.04	.10	VD	.11	Rej.		
41.	3	0	6	1	17	3	5	3	1	1	1	1	c	.63	.11	.37	MD	.53	Ret.	.63	.23
42.	6	4	4	2	7	3	17	5	0	0	0	0	d	.63	.18	.41	MD	.45	Ret.	.59	.14
43.	5	2	3	2	3	2	18	6	2	2	2	2	d	.67	.22	.45	MD	.45	Ret.	.55	.25
44.	18	7	6	2	7	5	5	3	2	13	2	1	a	.67	.26	.47	MD	.41	Ret.	.53	.25
45.	9	6	1	7	4	4	5	4	1	1	1	1	b	.70	.11	.41	MD	.59	Ret.	.59	.24
46.	5	3	5	2	3	0	1	0	3	2	3	2	b	.18	.07	.13	VD	.11	Rej.		
47.	6	5	16	1	2	0	16	4	2	1	2	1	b	.59	.04	.32	D	.55	Ret.	.68	.22
48.	19	4	9	2	5	1	3	3	6	3	6	3	a	.70	.15	.43	MD	.55	Ret.	.76	.18
49.	2	0	1	0	1	0	1	2	2	1	12	1	f	.44	.04	.24	D	.40	Ret.	.76	.18
50.	15	1	4	2	2	1	4	3	2	0	2	0	a	.56	.04	.30	D	.52	Ret.	.70	.21
51	5	2	5	3	3	0	2	1	1	1	1	1	a	.18	.07	.13	VD	.11			
52.	2	1	2	2	15	1	1	1	1	2	0	1	c	.56	.04	.30	D	.52	Ret.	.97	.20
53.	5	4	5	4	1	0	14	1	2	0	2	0	d	.52	.04	.28	D	.48	Ret.	.72	.25
54.	3	1	3	2	19	6	3	3	1	1	1	1	c	.70	.22	.46	MD	.48	Ret.	.54	.23
55.	4	4	5	4	16	4	3	1	3	1	3	11	c	.59	.15	.37	D	.44	Ret.	.63	.23
56.	6	3	5	2	5	3	18	2	2	1	2	11	d	.67	.07	.37	D	.60	Ret.	.63	.23
57.	16	2	2	6	2	2	5	3	2	0	2	0	a	.59	.07	.33	D	.52	Ret.	.67	.22
58.	17	3	11	6	3	2	6	2	6	2	1	6	a	.63	.11	.37	D	.52	Ret.	.63	.23
59.	5	4	4	5	2	0	5	1	1	0	1	0	d	.18	.04	.11	VD	.14	Rej.		
60.	4	1	1	4	5	1	15	2	2	2	2	12	d	.63	.11	.37	D	.52	Ret.	.63	.23
61.	16	3	2	5	3	1	17	3	13	1	3	1	d	.63	.11	.37	D	.52	Ret.	.63	.23
62.	5	3	18	10	3	3	4	2	2	1	2	1	b	.67	.37	.52	MD	.30	Ret.	.48	.25
63	18	3	3	0	1	1	14	2	2	0	2	0	a	.67	.11	.39	D	.56	Ret.	.61	.25
64.	3	0	7	4	0	2	4	0	1	0	1	0	b	.63	.15	.39	D	.48	Ret.	.61	.25
65.	16	4	5	1	2	2	3	3	2	1	2	11	a	.59	.15	.37	D	.44	Ret.	.63	.23
66.	5	1	2	1	1	0	18	2	2	1	3	0	d	.67	.07	.37	D	.60	Ret.	.63	.23
67.	2	1	1	0	2	0	0	1	3	1	1	1	e	.11	.04	.08	VD	.07	Rej.		
68.	1	1	14	2	0	0	4	13	5	0	0	0	b	.52	.07	.30	D	.45	Ret.	.70	.21
69.	4	2	6	4	2	0	16	3	1	11	10	0	d	.59	.11	.35	D	.48	Ret.	.65	.23
70.	6	3	13	6	12	2	16	2	4	2	1	0	d	.59	.07	.33	D	.52	Ret.	.41	.24
71.	1	1	2	1	4	0	2	2	4	0	3	2	f	.11	.07	.10	VD	.04	Rej.		
72.	12	1	1	2	1	1	1	2	2	11	5	0	a	.44	.04	.24	D	.40	Ret.	.76	.18
73.	5	2	2	5	4	2	16	4	4	0	2	10	d	.59	.15	.37	D	.44	Ret.	.63	.23
74.	4	4	4	6	4	0	14	2	1	0	2	1	d	.15	.07	.11	VD	.08	Rej.	.89	.10
75.	16	5	5	6	3	1	5	11	14	10	11	10	a	.59	.18	.39	D	.41	Ret.	.61	.24
76.	14	1	1	3	1	1	2	10	2	2	1	1	a	.52	.04	.28	D	.48	Ret.	.72	.20
77.	16	1	11	6	2	1	3	2	1	1	2	1	a	.59	.04	.32	D	.55	Ret.	.68	.22
78.	2	2	2	3	2	1	2	0	1	0	1	2	c	.07	.04	.06	VD	.03	Rej.		
79.	15	4	4	5	5	1	5	1	1	5	3	11	a	.56	.15	.36	D	.41	Ret.	.64	.23
80.	3	2	2	4	3	2	4	10	4	1	14	2	f	.52	.04	.30	D	.45	Ret.	.70	.21
Total																26.24		31.57		15.10	
Mean																.385		.464			
Reliability																.92					

APPENDIX K

Computation of the Reliability Coefficient (r)

$$r = \frac{k}{k-1}\left(1 - \frac{6\sum pq}{(kd)^2}\right)$$

$$r = \frac{68}{68-1}\left(1 - \frac{6(15.10)}{(68 \times 0.46)^2}\right)$$

$$r = \frac{68}{67}\left(1 - \frac{90.6}{(31.28)^2}\right)$$

$$r = 1.015\ (1 - 0.0926)$$

$$r = 0.92$$

APPENDIX L

Interpretation of the Reading Ease Score and Human Interest Score of the Flesch Formula

Reading Ease Score

RES	Descprition Style	Corrected Grade Level
90-100	Very Easy	5th grade
89-90	Easy	6th grade
70-80	Fairly Easy	1st-2nd yr (HS)
60-70	Standard	3rd-4th yr (HS)
50-60	Fairly Difficult	1st-2nd yr (Col)
30-50	Difficult	3rd-4th yr (Col)
0 -30	Very Difficult	College Graduate

Human Interest Scale

HIS	Description of Style
60-100	Dramatic
40-60	Highly Interesting
20-30	Interesting
10-20	Midly Interesting
0 -10	Dull

APPENDIX M

Computation of the RES Score

Pages	No of Words	No.of Sentences	No.of Syllables
1	100	7	168
2	100	10	185
3	100	11	210
4	100	10	160
5	100	8	195
6	100	9	176
7	100	8	189
8	100	6	144
9	100	7	170
10	100	8	164
11	100	8	178
12	100	6	155
13	100	4	142
14	100	8	163
15	100	5	145
	1500	115	2544

Ave. sentence lenght $\frac{1500}{115} = 13.04$

Ave. word lenght $\frac{2544}{15} = 169.5$

RES = (206.835 - [1.015 x 13.04 + 0.846 x 169.5]

= 206.835 - 13.24 + 143.40

= 206.835 - 156.637

RES = 50.20

APPENDIX N

Computation of the Human Interest Score

Pages	No. Of Personal Words	No. of Personal Sentence
1/1/2	1	2
1/1/6	1	4
1/1/10	2	2
1/2/1	3	4
1/2/4	1	2
1/2/5	2	5
1/2/8	3	6
1/2/11	1	2
1/2/15	3	4
1/3/3	2	3
1/3/8	4	3
1/3/10	2	3
1/4/3	1	4
1/4/6	2	4
1/4/8	1	5
Total	29	53

% Personal words
29/1500 = 1.93%

% Personal sentence
53/115 = 46%

HIS = 1.95/100 words
+46 x .314 = 21.4
Interesting

APPENDIX O –1

Computation of Means and t-value (Table 5)

$$\bar{x}_{ET} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{362}{25} = 14.48 \qquad \bar{x}_{CT} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{358}{25} = 14.32$$

$$s_{E1}^2 = \frac{n\sum x_{E1}^2 - (\sum x_{E1})^2}{n(n-1)} \qquad s_{C1}^2 = \frac{n\sum x_{C1}^2 - (\sum x_{C1})^2}{n(n-1)}$$

$$= \frac{25(5538) - (362)^2}{25(25-1)} \qquad = \frac{25(5418) - (358)^2}{25(25-1)}$$

$$= \frac{138450 - 131044}{25(24)} \qquad = \frac{135450 - 128164}{25(24)}$$

$$= \frac{7406}{600} \qquad = \frac{7286}{600}$$

$$= 12.34 \qquad = 12.14$$

$$t = \frac{\bar{x}_{E1} - \bar{x}_{C1} - \delta}{\sqrt{\frac{(n_{E1}-1)s_{E1}^2 + (n_{C1}-1)s_{C1}^2}{n_{E1} + n_{C1} - 2}\left[\frac{1}{n_{E1}} + \frac{1}{n_{C1}}\right]}}$$

$$t = \frac{14.48 - 14.32}{\sqrt{\frac{(25-1)12.34 + (25-1)12.14}{25 + 25 - 2}\left[\frac{1}{25} + \frac{1}{25}\right]}}$$

$$t = \frac{0.16}{\sqrt{\frac{27.168 + 291.36}{48}(0.04 + 0.04)}}$$

$t = 0.16$ $\qquad$ t_{05}, $df = 48$, is 2.00 significant, $p < .05$

APPENDIX O - 2

Computation of Means and t-value (Table 6)

$\Sigma D_e = 191$ $$\bar{D}_e = \frac{\Sigma D}{N} = \frac{191}{25} = 7.64$$

$\Sigma D_e^2 = 1845$ $N = 25$

$$S_{De} = \sqrt{\frac{N\Sigma D_e^2 - (\Sigma D_e)^2}{N(N-1)}} = \sqrt{\frac{25(1845) - (191)^2}{25(25-1)}}$$

$$= \sqrt{\frac{46125 - 36481}{600}} = \sqrt{\frac{9644}{600}}$$

$S_{De} = 4.01$

$S_{De}^2 = 16.0733$

$$t = \frac{\bar{D}_e}{S_{De}/\sqrt{N}} = \frac{7.64}{4.01/\sqrt{25}} = \frac{7.64}{4.01/5} = 9.526$$

t_{05}, $df = 24$, is 2.064 significant, $p < .05$

APPENDIX O - 3

Computation of Means and t-value (Table 7)

$\Sigma D_c = 161$ $\qquad$ $\bar{D}_c = \dfrac{\Sigma D}{N} = \dfrac{191}{25} = 6.44$

$\Sigma D_c^2 = 1473$ $\qquad$ $N = 25$

$$S_{Dc} = \sqrt{\frac{N\Sigma D_c^2 - (\Sigma D_c)^2}{N(N-1)}} = \sqrt{\frac{25(1473) - (161)^2}{25(25-1)}}$$

$$= \sqrt{\frac{36825 - 25421}{600}} = \sqrt{\frac{11404}{600}}$$

$S_{Dc} = 4.26$

$S_{Dc}^2 = 18.173$

$$t = \frac{\bar{D}_c}{S_{Dc} / \sqrt{N}} = \frac{6.44}{4.26 / \sqrt{25}} = \frac{6.44}{4.26/5} = 7.56$$

t_{05}, $df = 24$, is 2.064 significant, $p < .05$

APPENDIX O – 4

Computation of Means and t-value (Table 8)

$$\bar{x}_{E2} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{553}{25} = 22.12 \qquad \bar{x}_{C2} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{519}{25} = 20.76$$

$$s_{E2}^2 = \frac{n\sum x_{E2}^2 - (\sum x_{E2})^2}{n(n-1)} \qquad s_{C2}^2 = \frac{n\sum x_{C2}^2 - (\sum x_{C2})^2}{n(n-1)}$$

$$= \frac{25(12745) - (553)^2}{25(25-1)} \qquad = \frac{25(11275) - (519)^2}{25(25-1)}$$

$$= \frac{318625 - 305809}{25(24)} \qquad = \frac{281875 - 269361}{25(24)}$$

$$= \frac{12816}{600} \qquad = \frac{12514}{600}$$

$$= 21.36 \qquad = 20.86$$

$$t = \frac{\bar{x}_{E2} - \bar{x}_{C2} - \delta}{\sqrt{\frac{(n_{E2}-1)s_{E2}^2 + (n_{C2}-1)s_{C2}^2}{n_{E2}+n_{C2}-2}\left[\frac{1}{n_{E2}} + \frac{1}{n_{C2}}\right]}}$$

$$t = \frac{22.12 - 20.76}{\sqrt{\frac{(25-1)21.36 + (25-1)20.86}{25+25-2}\left[\frac{1}{25} + \frac{1}{25}\right]}}$$

$$t = \frac{1.36}{\sqrt{\frac{512.64 + 500.64}{48}(0.04 + 0.04)}}$$

$t = 1.05$ $\qquad$ t_{05}, $df = 48$, is 2.00 significant, $p < .05$

APPENDIX P

No. of Correct Responses and Index of Difficulty of Items and Topics

Sub Topics	No. of Correct Responses						Index of Difficulty %					
Integers	BSIT	DT	BSE	CE	TECH	BSIE	BSIT	DT	BSE	CE	TECH	BSIE
1.	17	13	16	14	9	10	15.3%					5.3%
2.	20	8	19	21	6	5		10%			6.3%	
3.	9	9	13	14	4	1			16%	16.3%		
Total	46	30	48	49	19	16				69.2		11.53%
Fraction												
4.	15	13	11	12	12	6	10.7%					6.3%
5.	8	10	9	2	15	11		10.3%			12.7%	
6.	9	8	10	7	11	2			10%	7%		
Total	32	31	30	21	38	19				57%		9.5%
Decimals												
7.	12	7	12	3	4	2	7.2%					3.2%
8.	11	8	11	6	6	4						
9.	3	3	3	7	4	2		5.8%			5%	
10.	3	3	3	5	0	4						
11.	7	8	7	13	11	4			7.2%	6.8%		
Total	36	29	36	34	25	16				35.2%		5.87%
Algebraic Exoression												
12.	7	4	7	5	3	1	4.4%					2.4%
13.	3	5	4	3	0	3						
14.	1	8	1	3	5	2		5.6%			3.6%	
15.	7	9	7	7	7	4						
16.	4	2	3	7	3	2			4.4%	5%		
Total	22	28	22	25	18	12				25.4%		4.23%
Exponents												
17.	3	11	4	9	5	2	11%					6%
18.	10	8	10	7	1	1						
19.	18	16	18	18	6	8		9.8%			4.8%	
20.	4	3	4	2	2	12						
21.	20	11	20	10	10	7			11.2%	9.2%		
Total	55	49	56	46	24	30				52%		8.7%

APPENDIX P (Cont'd.)

Exponents Notation										
22.	14	13	14	12	12	14	12.2			13.3
23.	6	15	16	16	14	18				
24.	11	16	15	11	16	14	13.7		7	
25.	14	11	14	11	13	10				
26.	13	12	13	12	13	13		14.5	13.5	
27.	15	15	15	19	0	11				
								74.2		
	73	82	87	81	42	80				12.37
Radicals										
28.	9	13	9	8	5	4	8.4		5.5	
29.	8	9	8	4	5	6				
30.	16	7	8	4	3	17				
31.	10	5	10	2	4	0	6.4		4.6	
32.	7	1	7	16	4	11				
33.	5	5	5	7	2	5				
34.	4	5	4	8	4	0		7.4	7.8	
35.	8	6	8	14	10	1				
	67	51	59	63	37	44		40.1		6.68
Solution of Linear Equation										
36.	11	9	11	14	5	10	7.5		5.7	
37.	5	6	5	8	3	0				
38.	6	3	6	5	10	21	5.5		5.8	
39.	12	13	12	4	10	6				
40.	8	9	8	4	5	11		8.8	7.9	
41.	3	10	3	8	2	3				
	45	50	53	43	35	34		43.3		7.22
Systems of Linear Equation										
42.	10	10	10	0	12	1				
43.	7	6	7	4	14	3	9.8		4.3	
44.	15	12	15	4	8	5				
45.	15	8	15	2	9	2				
46.	8	3	8	5	1	9	5.5		3.3	
47.	11	3	10	3	10	6				
48.	13	6	13	9	9	0				
49.	2	1	2	3	3	8		8.8	7.9	
50.	6	3	6	2	7	4				
51.	7	3	7	2	3	5				
52.	3	6	3	2	2	4				
	97	60	97	36	87	47		36.6		6.1

APPENDIX P (Cont'd.)

Quadratic Equations

53.	9	1	9	5	2	5	7.8					6.5
54.	4	15	4	6	2	0						
55.	8	10	8	4	4	2						
56.	9	8	9	10	3	0		5.5			6.6	
57.	8	4	8	8	2	6						
58.	10	5	11	8	8	1						
59.	9	2	9	6	11	11			7.8	6.3		
60.	5	6	5	7	4	2						
61.	9	4	9	10	13	3						
62.	8	6	8	6	2	11						
63.	11	2	11	6	6	2						
64.	3	2	2	4	1	13						
	62	44	62	51	53	52			40.5			6.75

Inequalities

65.	10	4	10	6	3	2	5.4					5.2
66.	6	11	6	10	3	3						
67.	3	12	3	1	4	1						
68.	2	10	2	7	3	10		6.4			6.1	
69.	6	2	6	9	12	10						
70.	9	0	9	8	6	1						
71.	2	4	2	3	4	5			5.5	4.9		
72.	3	12	3	10	6	2						
73.	7	6	7	6	14	12						
74.	8	4	8	6	1	1						
75.	11	14	12	2	14	13						
76.	5	2	5	2	4	11						
77.	7	3	7	5	2	2						
78.	4	2	4	2	11	3						
79.	9	6	9	0	6	3						
80.	5	5	5	2	5	2						
	87	103	88	79	98	83			33.56			5.59

APPENDIX Q

Key to Corrections Pretest/Posttest

1.	d
2.	b
3.	a
4.	b
5.	c
6.	b
7.	d
8.	b
9.	e
10.	e
11.	e
12.	e
13.	b
14.	c
15.	d
16.	b
17.	b
18.	a
19.	c
20.	b
21.	c
22.	a
23.	b
24.	e
25.	a
26.	c
27.	d
28.	e
29.	c
30.	a

CURRICULUM VITAE

CURRICULUM VITAE

NAME : ANESIA M. JAROMAY

ADDRESS : Canlapwas, Catbalogan, Samar

DATE OF BIRTH : June 18, 1954

PRESENT POSITION : Asst. Prof. 1

STATION : Samar State Polytechnic College
Catbalogan, Samar

RELEGION : Catholic

SEX : Female

CIVIL STATUS : Married

EDUCATIONAL BACKGROUND

Elementary : Almeria Elementary School
Almeria, Leyte

Secondary : Almeria High School
Almeria, Leyte

College : Colegio de San Jose Recoletos
Cebu City

Bachelor of Science in Chemical Engineering

Graduate : Samar State Polytechnic College
Catbalogan, Samar

Master of Arts in Teaching - Mathematics

CIVIL SERVICE ELIGIBILITIES

Chemical Engineers Board Examination, April, 1978

POSITIONS HELD

Instructor : Pasica Training Services
San Jose, Dinagat
Surigao del Norte

SST : SSPC, Catbalogan, Samar
1981-1982

Substitute vice Mrs. C. Oliva (On Study Leave)
College - BSIE & BSIT

SST : SSPC, Catbalogan, Samar
High School – 3rd Year & 2nd Year
1982 -1984

SST : SSPC, Catbalogan, Samar
High School – 2nd Year
1984 -1986

Instructor I : SSPC, Catbalogan, Samar
Incharge – Science Laboratory Stockroom
1986 - 1987

Instructor III : SSPC, Catbalogan, Samar
Incharge – Science Laboratory Stockroom
1987 - 1989

Instructor III : SSPC, Catbalogan, Samar
Incharge – Eng'g Laboratory Stockroom
1988 - 1990

Instructor III : SSPC, Catbalogan, Samar
College Instructor – BSIT
1990 – 1991

SEMINARS AND WORKSHOPS ATTENDED

One Day Conference on Administrative and Professional Matter, June 26, 1981, SSAT Conference Hall, SSAT, Catbalogan, Samar.

Regional FFP/FAHP/FFPCC Work Conference, November 23 – 25, 1981, SSAT Conference Hall, SSAT, Catbalogan, Samar.

Seminars on Techniques of Developing Reading Skills Among Students, SSAT, Catbalogan, Samar, October 12 –19, 1981.

Third Regional Youth Science Camp, SSAT, Catbalogan, Samar, August 1-7, 1982.

IPSED, MECS Regional Office No. VIII, UPSEC & NSTA Seminar Workshop on Science I – IV and Mathematics I – IV, SNS, Catbalogan, Samar, December 8 – 10, 1982.

Echo Youth Science Camp, SSAT, Catbalogan, Samar, November 11, 1984. (Camp Facilitator)

Seminar Workshop on Values Development, SSPC, Catbalogan, Samar, June 22, 1986.

Combined Basic Primary Course on Technological Creativity and Invention Development, SSPC, Catbalogan, Samar, March 15 – 16, 1986.

1986 Draft Constitution Trainor's Workshop, SSPC, Catbalogan, Samar, December 5, 1986.

Seminar Workshop on Research & Development, SSPC, Catbalogan, Samar, February 27, 1987.

Physics Olympics, Divine Word University, Tacloban City, May 28 – 29, 1988.

4th Philippine Chemistry Congress, Tacloban City, May 28 – 29, 1988.

Chemistry and Society, Peoples Center, Tacloban City, May 28 – 29, 1988.

Seminar Workshop on Values Education, SSPC, Catbalogan, Samar, July 15 – 16, 1988.

Regional Seminar Workshop for General Science and Chemistry Teachers, SSPC, Catbalogan, Samar, July 12 – 13, 1988.

10th National Physics Convention, DWU, Tacloban City, April 6 – 9, 1988.

Regional Science Seminar Workshop, SSPC, Catbalogan, Samar, July 11 – 13, 1988.

Science Quiz, SSPC, Catbalogan, Samar, October 30, 1988. (Facilitator)

Seminar Workshop on "Bayan Muna Bago ang Sarili", SSPC, Catbalogan, Samar, July 5 – 6, 1990.

Seminar Workshop on "The Role of Citizens and Government Agencies in a Clean and Honest Election", SSPC, Catbalogan, Samar, March 5, 1992.

Low Cost Laundry Soap Making, SSPC, Catbalogan, Samar, March 11, 1992.

Regional Seminar Workshop for Cooperating Teachers, SSPC, Catbalogan, Samar, November 29, 1993.

CO – CURRICULAR ACTIVITIES

Secretary	:	Pasica Training Services Personnel Organization San Jose, Dinagat Surigao del Norte
Sub- Camp II Adviser	:	Science Club SSPC, Catbalogan, Samar
Camp Facilitator	:	Science Camp SSPC, Catbalogan, Samar

Adviser : FFPCC - First Year Level
SSPC, Catbalogan, Samar

Auditor : Number Theory & Linear Algebra Classes
SY 1986 - 1987

AWARDS & DISTINCTION

Certificate of Commendation: Member Program & Invitation Committee
SSPC, Catbalogan, Samar

Certificate of Commendation: Judge, Talents Unlimited Contest
FFP/FAHP/FFPCC Week Celebration
SSAT, Catbalogan, Samar
October 11-13, 1982

Mahogany Service Award : SSPC, Catbalogan, Samar

Certificate of Training : Basic Computer Operation, Wordstar,
Lutos 123, and DBase III
November 21 – February 6, 1992.

LIST OF TABLES

LIST OF FIGURE